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INTRODUCTION TO STRUCTURAL MECHANICS

FOR BUILDING AND ARCHITECTURAL STUDENTS

By

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PREFACE TO THIRD EDITION

THE alterations in the calculations for timber struts and posts (page 298) are inserted by kind permission of the L.C.C.

T. J. R.
L. E. K.

PREFACE TO SECOND EDITION

As mentioned in the preface to the original edition of this book in 1944, the aim was to present, in simple language, the basic principles of building mechanics.

However, the application of these principles to modern structural regulations has not been forgotten.

British Standard 449 has laid down certain new working stresses. For example, an important change is the raising of the permissible working stress in bending from 8 tons/in.^{2} to 10 tons/in.^{2}, in the case of simple beams. Where it has appeared to be desirable, the new working stresses have been used in this edition, keeping in mind the general scope of the book. It must be remembered that all lists of stresses, etc., issued in 'Codes' and 'Standards' are subject to periodic revision.

The book remains a text-book of fundamental principles of structural mechanics. The authors wish to thank the British Constructional Steelwork Association and the British Steel Makers for permission to include the tables on pages 262–265 inclusive.

T. J. R.
L. E. K.

PREFACE TO FIRST EDITION

THE subject of 'Structural Mechanics' in the more advanced stages is adequately covered by the many excellent books dealing with the 'theory of structures' and the 'strength of materials.' The authors trust that the present volume, introductory and fundamental in character, will be of assistance to building and architectural students in the early stages of their professional training.

The aim of the book is to present, in simple language and in a logical sequence, the basic principles of building mechanics. The temptation to enlarge unduly on certain topics has been resisted. Too much detail in the early stages of a student's reading is apt to make him lose sight of the natural development of the subject, stage by stage.

An endeavour has been made to reduce the amount of mere arithmetical working in problems by selection of suitable data. It is difficult for an elementary student to keep unobscured the underlying principles involved in an exercise, when he has to deal with a mass of awkward figures.

The text is thoroughly interspersed with numerical examples and diagrams, the only effective method of sustaining interest and enthusiasm.

In addition to the exercises at chapter ends, a chapter is devoted to revision examples. To these examples abridged solutions are provided.

The last chapter contains test papers. Completely worked solutions to the numerical portions of these papers are supplied. Page references are given for points of theory raised in the papers. Readers are urged to attempt these question papers in the spirit of examination tests. Reference to the solutions should not be made until the paper attempted has been fully worked.

Students preparing for the Intermediate examination of the Royal Institute of British Architects, the Graduateship examination of the Institution of Structural Engineers, the examinations of the Chartered Surveyors' Institution, and similar examinations, should find the book helpful.

Teachers proposing to use the book as a class book in preliminary courses approved for National Certificates and Diplomas in Building will have no difficulty in sealing-up the answers and worked solutions, if this were considered to be desirable.

The book has not been written to comply with any one set of

building regulations. It has been necessary, however, to refer to typical regulations occasionally in order to demonstrate the relationship between theory and practice. Having grasped the basic principles of structural mechanics the reader will be in a position to apply his knowledge to the interpretation of any new regulations which may be issued from time to time by the British Standards Institution, etc.

The authors wish to acknowledge freely the many sources of the theoretical principles upon which the book is based. Examining and assessing experience has been an aid in the attempt to anticipate points of difficulty for the beginner. Practical experience has, likewise, assisted in placing necessary emphasis. Thanks are due to a number of firms for permission to publish data, photographs, etc. The firms include Messrs. Redpath, Brown & Co., Ltd., Messrs. A. Macklow-Smith & Co., Ltd. (for the testing apparatus in Chapter IX), and Messrs. Cussons Ltd. (Manchester), to whom the authors are indebted for the permission to include the illustrations of experimental apparatus given in Chapters I–XII. Fig. 238 is included by permission of Messrs. T. C. Howden & Co.

The photographs in Appendix II are published by permission of Messrs. Dawnays Ltd., and Messrs. The Quasi-Arc Company, Ltd.

Certain section tables are inserted by kind permission of the British Steelwork Association. The Institution of Structural Engineers kindly permitted the authors to make extracts from a report, particulars of which are given in Chapter XV.

A list of British Standard Specifications dealing with building construction is given in Appendix I. The list is included by the courtesy of the British Standards Institution. The authors wish to express their indebtedness to the London County Council for permission to quote from building regulations and to publish photographs of experimental apparatus.

Finally, it is desired to record appreciation of the interest shown by Principal Drury, M.Sc., in the production of the book, and to thank Mr. E. G. Warland, M.I.Struct.E., for preparing the cover design.

T. J. R.
L. E. K.

CONTENTS

ILLUSTRATIONS

CHAPTER I

COMPOSITION OF FORCES. RESULTANTS

Introduction.—' Structural mechanics ' is mainly concerned with forces, how they combine together, how they keep a body at rest and, in general, with the effect they have on the stability of the parts of the structure to which they are applied. In this subject we will have to consider also the effect a force has on the size and shape of the actual material upon which it acts.

The object of studying the subject of structural mechanics is to learn how to build structures with a view to both *economy* and *strength*.

Force

It is usual to define *force* as that which tends to alter the state of rest of a body or its uniform motion in a straight line. As building students we are chiefly concerned with bodies at rest. It must be realised that if a single force act on a body, it will produce motion in that body. Also, if a number of forces act on a body in such a manner as to be equivalent to a ' *net resultant force*,' the body cannot possibly remain at rest. From our point of view, the important fact to remember is that if any unit in a structure, e.g. a girder, is to remain in *equilibrium* (i.e. at rest), there must be no resultant force acting on it. It is our duty to provide for such a member a set of forces that shall satisfy the ' *laws of equilibrium*.'

Types of Forces met with in Structural Calculations

(*a*) In all practical problems there is one force which should always be carefully noted. This force is the ' *self-weight* ' of the member involved. For example, in beam problems the weight of the beam itself should always be considered in estimating the total load the beam has to carry. The load due to the weight of a member is usually termed a ' *dead load*.'

(*b*) Structural members have to support external loads which are known as ' *superimposed loads*.' In floor-design calculations, superimposed loads are frequently described as ' *live loads*.' The

following represents a typical load table for a reinforced concrete floor:

Dead load =	72 lb.	per sq.	foot.
Floor finish =	10 ,,	,,	,,
Live load =	224 ,,	,,	,,
Total =	306 lb.	per sq.	foot.

In this case we have a floor slab 6" thick with an additional top surface designed to resist wear. The term 'live' in this example merely indicates that the load of '224 lb. per sq. foot' *may* be applied all over the floor.

Strictly speaking, a '*live load*' is of a dynamic character, such as the load a pile receives when it is hit by the falling 'monkey.' A 'live load' of this type may be considerably more damaging to a structure than a 'dead load' of the same magnitude.

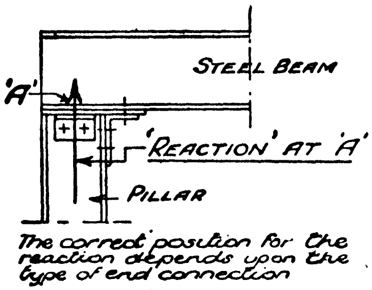


FIG. 1.—BEAM SUPPORT REACTION.

(*c*) When one structural member rests upon another, the force exerted by the supporting member is termed a '*reaction*.' Fig. 1 shows a steel beam simply supported at its end 'A.' The force exerted in this case by the pillar on the beam at 'A,' indicated in the diagram by an arrow, would be termed the '*reaction at A*.'

(*d*) In framed structures, some of the members will be pulling at their end connections and others will be exerting a thrust (Fig. 2). Those members which pull are termed '*ties*,' and those which push are known as '*struts*.' The arrow heads in Fig. 2 indicate the nature of the forces which the members exert at the joint. The reader

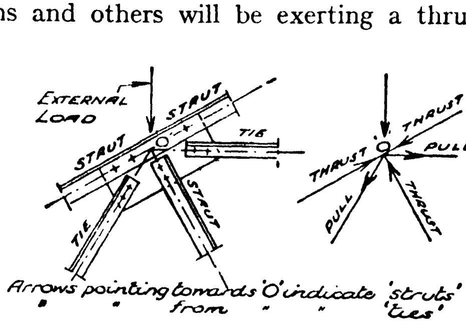


FIG. 2.—STRUTS AND TIES.

should carefully note the relationship between the way in which the respective arrows are pointing and the type of the member concerned.

(*e*) Walls sometimes have to be built to retain liquids, or granular materials like earth. Such structures are known as '*retaining walls.*' The forces involved in retaining-wall calculations are dealt with in Chapter XV.

(*f*) When a member is subjected to load the fibres of the material transmit the load from section to section throughout the

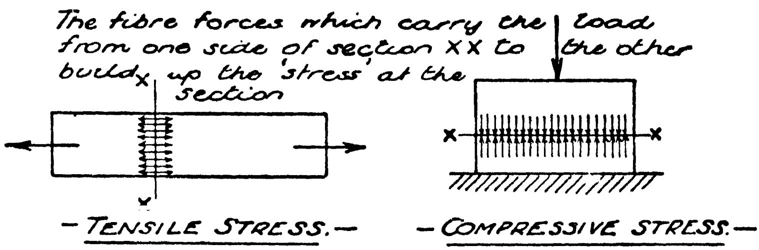


FIG. 3.—THE NATURE OF STRESS.

member. Such a system of internal force transmission is termed a '*stress.*' Chapter IX is concerned with the subject of 'stress.'

Units for Expressing Force Values

Two systems of units are employed.

(i) *Gravitational or Engineers' system.*

(ii) *Absolute system.*

The latter system is based upon Newton's laws of motion, and in this system a force is measured by the acceleration it produces in a given mass. This method of giving force values is not commonly used in building calculations and will not be further referred to.

Gravitational System.—Every body is attracted towards the earth's centre by a force which depends upon (i) the amount of matter in the body, i.e. its '*mass,*' and (ii) the distance the body is from the centre.

We may ignore the slight difference in the '*force of gravity*' which a given mass would experience at different positions on the earth's surface and which is occasioned by the fact of the earth not being a perfect sphere. For all practical purposes we may

regard the mass of a body as the only factor which influences the value of the earth's pull on it.

In London there is stored a piece of platinum the mass of which is defined to be '*one pound.*' If we were to measure the pull of gravity on this standard unit of mass we would get the force value known as '*one pound weight.*' From this primary unit the system of force units commonly used in structural calculations is built up.

It is usual to omit the word '*weight*' in expressing force values. When we say, for example, that the force in a tie-bar in a steel frame is '30 *cwts.*', we mean it is '30 × 112 times' as great as the standard unit of force, viz. '*one pound weight.*'

Resultant of a Force System.—The 'resultant' of a system of forces (i.e. of a specified number of given forces) is the single force which we could replace for the given system without altering the net effect the system has on the state of rest ('*equilibrium*') of the body upon which it acts (see Fig. 4 (*a*)).

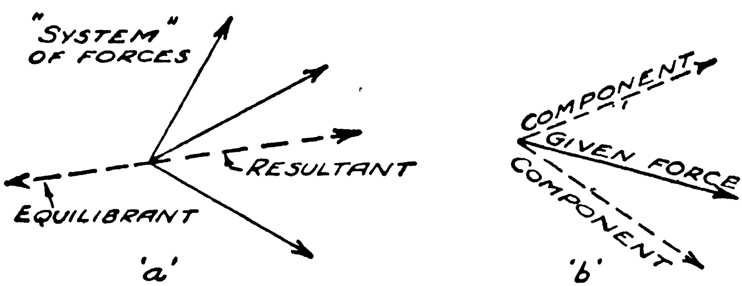


FIG 4.—RESULTANT, EQUILIBRANT AND COMPONENT.

It is often important to be able to substitute one hypothetical force for an existing group of forces. In all problems of the '*equilibrium*' type, this is a permissible substitution.

Equilibrant.—The '*equilibrant*' of a system of forces is the force which would have to be introduced into the system in order to bring it into the state of equilibrium. It is clear that the 'equilibrant' of a system must balance the *resultant* of that system. The relationship between these two important forces is, therefore, that they are '*equal in magnitude*' and act in '*exactly opposite directions.*' Both resultant and equilibrant will be of zero value for a system in equilibrium.

Components.—The components of a force are the separate forces which acting all together will have the given force as their resultant.

The operation of resolving a force into its components, termed '*resolution,*' is frequently employed in structural calculations. Fig. 4 (*b*) illustrates a force resolved into two components.

Determination of Resultants

It is intended in this book to consider only the type of force system in which all the forces act in the same plane. Such a system is known as a '*co-planar system of forces.*' It is the type which commonly occurs in building calculations.

Force systems may be divided into two classes:

(i) '*Concurrent systems,*' in which all the '*lines of action*' of the various forces pass through one common point.

(ii) '*Non-concurrent systems,*' in which the lines of action have no common point of concurrence.

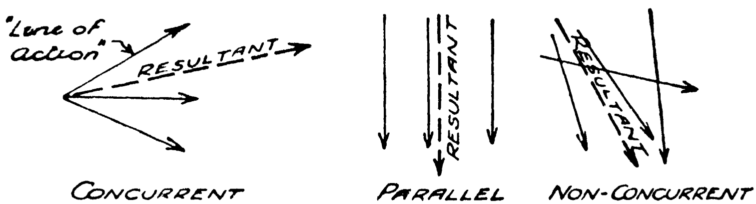


FIG. 5.—FORCE SYSTEMS WITH TYPICAL RESULTANTS.

Fig. 5 illustrates these two main types and also the special important case of the '*parallel system.*'

Problems in structural work are solved either by methods of calculation or by '*graphical methods.*' Graphical methods are widely used and will be considered first.

Graphical Representation of a Force

A force has '*magnitude,*' '*direction,*' and '*position.*' For example, we might have a force of 1000 lb. (*magnitude*) acting vertically downwards (*direction*) at the apex of a roof truss (*position*). It is possible to represent these three properties by a straight line drawn to a convenient force scale (see Fig. 6).

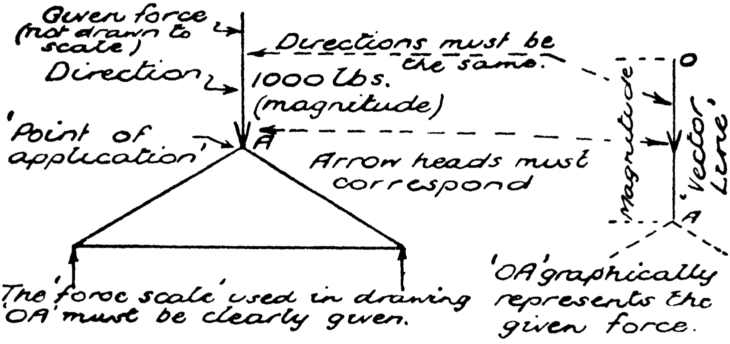


FIG. 6.—GRAPHICAL REPRESENTATION OF A FORCE

EXAMPLE.—*Represent graphically a force of 400 lb. acting, at a given point, horizontally towards the right.*

In Fig. 7, the original scale chosen was $\frac{3}{4}'' = 100$ lb. The length of the '*vector line*' representing the force was therefore

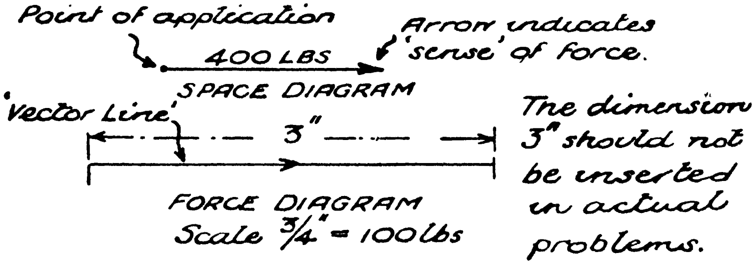


FIG. 7.—A FORCE REPRESENTED BY A STRAIGHT LINE.

'3 ins.' The reader should not calculate the length of line required but use the side of his scale rule marked off in three-quarter inch graduations and plot '4' of these graduations in this case. Lines drawn to scale to represent force values will be termed '*vector lines.*' A '*vector quantity*' is one (like a force) which has '*direction*' as well as '*magnitude.*'

It is absolutely essential in graphical work that **a vector line shall be drawn parallel to the line of action of the force it represents.** In drawing a vector line, as far as is possible, the pencil point should travel in the direction of the '*sense*' of the given force, as indicated by the arrow head in the space diagram.

Always indicate clearly, near the diagram concerned, the scale used in its construction.

Resultant of a Co-linear Force System

EXAMPLE.—*Fig. 8 shows three horizontal forces pulling on a ring. Find the resultant pull on the ring* (a) *by calculation,* (b) *by 'graphics.'*

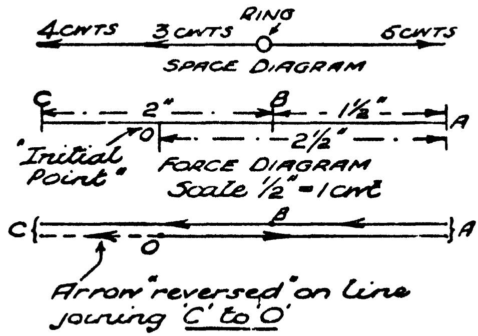


FIG. 8.—GRAPHICAL SOLUTION

In graphical work a *space diagram* should always be drawn, showing the correct directions of the forces acting, with indications of the force values where known. The forces are not represented to scale in a space diagram, but the diagram may require a linear scale to set out correctly the various force directions (see page 36).

(*a*) Total force to right = 5 cwts.
Total force to left = (3 + 4) cwts.
Net force to left = (3 + 4 − 5) cwts. = 2 cwts.
∴ Resultant = 2 cwts., acting horizontally towards the left.

(*b*) From a convenient initial point 'O' draw a vector line 'OA,' horizontally towards the right, to represent the '5 cwts.' force to a suitable scale. To the same scale draw 'AB' (to left) to represent '3 cwts.' and 'BC' (to left) to represent '4 cwts.'

Join the final point of the diagram (C) to the initial point (O) and reverse the arrow on this line.

'OC' will represent the resultant of the three given forces. The reader will note that this agrees with the calculated result.

This example has been explained in detail because the rule given in heavy type above will be shown later to be applicable to any system of forces acting at a point.

Resultant of Two Intersecting Forces

The solution of this case is effected by means of a theorem known as the '*parallelogram of forces.*' The experimental verification of the theorem may be conducted by apparatus such as that illustrated in Fig. 9.

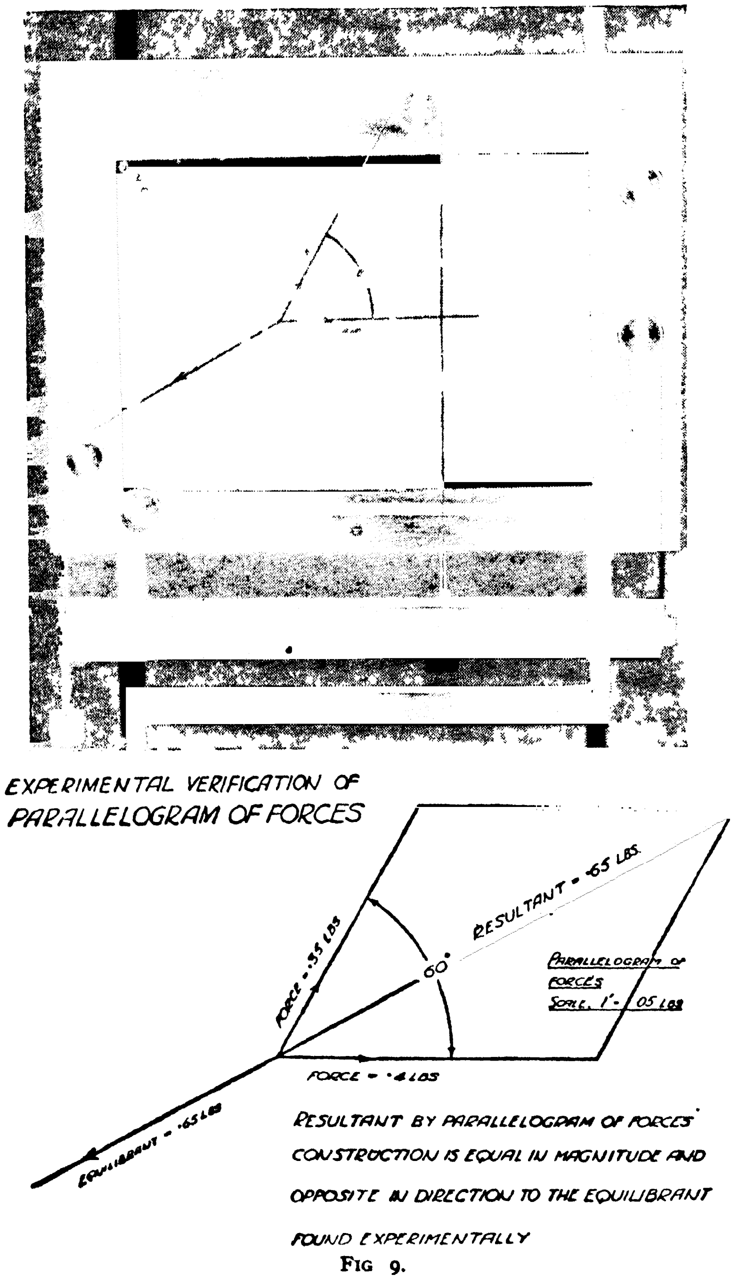

EXPERIMENTAL VERIFICATION OF
PARALLELOGRAM OF FORCES
RESULTANT = ·65 LBS
FORCE = ·35 LBS
60°
PARALLELOGRAM OF
FORCES
SCALE. 1"-·05 LBS
FORCE = ·4 LBS
EQUILIBRANT = ·65 LBS
RESULTANT BY PARALLELOGRAM OF FORCES
CONSTRUCTION IS EQUAL IN MAGNITUDE AND
OPPOSITE IN DIRECTION TO THE EQUILIBRANT
FOUND EXPERIMENTALLY

FIG 9.

Rule.—To find the resultant of two forces 'P' and 'Q' (Fig. 10) proceed as follows. From a convenient point 'O' draw 'OA' (to suitable scale) to represent force 'P' and also draw 'OB' (to same scale) to represent force 'Q.' Complete the parallelogram 'OACB' and draw-in the diagonal 'OC.' 'OC' wili then represent (to the chosen force scale) the resultant of the forces 'P' and 'Q.'

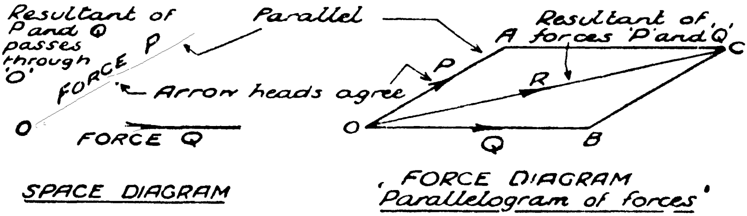


FIG. 10.—RESULTANT OF TWO INTERSECTING FORCES.

EXAMPLE.—*A peg, fixed in the ground, has two ropes attached to it, both ropes being parallel to the ground. The angle between the ropes is* 60°. *In one rope there is a tension* (*i.e. pull*) *of* 70 *lb. and in the other a tension of* 80 *lb. Find the resultant pull on the peg.*

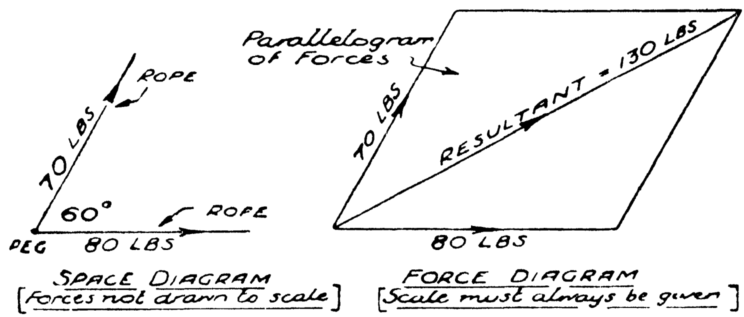


FIG. 11.—PARALLELOGRAM OF FORCES

By means of the parallelogram of forces shown in Fig. 11, the resultant is found to be a force of 130 lb., acting in the direction indicated.

In order to remove all pull off the peg a force of 130 lb. acting *in the opposite direction* to that of the resultant would have to be applied. This force is, of course, the '*equilibrant*' of the system.

Special Cases.—Fig. 12 shows a '*thrust*' and a '*pull*' acting at a point. To use the '*parallelogram of forces*' we must reduce *the system either to two* 'pulls' or to two 'thrusts.' Such cases may be solved more conveniently by the method given in next paragraph.

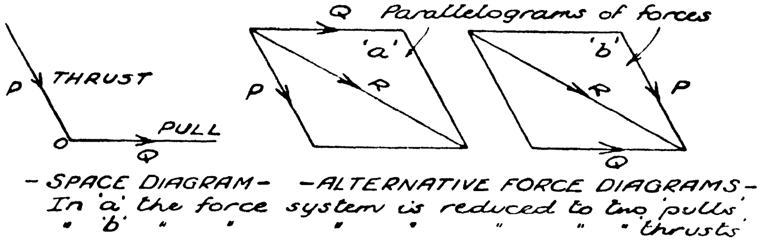


FIG. 12.—REDUCTION OF FORCE SYSTEM.

Second Method for Two Intersecting Forces

The vector line 'OC,' which represents the resultant in Fig. 10, may be obtained as follows: draw 'OA' (Fig. 13) to represent

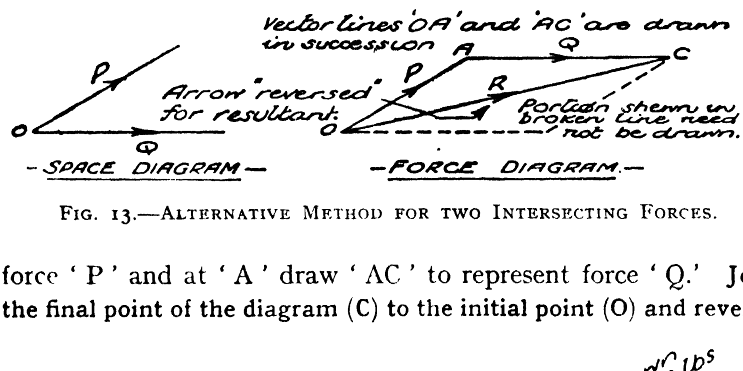


FIG. 13.—ALTERNATIVE METHOD FOR TWO INTERSECTING FORCES.

force 'P' and at 'A' draw 'AC' to represent force 'Q.' Join **the final point of the diagram (C) to the initial point (O) and reverse**

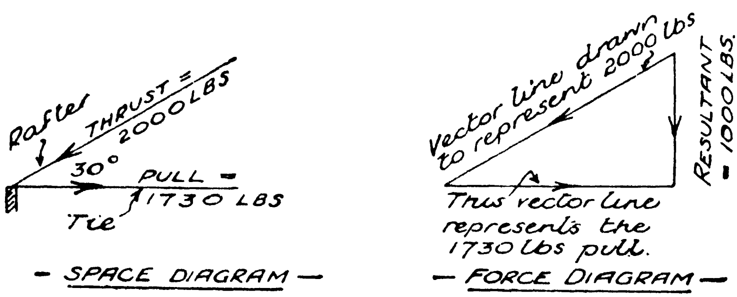


FIG. 14.—COMPOUNDING A THRUST AND A PULL.

the arrow on this line. This rule agrees with that given for co-linear forces on page 7.

EXAMPLE.—*In Fig.* 14 *the rafter exerts a thrust of* 2000 *lb. and the horizontal tie pulls with* 1730 *lb. force. Find the resultant force the truss transmits to its end support.*

The force diagram in Fig. 14 indicates that the support will have to carry a vertical load of 1000 lb.

Note.—Fig. 15 shows a body pulled by two strings. The strings do not actually intersect. In such cases the lines of action must be produced until they do intersect. The parallelogram of forces (or alternative method) may then be applied in the usual manner. The resultant will pass through the intersection point of the two lines of action. In all problems of this type, concerning equilibrium, a force may be assumed to be acting at any point in its own line of action.

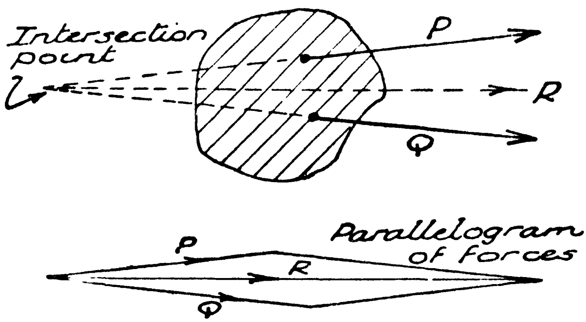


FIG 15.—PRODUCTION OF LINES OF ACTION.

Resultant of any Number of Concurrent Forces

Fig. 16 illustrates how the previous method of successive vector line construction can be applied to any number of forces. 'OB' represents R_{PQ}, the resultant of forces 'P' and 'Q.' This resultant force is then combined with force 'S,' giving 'OC' as the vector line representing the resultant of forces 'P,' 'Q' and 'S.' This procedure is continued until all the forces are taken. 'OD' represents the final resultant in the example taken. In practical examples there will be no need to draw in the intermediate resultant vector lines 'OB,' 'OC,' etc. It will be necessary simply to construct the polygonal outline 'OABCD.'

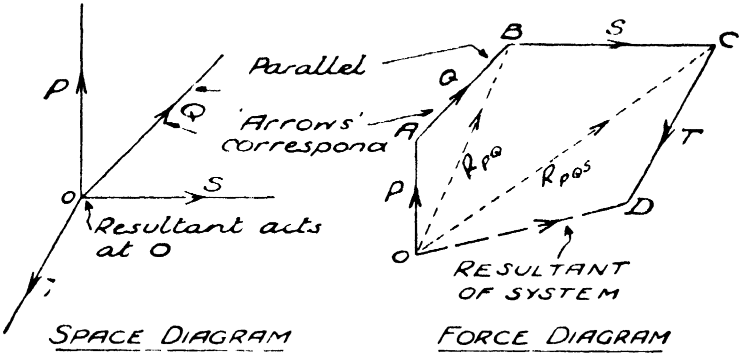


FIG. 16.—RESULTANT OF A CONCURRENT SYSTEM.

The rule is therefore the same as for the previous force systems considered, viz. : **draw vector lines in succession to represent the forces, join the final point of diagram to the initial point and reverse the arrow on this vector line.**

EXAMPLE.—*Find the resultant of the concurrent force system given in Fig. 17.*

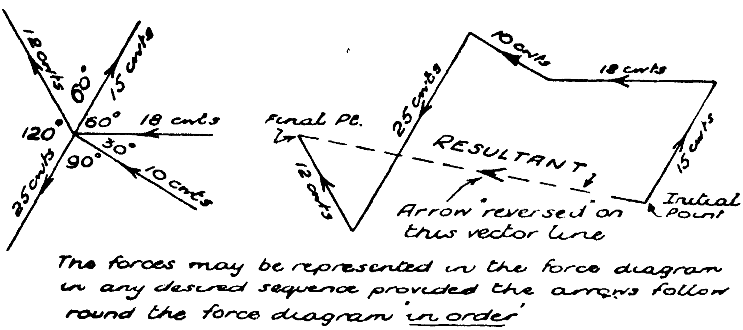


FIG 17—CONCURRENT SYSTEM

The resultant is a force of 38·26 cwts. acting in the direction indicated in the force diagram. Its ' point of application ' is the point of concurrence of the forces forming the system.

Recapitulation.—The reader should note carefully the following important points :

(i) **It is essential to construct the vector lines in a force diagram parallel to the respective forces they represent in the space diagram.**

(ii) **The arrow head on a vector line must agree ' in sense ' with**

that indicated in the space diagram for the particular force represented, i.e. the two arrows must point in the same direction.

(iii) The arrow heads must follow round the force diagram ' in order ' (i.e. all point ahead as you proceed round the diagram), except on the vector line which represents the resultant of the system. In this case it is directly ' reversed ' to all the others.

(iv) The forces given in the space diagram may be represented in any sequence when constructing the force diagram. The vector lines of the force diagram may cross one another.

Calculation Methods

As an alternative to graphical methods, methods involving the employment of simple trigonometrical functions and formulæ may be used for the solution of problems requiring the compounding of forces.

Readers unacquainted with trigonometry should omit the following paragraphs and continue with the study of the graphical methods given in the next and subsequent chapters.

If we can express by a formula the length of the diagonal of a parallelogram, or the closing line of a polygonal diagram, there will be no need to draw such diagrams to scale.

Two Intersecting Forces

In Fig. 18, if ' θ ' be the angle ' AOB,' the length ' OC ' of the parallelogram is given by the formula :

$$OC^2 = OA^2 + OB^2 + 2.OA.OB. \cos \theta.$$

Expressed in terms of force values this formula becomes :

$$R^2 = P^2 + Q^2 + 2.\ P\ Q.\ \cos \theta.$$

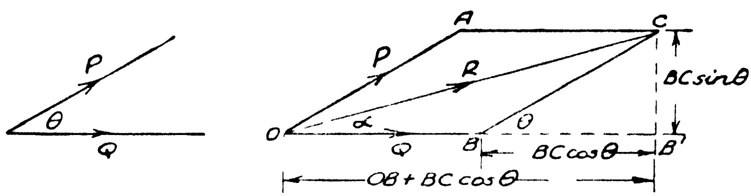


FIG. 18.—RESULTANT BY CALCULATION METHOD.

Applying this method to the example given in Fig. 11 we have :

$$\begin{aligned} R^2 &= 70^2 + 80^2 + 2 \times 70 \times 80 \times \cos 60^\circ \\ &= 4900 + 6400 + 2 \times 70 \times 80 \times \cdot 5 \\ &= 16900 \\ R &= 130 \text{ lb.} \end{aligned}$$

If forces ' P ' and ' Q ' act at right angles to one another we get the simple formula : $R^2 = P^2 + Q^2$.

Referring to Fig. 18, let ' α ' be the angle the diagonal ' OC ' makes with the side ' OB.'

$$\tan \alpha = \frac{CB'}{OB'} = \frac{BC \sin \theta}{OB + BC \cos \theta}.$$

Expressing in force values :

$$\tan \alpha = \frac{P \sin \theta}{Q + P \cos \theta}.$$

Applying this formula to the case given in Fig. 11 :

$$\tan \alpha = \frac{70 \sin 60^\circ}{80 + 70 \cos 60^\circ} = \frac{70 \times \cdot 866}{80 + 70 \times \cdot 5} = \frac{60 \cdot 62}{115} = \cdot 5271.$$

$\therefore \alpha = 27^\circ\ 48'$.

The resultant makes an angle of $27^\circ\ 48'$ with the 80-lb. force.

When Chapter II, dealing with the '*resolution of forces,*' has been studied, the formula given above for ' tan α ' will be more readily appreciated. We will then also be in a position to consider the derivation of the resultant of a system of several concurrent forces by the calculation method.

The examples given in Exercises 1 are intended to be solved by graphical methods. Calculation methods may be employed, if desired, as a check on the graphical work. The answers given at the end of the book have been *calculated.* Results obtained by graphical methods are sufficiently accurate for all practical purposes, provided the various diagrams are carefully drawn.

Exercises 1

(1) Fig. 19 shows a bolt to which are attached two chains. Find the magnitude and direction of the resultant pull on the bolt.

(2) A vertical post has fixed to it, at the top, two horizontal wires in which the pulls are respectively 80 lb. and 60 lb. The angle between the wires is 90°. Find the magnitude of the result-

ant pull on the post. Obtain the angle between the line of action of the resultant force and that of the 60-lb. force.

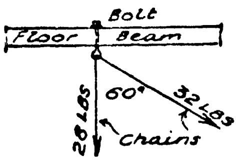


FIG. 19.

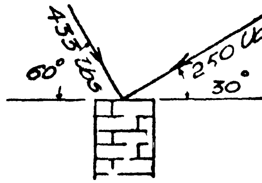


FIG. 20.

(3) A wall supports two thrusts in the manner indicated in Fig. 20. Show that the wall will have no tendency to move horizontally. Find the vertical load the wall has to support.

(4) A rope passes over a pulley (which may be assumed to have frictionless bearings) placed at the end of the jib of a crane (Fig. 21). Find the resultant force on the jib end due to the rope.

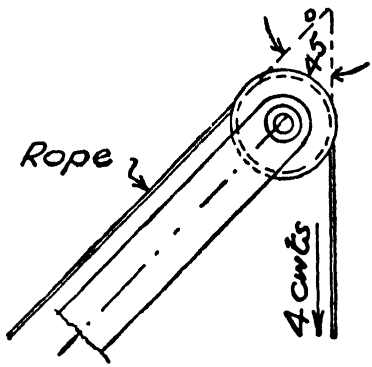


FIG. 21.

(*Note.*—The tension (i.e. the pull) in the rope may be taken to be constant so that at the end of the jib there are two forces, each of value 4 cwts., acting in the respective rope directions. Produce these rope directions to intersect.)

(5) Find the resultant force in each of the cases given in Fig. 22.

(6) In Fig. 23 is shown a vertical dead load of 1200 lb. and a

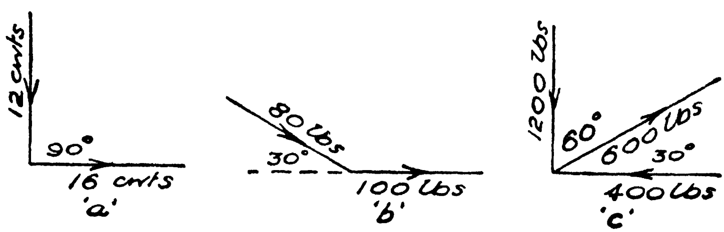


FIG. 22.

wind load of 1000 lb., acting at a rafter joint in a roof truss. Find the resultant load at the joint.

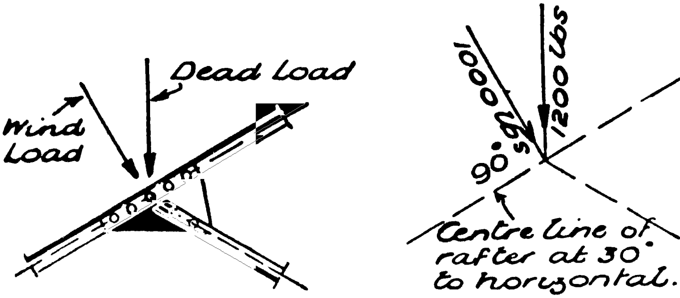


FIG 23.

(7) A particle is acted upon by three forces as follows: (i) 10 lb. vertically downwards, (ii) 15 lb. horizontally towards the right, (iii) 20 lb. acting towards the right and making an angle of 30° with, and above, the horizontal. Prove (i) that the particle has no tendency to move vertically, (ii) that it will begin to move horizontally towards the right as if a single force of 32·32 lb. were acting on it.

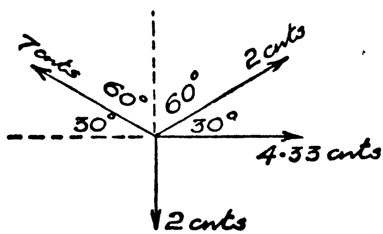


FIG. 24.

(8) Find the equilibrant of the concurrent force system given in Fig. 24.

(9) At the apex joint of a truss (Fig. 25) three forces are acting: (i) a dead load of 1600 lb., (ii) a positive wind load of 900 lb., and (iii) a negative (or suction) wind load of 600 lb. Find the magnitude of the resultant load at the joint. (Wind loads act at right angles to the roof slope.)

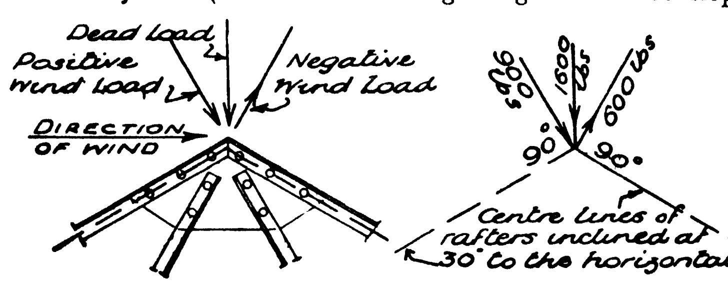


FIG. 25.

(10) Fig. 26 illustrates a riveted connection in which the rivets, owing to the nature of the applied reaction load, are subjected to two simultaneous loads. Taking rivet 'A,' which carries loads of 1 ton and 1·4 tons in the respective directions shown, determine the resultant load it must be capable of supporting.

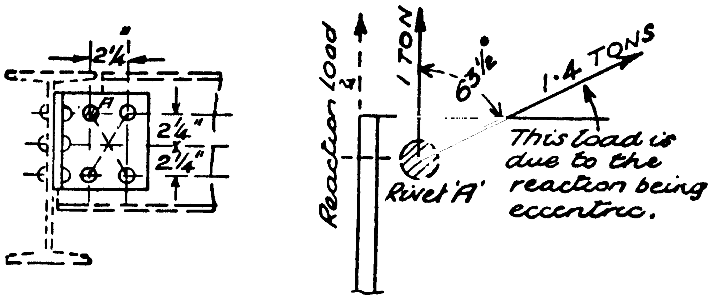


FIG. 26.—ECCENTRIC LOAD ON RIVET GROUP.

(11) The diagram given in Fig. 27 represents the section of a retaining wall. The earth thrust and the self-weight of the wall are computed on the basis of one-foot length of wall. Obtain the resultant force on the wall per foot of length. Determine the distance, from the vertical back of the wall, at which the resultant cuts the wall base.

(Produce the line of action of the earth thrust to cut the vertical line of action of the weight of the wall (see Fig. 15).)

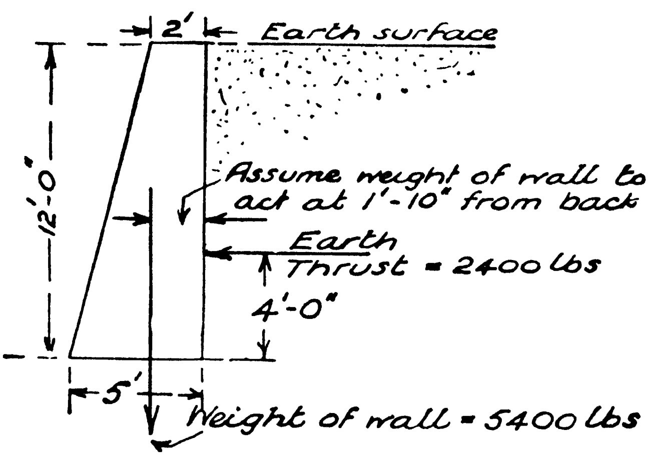


FIG. 27.—RETAINING WALL.

CHAPTER II

RESOLUTION OF FORCES. RECTANGULAR COMPONENTS

In order to solve structural problems, not only must we be able to *compound* several forces into one equivalent resultant force, but we should be familiar with the methods whereby a single force may be *resolved* into two or more components.

A given force system has only one resultant force, but a single force may be resolved into component forces in an indefinite number of ways. It is necessary, therefore, to specify some particulars of the required components.

The usual problem is to have to resolve a given force into *two* components whose directions are specified. The most important case of all is when these two components are themselves at *right angles* to one another.

EXAMPLE.—*A force of* 6 *cwts. acts, at a certain point, vertically downwards. Resolve this force into two components, one acting at* 30° *to the left and the other at* 45° *to the right of the given force direction* (*Fig.* 28).

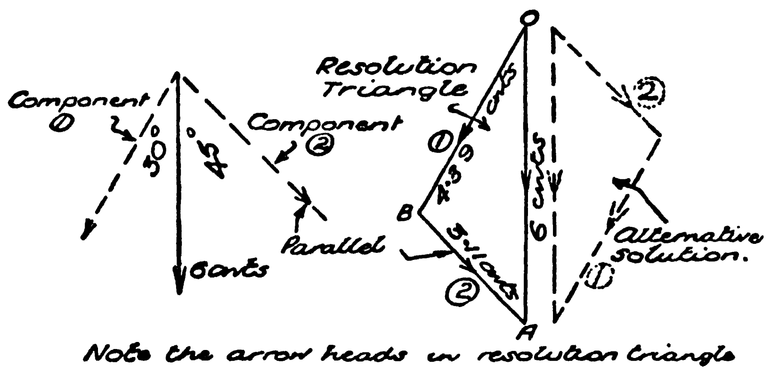


FIG. 28.—COMPONENTS OF A FORCE.

First of all, draw a space diagram showing the lines of action of the forces concerned. An arrow head should be placed on the given force. The correct placing of the arrow heads on the com-

ponent lines is explained below. Now draw a vector line 'OA,' to a suitable force scale, to represent the '6 cwts. force.'

On this line it is essential to place an arrow head (i.e. pointing downwards) to indicate the '*sense*' of the force. Construct a triangle having the drawn vector line 'OA' for one side and having the other two sides respectively parallel to the required directions of the components. Place the arrow heads on these latter two sides, '*in order*' with one another but '*reversed*' in respect to the arrow head on the '6 cwts. force' vector line. The magnitude of each component may now be scaled off the 'resolution triangle,' being represented by 'OB' and 'BA' respectively. From Fig. 28 we see that the components are respectively 4·39 cwts. and 3·11 cwts., acting in the directions indicated. It is clear, by the rule given on page 10, that these two components have the given force of '6 cwts.' as their resultant.

Rectangular Components

When a force is resolved into two components, and these two components are at right angles to one another, they are termed

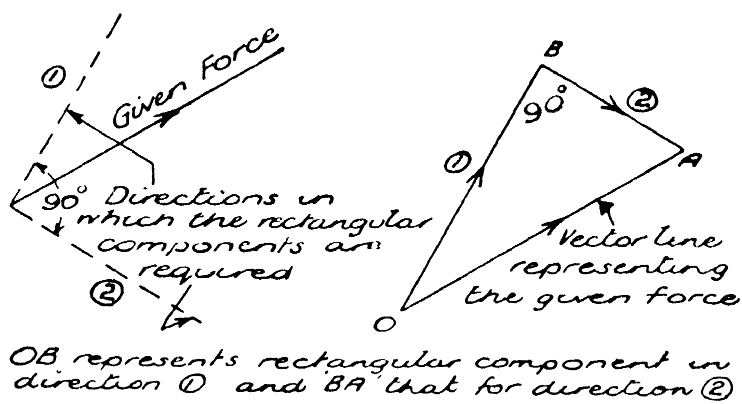


FIG. 29.—RECTANGULAR COMPONENTS.

rectangular components.' The term '*resolved parts*' is also sometimes used in this case. It is important to remember, when considering one rectangular component of a force, that with this component there is always associated another, acting in a direction at right angles to it (Fig. 29).

Important Property possessed by Rectangular Components

In Fig. 30, the force 'F' exerts its full forward effect on the slider block only when it is applied horizontally. As the angle made by force 'F' with the horizontal increases, the effective forward pull on the block decreases. When force 'F' acts vertically it has no tendency to move the block forward horizontally.

A force, therefore, has no effect in a direction at right angles to its own line of action.

If we resolve a force into two components at right angles to one another, each component has no effect in the direction of the other. In such a case each component represents the *total effect* the original given force has in the particular component direction. Rectangular components are not merely components—each rectangular component represents a *net effective value* of the given force.

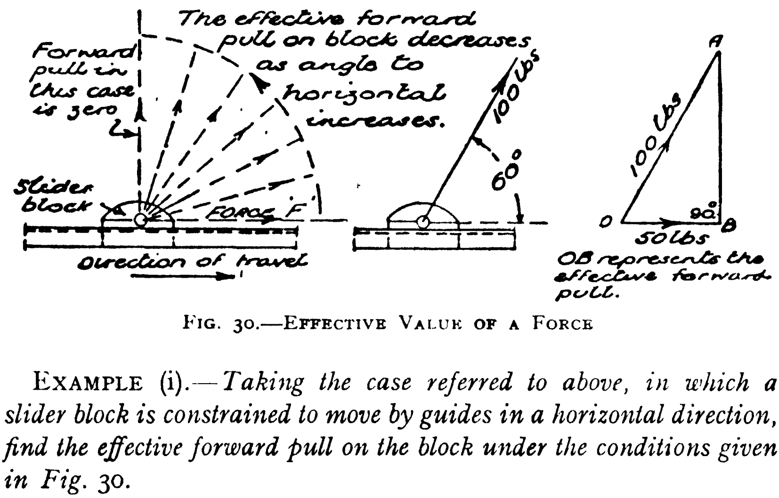


FIG. 30.—EFFECTIVE VALUE OF A FORCE

EXAMPLE (i).—*Taking the case referred to above, in which a slider block is constrained to move by guides in a horizontal direction, find the effective forward pull on the block under the conditions given in Fig.* 30.

Method.—Draw 'OA' to represent the given force of '100 lb.' On 'OA,' as hypotenuse, construct a right-angled triangle 'OAB,' *with one side 'OB' in the direction in which the effective force is required*, i.e. horizontal in this case. 'OB' will represent the required forward pull on the block. The value of the pull will be found to be 50 lb. If the resistance to forward motion, due to friction, etc., should equal 50 lb., the block would not move forward in spite of the fact that the total force acting on it is 100 lb.

EXAMPLE (ii).—*What tendency has the force given in Fig.* 31 (a) *to lift the body at right angles to the inclined plane,* (b) *to pull the body up the plane ?*

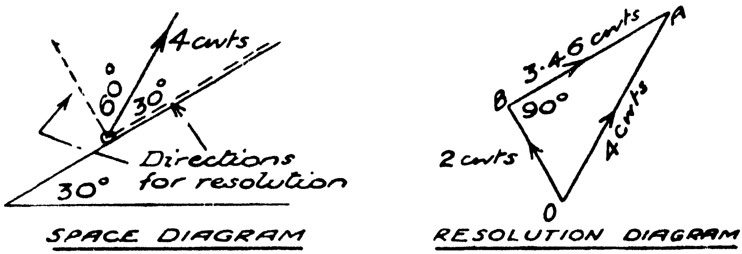


FIG. 31.—EFFECTIVE FORCE VALUES.

Note the construction of the right-angled triangle in Fig. 31, with the sides containing the right angle drawn in the directions referred to in the question. Vector line ' OB ' scales ' 2 cwts.' and represents the lifting tendency at right angles to the plane. ' BA,' which is parallel to the plane, represents the up-plane tendency and scales 3·46 cwts.

EXAMPLE (iii).—*Fig.* 32 *shows a block of stone weighing* 200 *lb. resting on an inclined banker. Find the effective down-plane weight of the block.* In this case we must construct a right-angled triangle with one of the sides (containing the right angle) in the down-plane direction. As indicated in Fig. 32, the effective force of gravity down the plane is 68·4 lb.

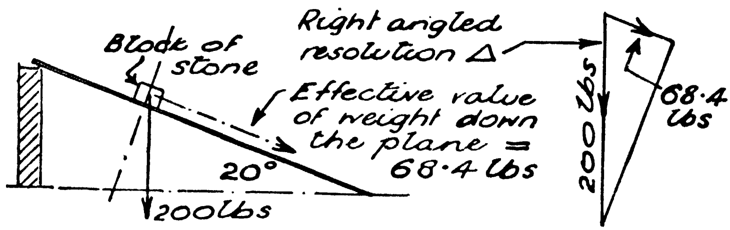


FIG. 32.—RESOLUTION OF SELF-WEIGHT.

Horizontal and Vertical Components

Very often in structural problems the rectangular components are required to be in the horizontal and vertical directions respectively.

It is not always convenient to find these components graphically, by the construction of resolution triangles. A simple formula involving the trigonometrical *cosine* is often employed (see Fig. 33).

Rule.—**To find the effective value of a force in a direction making an angle ' θ ' with its own line of action, multiply the force by the cosine of ' θ.'**

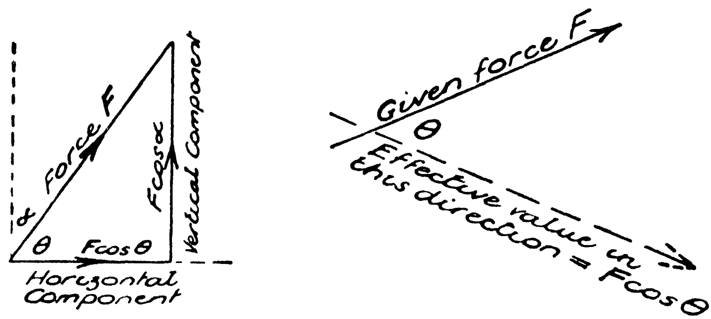


FIG. 33.—FORMULA FOR RECTANGULAR COMPONENTS.

In Fig. 33 (left)—where ' θ ' is the angle made by the force ' F ' with the horizontal—the horizontal component is ' F cos θ.' Similarly, the vertical component is ' F cos α ' where ' α ' is the angle made by force ' F ' with the vertical. As $\alpha = 90° - \theta$, we may express the vertical component, if desired, as ' F sin θ.' The reader is advised, at first, to use only the *cosine rule* in finding rectangular components and to ascertain in each problem the angle between the given force direction and that in which the effective component value is required.

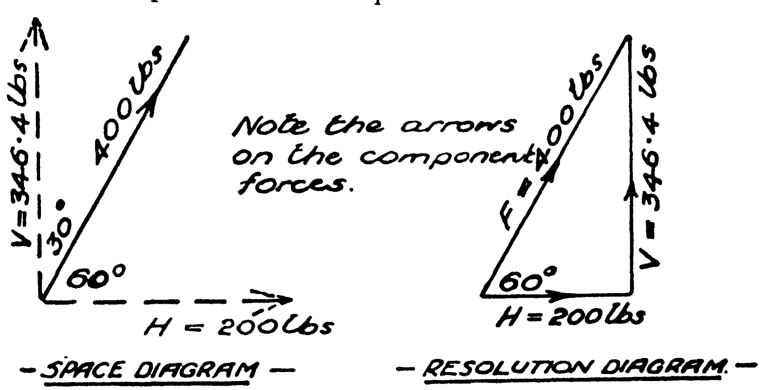


FIG. 34.—DETERMINATION OF COMPONENTS.

EXAMPLE.—*A force of* 400 *lb. acts at an angle of* 60° *to the horizontal* (*Fig.* 34). *Find its horizontal and vertical components.*

Angle between the given force direction and the horizontal = 60°. Cos 60° = ·5. Using the cosine rule, the horizontal component = F cos θ = (400 × ·5) lb. = 200 lb.

Angle between ' F ' and the vertical = 30°. Cos 30° = ·866, therefore the vertical component = F cos θ = F cos 30° = (400 × ·866) lb. = 346·4 lb.

There should be no difficulty in determining, in any given resolution example, whether the horizontal component acts towards the left or the right, or whether the vertical component acts upwards or downwards. Apart from the rules already given for arrow heads, imagine a particle to be acted upon by the given force. The general tendencies of motion of the particle, left or right, up or down, can be readily decided upon.

EXAMPLE.—*Fig.* 35 *shows a buttress subjected to an arch thrust. Find the tendency the thrust has* (i) *to move the abutment horizontally.* (ii) *to increase the load on the buttress foundation.*

The problem is solved graphically in Fig. 35.

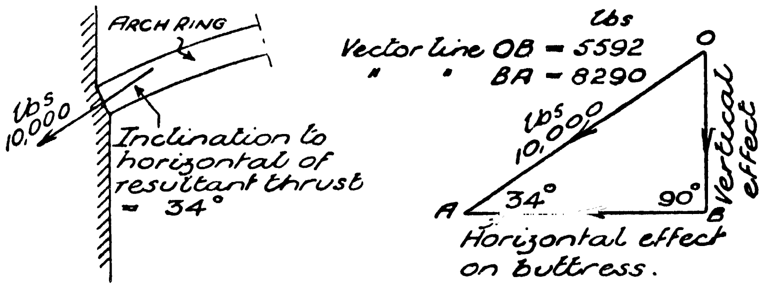


FIG. 35.—THRUST OF AN ARCH.

Using the cosine rule :

(i) horizontal component of force = F cos θ = (10000 × cos 34°) lb. = (10000 × ·829) lb. = 8290 lb.

(ii) vertical component of force = F cos θ = (10000 × cos 56°) lb. = (10000 × ·5592) lb. = 5592 lb.

Concurrent Force System reduced to Rectangular Components

The net or resultant horizontal effect of a system of forces will be the *algebraic* sum of the horizontal components of the various

forces forming the system. Similarly the algebraic addition of the vertical components will give the real vertical effect of the system.

'*Algebraic addition*' simply means: *adding together all the components in one direction (say, to the right) and then adding together all the components in the opposite direction (i.e. the left), and finally subtracting the smaller total from the larger.*

EXAMPLE.—*Find graphically, and by the cosine rule, the resultant horizontal and vertical components, respectively, of the force system given in Fig 36.*

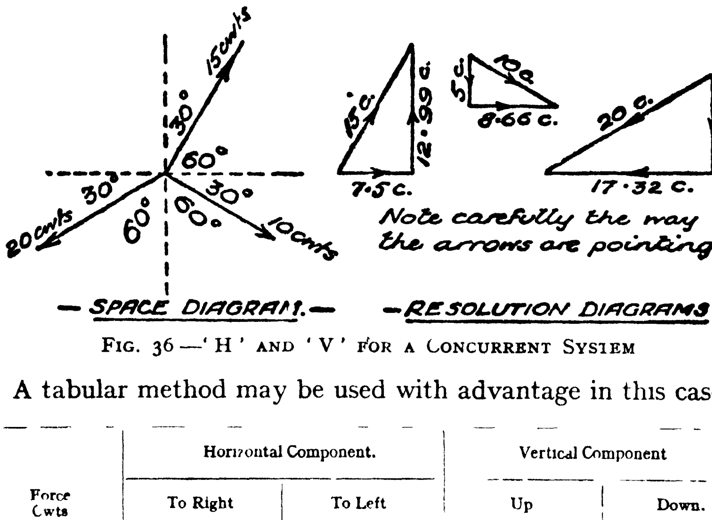


FIG. 36—'H' AND 'V' FOR A CONCURRENT SYSTEM

A tabular method may be used with advantage in this case.

Force Cwts	Horizontal Component.		Vertical Component	
	To Right	To Left	Up	Down.
	Cwts	Cwts	Cwts	Cwts
15	7·50	—	12·99	—
10	8·66	—	—	5·00
20	—	17 32	—	10·00
TOTALS	16·16	17·32	12·99	15·00

Net horizontal force $= (17{\cdot}32 - 16{\cdot}16)$ cwts. to left
$= 1{\cdot}16$ cwts. to left.

Net vertical force $= (15{\cdot}00 - 12{\cdot}99)$ cwts. down
$= 2{\cdot}01$ cwts. down.

It is sometimes convenient to adopt a convention of signs, plus and minus, to indicate force direction.

Fig. 37 shows the usual convention. Applying these signs, and

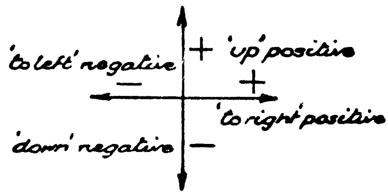


FIG 37.—CONVENTION OF SIGNS.

employing the symbols 'H' and 'V' respectively for the total of horizontal and vertical components, we have :

$H = (7{\cdot}5 + 8{\cdot}66 - 17{\cdot}32)$ cwts. $= -1{\cdot}16$ cwts.,
i.e. $1{\cdot}16$ cwts. acting towards the left.

$V = (12{\cdot}99 - 5 - 10)$ cwts. $= -2{\cdot}01$ cwts.,
i.e. $2{\cdot}01$ cwts acting downwards.

The figures in the table may be filled in by graphical or by calculation methods.

The usual practical method for dealing with such problems as these is to use the 'cosine rule' with the prefixing of signs.

$$\begin{aligned}
\text{Thus } H &= 15 \cos 60° + 10 \cos 30° - 20 \cos 30° \\
&= (15 \times {\cdot}5) + (10 \times {\cdot}866) - (20 \times {\cdot}866) \text{ cwts.} \\
&= (7{\cdot}5 + 8{\cdot}66 - 17{\cdot}32) \text{ cwts.} \\
&= -1{\cdot}16 \text{ cwts., i.e. } 1{\cdot}16 \text{ cwts. to left.} \\
V &= 15 \cos 30° - 10 \cos 60° - 20 \cos 60° \\
&= (15 \times {\cdot}866) - (10 \times {\cdot}5) - (20 \times {\cdot}5) \text{ cwts.} \\
&= (12{\cdot}99 - 5 - 10) \text{ cwts.} \\
&= -2{\cdot}01 \text{ cwts., i.e. } 2{\cdot}01 \text{ cwts. downwards.}
\end{aligned}$$

EXAMPLE. - *Fig. 38 illustrates a portion of a retaining wall acted upon by three forces, viz. its own weight and two earth thrusts. Obtain* (i) *the resultant force tending to cause the wall to slide over its foundation,* (ii) *the total vertical thrust on the subsoil under the wall base.*

The forces in this example are not concurrent but the principles of horizontal and vertical resolution may be applied.

Solution by Graphical Method.—The only inclined force is the '8000 lb.' earth thrust. By the resolution diagram of Fig. 38, the horizontal and vertical components of this force are '6028 lb.'

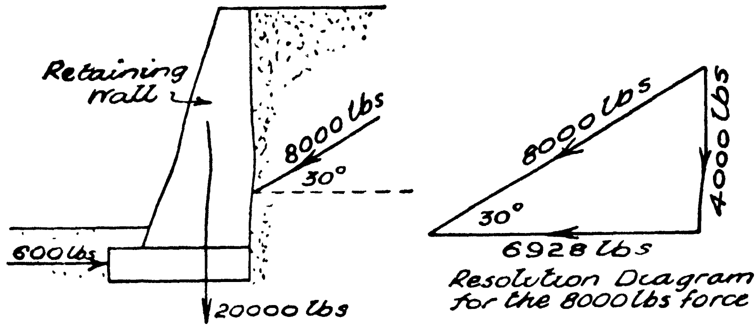


FIG. 38.—NON-CONCURRENT SYSTEM.

and '4000 lb.' respectively. The other two forces require no resolution.

(i) Total horizontal component of system tending to cause sliding = (6928 − 600) lb. = 6328 lb.

(ii) Total vertical thrust on subsoil = (20000 + 4000) lb. = 24,000 lb.

Solution by Cosine Rule:

(i) Total horizontal component = (8000 cos 30° − 600) lb.
= (8000 × ·866 − 600) lb.
= (6928 − 600) lb. = 6328 lb.

(ii) Total vertical component = (20000 + 8000 cos 60°)
= (20000 + 8000 × ·5) lb.
= (20000 + 4000) lb.
= 24,000 lb.

This type of problem is further considered in Chapter XV.

Resultant of a Concurrent System of Forces

The resultant of a concurrent system of forces may be conveniently obtained by means of rectangular components. It is usual to arrange for the axes of resolution to be in the horizontal and vertical directions respectively.

Rule.—Resolve all the forces into their horizontal and vertical components (see Fig. 39). Find 'H' and 'V,' the *algebraic sum* of the components in each case respectively. Using the parallelogram of forces law, compound 'H' and 'V' into their resultant 'R.'

Alternatively, we may obtain ' R ' by the expression :

$$R^2 = H^2 + V^2 \quad \text{or} \quad R = \sqrt{H^2 + V^2}.$$

The graphical method will give the direction of the resultant. If the formula above be used, the direction of the resultant may be obtained by trigonometry.

Let ' θ ' be inclination to the horizontal of the resultant. Then $\tan \theta = V \div H$, i.e. $\frac{V}{H}$.

EXAMPLE.—*Find the resultant of the concurrent force system given in Fig.* 39, *by means of horizontal and vertical components.*

The total horizontal and vertical components, as obtained in Fig. 39, are ·884 cwts. *to left* and ·932 cwts. *upwards*, respectively. It is left as an exercise for the reader to check the separate components by the cosine rule.

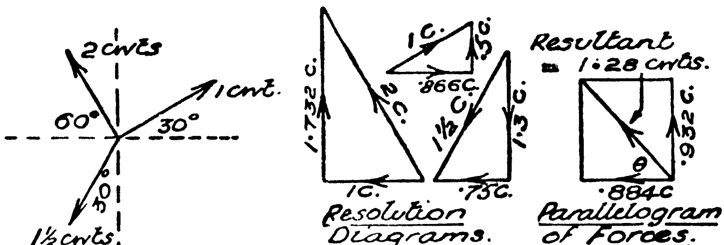


FIG 39.—RESULTANT OF CONCURRENT SYSTEM.

The graphical solution gives 1·28 cwts. as the resultant, acting in the direction indicated in the parallelogram of forces. By calculation :

$$R^2 = H^2 + V^2, \text{ i.e. } R^2 = \cdot 884^2 + \cdot 932^2$$
$$\therefore R = \sqrt{\cdot 884^2 + \cdot 932^2} \text{ cwts.}$$
$$= 1\cdot 28 \text{ cwts.}$$

If the resultant act at ' θ ' to the horizontal,

$$\tan \theta = V/H = \frac{\cdot 932}{\cdot 884} = 1\cdot 054$$
$$\therefore \theta \text{ (from trig. tables) } = 46\tfrac{1}{2}^\circ \text{ nearly.}$$

It is advisable, in the calculation method, to draw a sketch diagram showing ' H ' and ' V ' in their correct directions, so as to avoid any risk of error in the direction of ' R.'

Exercises 2

(1) Resolve each of the forces given in Fig. 40 into components which act in the directions of the respective broken lines shown

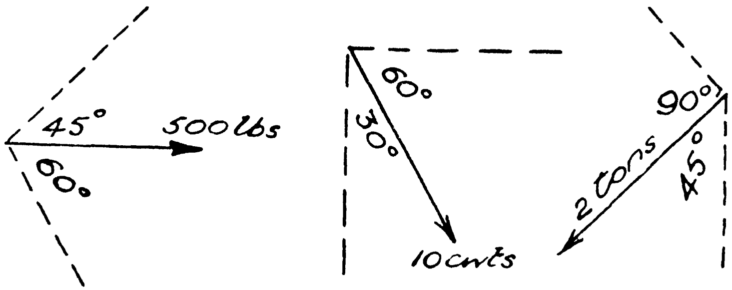


Fig 40

(Note : *The cosine rule must not be used unless the two required components are at right angles to one another.*)

(2) A skip is pulled along its rail track in the manner indicated in Fig 41. Find the effective tractive force in the direction of travel.

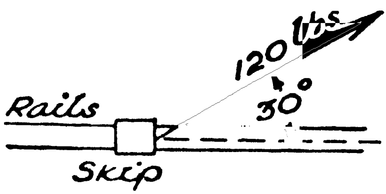


Fig 41

(3) In Fig 42 the peg is inclined at 30° to the vertical and the rope makes an angle of 45° with the horizontal. If the friction of the earth round the peg can only exert a resistance to with-

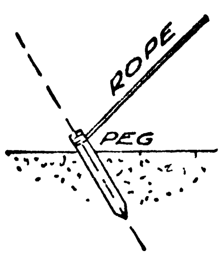


Fig 42

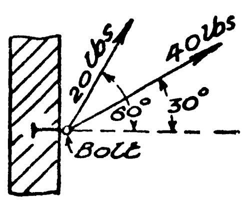


Fig 43.

drawal (in the direction of the length of the peg) of 40 lb., find the maximum safe pull in the rope.

(4) What is the effective horizontal withdrawal force on the bolt given in Fig. 43 ?

(5) Find the horizontal and vertical components of each of the forces given in Fig. 44.

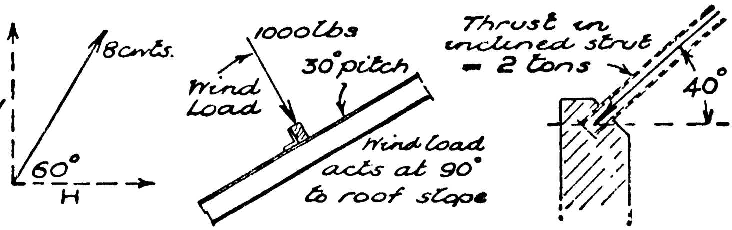


FIG. 44.

(6) Calculate the total horizontal and vertical components respectively of the concurrent system of forces shown in Fig. 45.

(7) Find the magnitude and direction of the resultant of the system of dead and wind loads shown in Fig. 46 : (i) by graphical construction, (ii) by calculation of components.

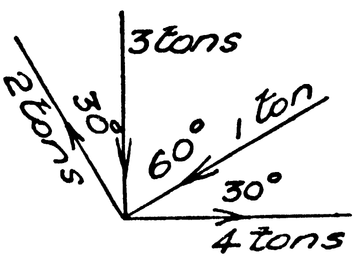


FIG. 45.

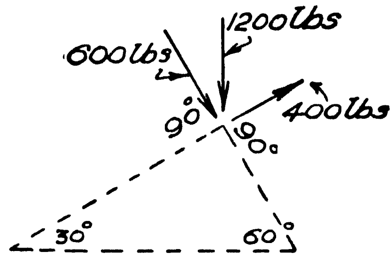


FIG. 46.

(8) A retaining wall is subjected to a resultant thrust of 10,000 lb. for every foot length of wall. The resultant thrust makes an angle of 70° with the horizontal. Calculate the resistance to sliding which the foundation must be able to provide if it is to exceed the horizontal sliding tendency by 50 per cent.

(9) A single timber shore is supported at the bottom by a horizontal timber balk. The thrust in the shore is 2000 lb. If the connection at the bottom of the shore has a safe resistance

to horizontal movement of 1000 lb., find the minimum permissible angle the shore may make with the horizontal.

(10) The main rafters in a roof truss are inclined at 30° to the horizontal. The truss is subjected to a resultant wind load, on the left slope, of 6000 lb. The left end of the truss is supported on a roller and provides no resistance to horizontal movement. Calculate the horizontal force which the right end reaction must provide to maintain equilibrium. (A wind load acts at right angles to the roof slope.)

CHAPTER III

CONCURRENT FORCES. GRAPHICAL LAWS OF EQUILIBRIUM

Bow's Notation

It is essential that the reader should become familiar with an extremely useful method of designating forces. We have hitherto described forces as 'force P' or 'force (1),' etc. An engineer named 'Bow' devised the following notation scheme, which is largely used in the solution of graphical problems.

In each space, formed by the various lines of action of the forces in the 'space diagram,' a capital letter is placed. Any given force is then described by the pair of letters which lie on either side of its line of action. Fig. 47 illustrates the use of this method and gives the notation for each force, corresponding to the two possible sequences for taking the forces, viz. 'clockwise' and 'anticlockwise.'

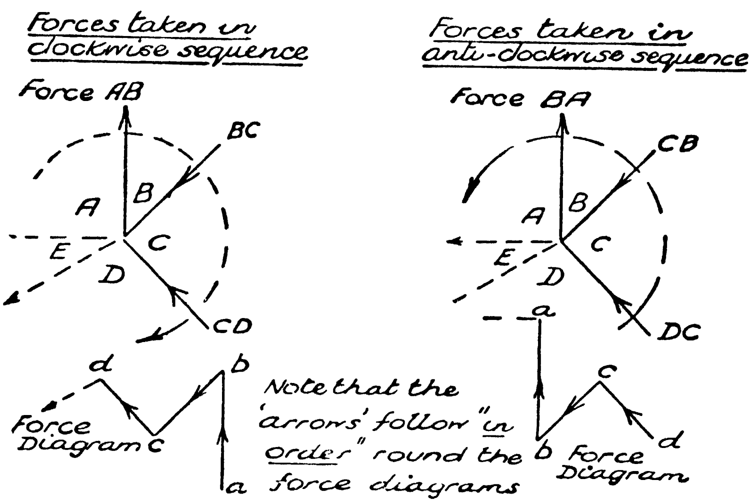


FIG 47.—Bow's Notation

It is extremely important to note the order of the precedence of the capital letters, e.g. it would be a serious error to confuse 'AB' with 'BA' in practical problems.

In the force diagram, corresponding small letters are used *at the ends of the vector lines.* Thus '*ab*' in the force diagram would be drawn parallel to, and represent to scale, the force 'AB' in the space diagram. Similarly, '*ba*' would represent 'BA,' in magnitude and direction.

Triangle of Forces

The two upper diagrams in Fig. 48 show the derivation of 'R,' the resultant of forces 'P' and 'Q,' by means of the type of vector diagram explained in Chapter I. A force 'E' ('*equilibrant*'), equal in magnitude to 'R' and acting in the opposite direction, would balance 'R' and therefore balance 'P' and 'Q.' Forces 'P,' 'Q' and 'E' *acting together* would therefore constitute a system of three concurrent forces in equilibrium. It will be clear that if we draw vector lines in succession to represent forces 'P,' 'Q' and 'E,' a *triangle will be formed.* This triangle is known as the 'triangle of forces' for the three given forces.

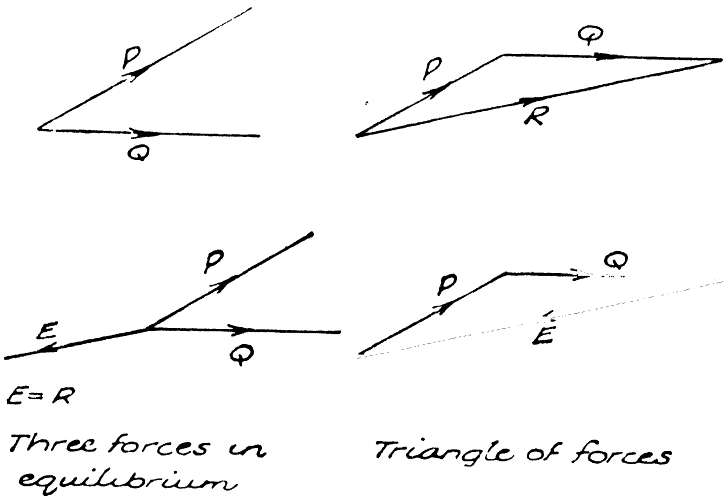


FIG 48.—THREE FORCES IN EQUILIBRIUM.

Law.—**If three forces, acting at a point, be in equilibrium, they may be represented in magnitude and direction by the three sides of a triangle, taken in order.** The 'triangle of forces' law is best expressed for structural calculations in the terms given, and not in the more usual converse form, which gives the necessary

qualification for three concurrent forces to be in equilibrium, viz. that a '*triangle should be formed.*'

Any triangle whose sides are parallel to the lines of action of the three given forces will have its sides respectively proportional to the magnitudes of the forces. If, therefore, we represent one of the forces to a certain scale, the remaining sides of the triangle will represent the two other respective forces to the same scale (Fig. 49).

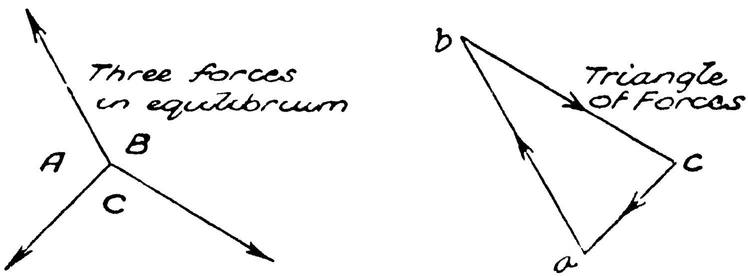


FIG. 49 —THE TRIANGLE OF FORCES.

The usual problem, involving the employment of the 'triangle of forces,' is to be given one of the three concurrent forces completely, but only the lines of action of the other two. The law enables us to find (i) the magnitudes of the two unknown forces, (ii) the '*sense*' in which the unknown forces act, i.e. whether they act respectively '*towards*' or '*away from*' the point of concurrence. The latter information decides whether the corresponding member is a 'strut' or a 'tie.'

Experimental Verification of Law

The triangle of forces law may be experimentally verified by means of the apparatus shown in Fig. 50.

The three concurrent forces are represented by (i) the suspended weight, ·5 lb., (ii) the pull in the left-hand string, ·45 lb., and (iii) the pull in the right-hand string, ·4 lb. The string pulls are obtained from the corresponding weights suspended over the side pulleys, on the assumption that the friction at the pulley bearings may be neglected.

As shown in Fig. 50, a space diagram is drawn for the three forces. The ·5 lb. is represented by a vertical vector line drawn

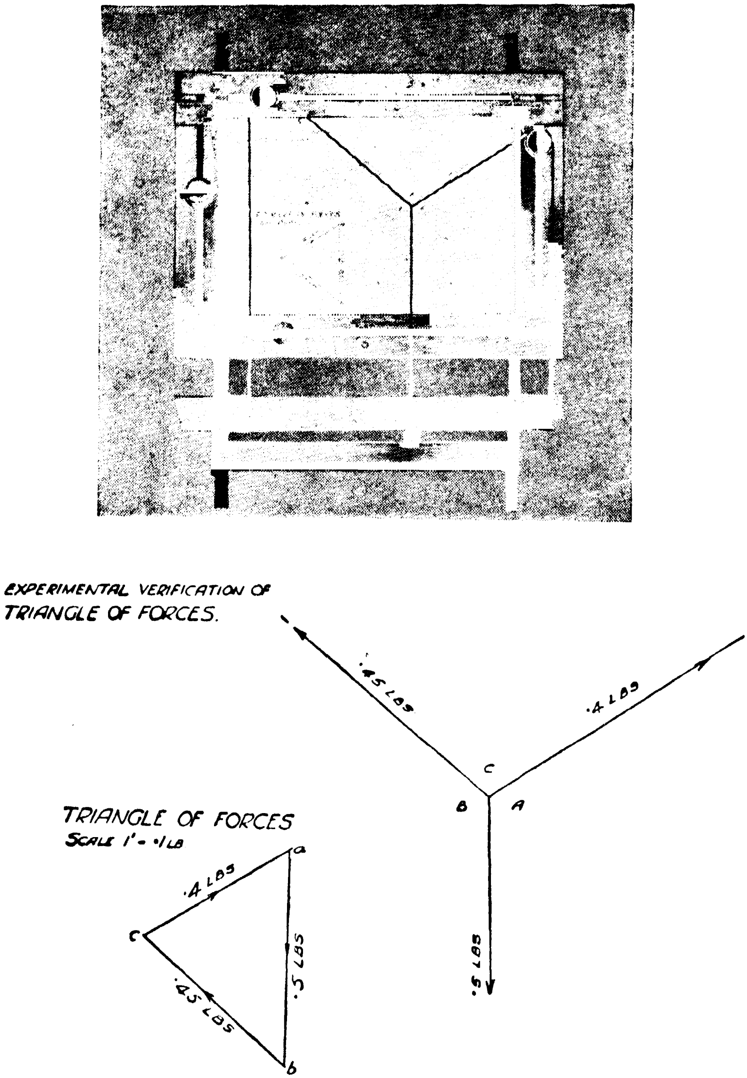


FIG 50.

to a suitable scale. Vector lines are then drawn *in succession* to represent the string pulls of ·45 lb. and ·4 lb. respectively. It will be seen that the force diagram thus constructed is a 'triangle.' The reader will note the employment of '*Bow's notation*' in the

drawing of the force diagrams in the experiment and in Fig. 49 respectively.

Examples on the 'Triangle of Forces'

The law has many applications in structural problems, and a few typical examples will now be considered.

EXAMPLE (i).—*Find the tension in each rope in the example given in Fig.* 51.

Step (i).—Draw a space diagram showing the correct disposition in space of the forces concerned. In this case, the angles '30°' and '60°' determine the rope directions.

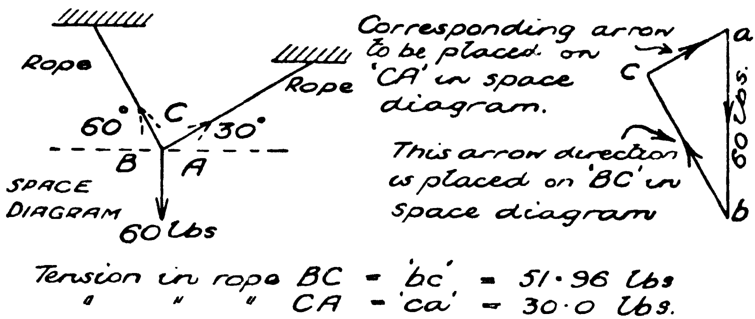


FIG. 51.—EXAMPLE ON TRIANGLE OF FORCES.

Step (ii).—Insert Bow's letters in the spaces formed by the three forces involved in the example. In this solution, 'AB' denotes the weight '60 lb.,' 'BC' and 'CA' denote, respectively, the rope tensions.

Step (iii).—Choose a convenient force scale and draw '*ab*' to represent the weight, '60 lb.' Force 'AB' acts vertically downwards, hence the vector line '*ab*' should be drawn in a downward direction by the pencil point. The first letter in 'AB' is 'A,' hence the letter to be placed *where the pencil point starts* is '*a*.' If we had decided to take the forces in anticlockwise sequence, the weight would be represented by 'BA,' and the top of the vector line representing this force would have been '*b*.'

Step (iv).—Complete the triangle of forces by drawing '*bc*' from '*b*' parallel to force 'BC,' and '*ac*' from '*a*' parallel to force 'CA.' It is necessary to draw the last vector line *from* '*a*' in order to fix the point '*c*.'

Step (v).—We now have a triangle with its sides parallel to the given forces and the force scale for one of the sides known. To obtain the magnitudes of the unknown tensions we simply have to scale off (to force scale used for constructing '*ab*') the vector lines '*bc*' and '*ca*.'

EXAMPLE (ii).—*A weight of* 2 *cwts., suspended from the end of a crane jib, is pulled by a horizontal force in the manner shown in Fig.* 52. *Determine the value of the horizontal force and the tension in the upper portion of the chain.*

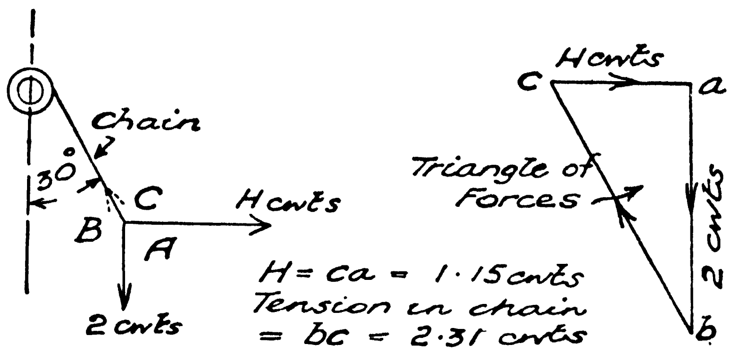


FIG 52.

The problem is solved in Fig. 52. The reader should follow the solution, step by step, as indicated in the previously worked example.

EXAMPLE (iii).—*A wall bracket is used to carry a weight of* 100 *lb. as indicated in Fig.* 53. *Determine the force in each projecting arm of the bracket and state its nature.*

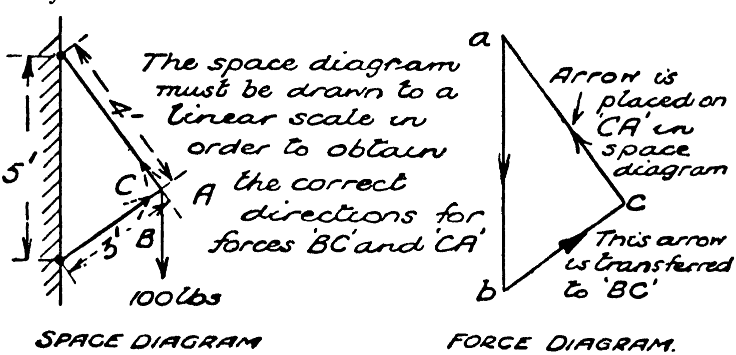


FIG. 53.

In this example we require a linear scale for the space diagram in order to construct correctly the directions of the three forces acting at the extremity of the bracket. The arrow directions, derived from the triangle of forces, are inserted in the space diagram on the appropriate members, *near the junction of the three force lines*. Member 'BC' is a strut (thrust = 60 lb.) and member 'CA' is a tie (pull = 80 lb.).

EXAMPLE (iv).—*Determine the force in the principal rafter and the tension in the tie of the roof truss given in Fig.* 54.

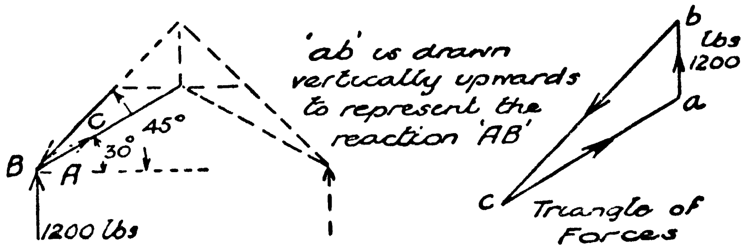


FIG. 54.

The principal rafter is a strut and exerts a thrust of 4016 lb. The tension in the inclined tie is 3279 lb.

Theorem relating to Three Forces in Equilibrium

The following theorem will be found useful in solving problems involving three forces in equilibrium.

If three forces acting on a body keep it in equilibrium, they must either be parallel forces or all their lines of action must pass through one common point.

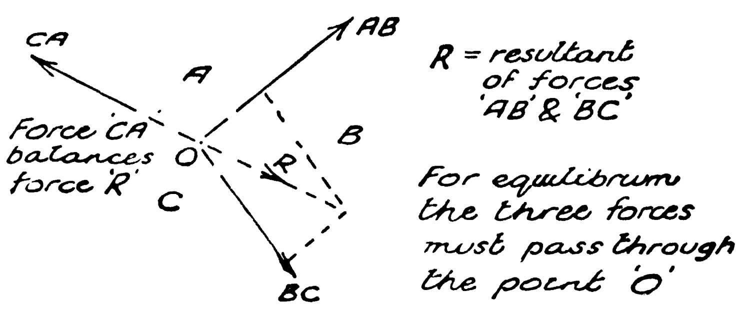


FIG. 55.

We are only concerned with the latter part of the theorem in this chapter. Consider the three forces 'AB,' 'BC,' and 'CA' in Fig. 55. We may replace 'AB' and 'BC' by their resultant 'R.' Force 'CA' will therefore balance 'R,' i.e. it will act in exactly the opposite direction. Clearly, then, it will pass through 'O,' the point in which forces 'AB' and 'BC' intersect.

EXAMPLE (i).—*In Fig. 56 it may be assumed that the reaction at the left end of the truss is vertical. Determine the value of each reaction for the conditions given.*

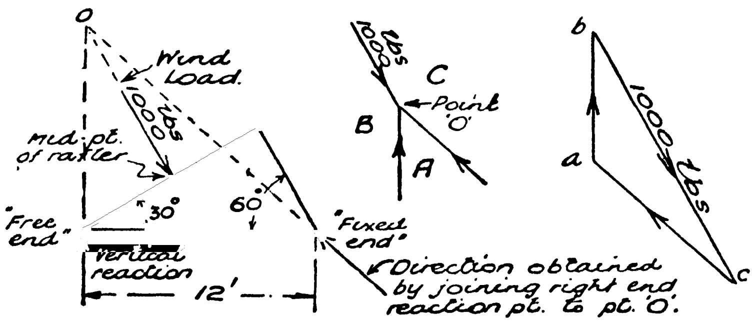


FIG. 56.

The direction of the reaction at the right end of the truss not being given, we make use of the foregoing theorem to determine it. The problem then reduces to a system of three forces (acting at point 'O') in equilibrium, and is solved as in Fig. 56 by the 'triangle of forces.'

The left-end reaction = 433 lb. The right-end reaction acts in the direction indicated and its magnitude = 661 lb.

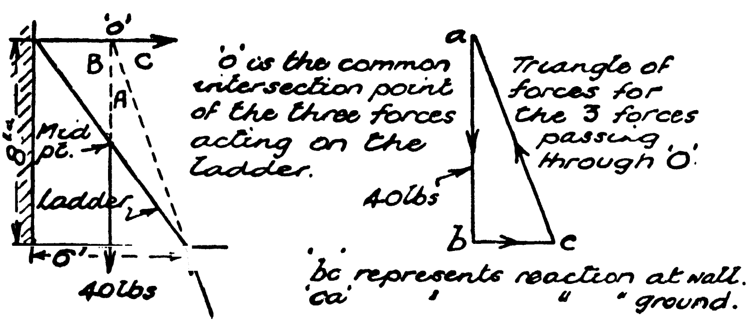


FIG. 57.

EXAMPLE (ii).— *A ladder (Fig. 57) rests against a smooth wall. Taking the conditions given, determine the reactions at the wall and at the ground respectively.*

As the wall is smooth and cannot exert any frictional force on the ladder end, the reaction at the wall is horizontal. The lines of action of this reaction and the weight respectively intersect at the point 'O.' Hence the ground reaction passes through 'O.' A triangle of forces is now drawn for the three forces, in equilibrium, passing through 'O.' The direction of the ground reaction is as given in the space diagram, its magnitude = *ca* = 42·7 lb. The wall reaction = *bc* = 15 lb.

A method of solution, by calculation, of problems involving three forces in equilibrium is given in Chapter V.

Polygon of Forces

The graphical method for finding the resultant of a concurrent force system is explained on page 12. Let us suppose that we applied this construction to a force system known to be in equilibrium--a system which has, of course, no resultant force. It is clear that there must be no distance between the final and initial points of the diagram, as the resultant equals zero. It appears, therefore, that for a system in equilibrium the force diagram must be a closed figure (Fig. 58). We have, thus, the following law of equilibrium, known as the '**polygon of forces**' law.

'If a number of forces acting at a point be in equilibrium, the forces may be represented in magnitude and direction by the sides, taken in order, of a certain closed polygon.'

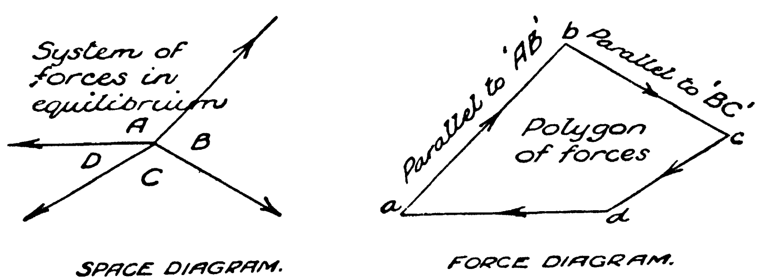
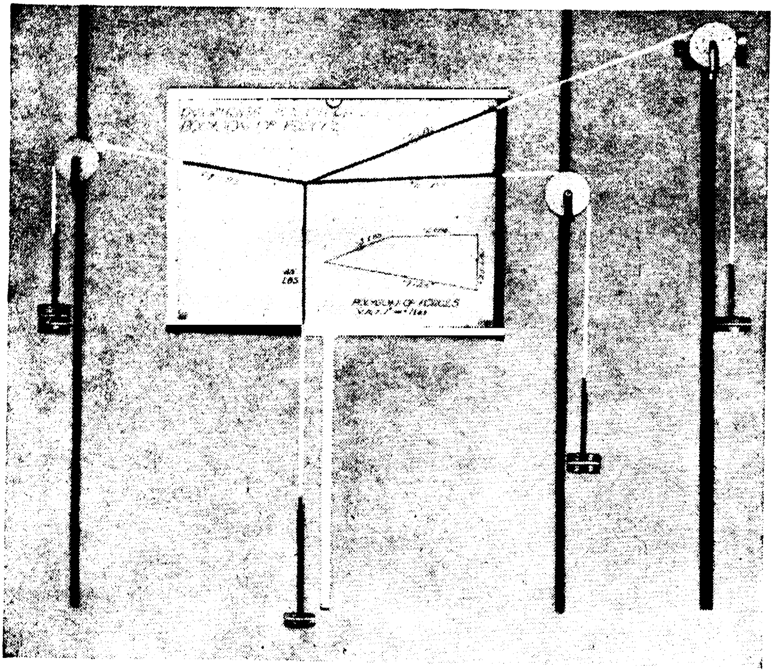


FIG. 58.—THE POLYGON OF FORCES.



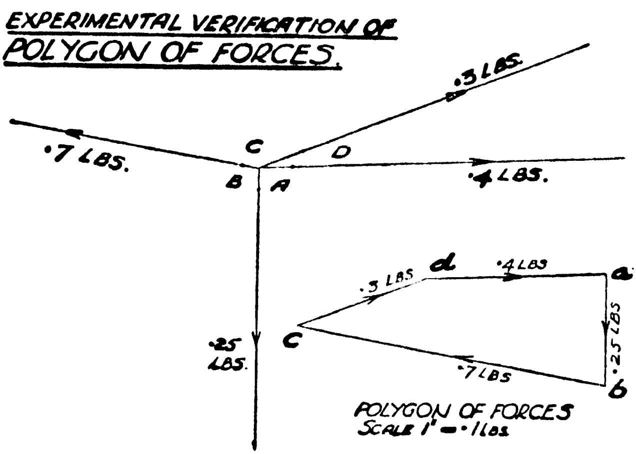

EXPERIMENTAL VERIFICATION OF
POLYGON OF FORCES.
·3 LBS.
·7 LBS.
C
D
B
A
·4 LBS.
·25 LBS.
·3 LBS
d
·4 LBS
a
c
·25 LBS
·7 LBS
b
POLYGON OF FORCES
SCALE 1" = ·1 LBS.

FIG. 59

The word '*certain*' has been introduced into the definition to ensure that the reader will not assume that *any* polygon whose sides are parallel to the given force directions will necessarily represent all the forces to some given scale.

The terms of this extremely important law have been expressed in the form required for application to structural problems. Simply, the law states that if we draw vector lines, one at the end of the other, to represent the forces (taken in any convenient sequence), the force diagram formed will be a *closed polygon*, provided the force system be in equilibrium. The vector lines must be drawn with the pencil point travelling in the *sense* indicated by the arrows on the corresponding forces in the space diagram.

The usual problem is to have a number of forces, in equilibrium, acting at a point (such as a joint in a roof truss), the lines of action, but not the magnitudes, of all the forces being known. Not more than *two* forces must be unknown in magnitude. The 'polygon of forces' law is applied to determine the unknown magnitudes.

The polygon of forces law may be verified by apparatus such as that shown in Fig. 59. The special wall-board apparatus, given in Fig. 50, for the 'triangle of forces' law, is also suitable.

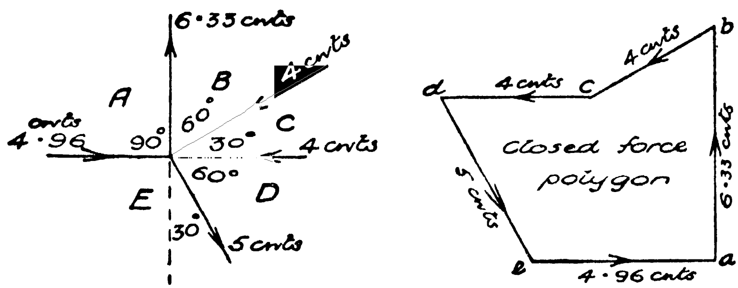


FIG. 60.—SYSTEM IN EQUILIBRIUM

In actual experiments of the type indicated fine string should be used as the weight of the string is neglected.

EXAMPLE (i).—*Test for equilibrium the system of forces given in Fig.* 60.

It will be seen, from the figure given, that the force diagram closes, hence the system is in equilibrium.

It is usual in employing Bow's notation to take the forces in a clockwise sequence around the common intersection point, though this is not essential. After drawing '*ab*' upwards to represent force 'AB,' i.e. '6·33 cwts.,' '*bc*' is drawn *from* '*b*' to represent 'BC,' i.e. '4 cwts.' The vector line '*ea*' eventually closes the diagram.

Note (i) the correspondence of letters throughout, e.g. '*de*,' is *parallel* to force 'DE'; (ii) the arrows go round the figure without reversal, i.e. they are '*in order*'; (iii) it would not matter if the vector lines in the polygon crossed one another.

EXAMPLE (ii).—*Fig.* 61 *represents four concurrent forces acting in a structural joint. The four forces are given in direction, but two are unknown in magnitude. Find the magnitude of the latter two forces.*

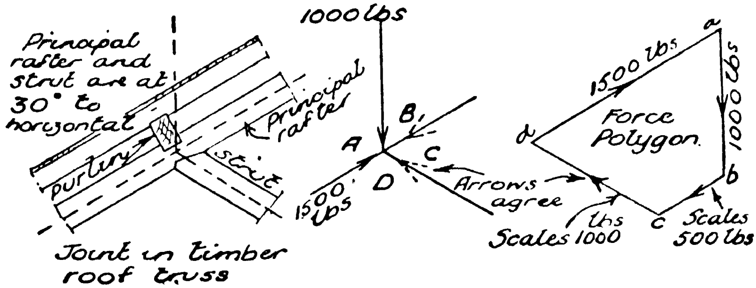


FIG. 61.—JOINT IN TIMBER TRUSS.

This problem, which is solved in Fig. 61, is the basic problem involved in *stress diagram* construction (which is considered in Chapter VIII).

Vector line '*da*' is drawn to represent force 'DA' (1500 lb.) and '*ab*' is drawn from '*a*' to represent force 'AB' (1000 lb.). We cannot draw '*bc*' directly because force 'BC' is unknown. If we draw from '*b*' a line parallel to 'BC' and from '*d*' a line parallel to 'CD,' the point '*c*' will be where these two lines intersect. Vector line '*bc*' (to scale of force diagram) scales '500 lb.' and '*cd*' scales '1000 lb.' The arrows are continued round the force diagram '*in order*' and transferred to the space diagram.

'BC' is a strut with a thrust of 500 lb., and 'CD' is a strut with a thrust of 1000 lb.

EXERCISES 3

(1) Determine the force in each rafter of the truss shown in Fig. 62. Verify that the rafters act as struts.

(2) Find the force in the tie member of the truss (Fig. 62) by drawing a triangle of forces (i) for the left-end reaction point, (ii) for the right-end reaction point. The force in each rafter must first be found or the answers for question (1) assumed.

(3) Obtain the forces, in the 'tie' and the 'jib' respectively, of the jib crane represented by the outline diagram given in Fig. 63. Is the jib in tension or compression ?

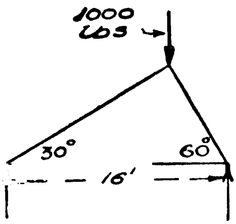


FIG. 62.

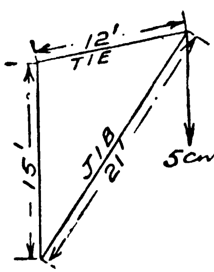


FIG. 63.

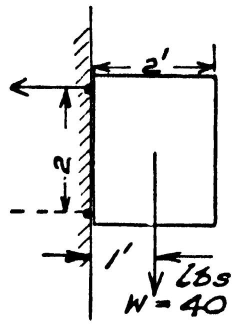


FIG. 64.

(4) Assuming, in the case of the jib crane given in Fig. 63, that the rope carrying the weight passed over a free-running pulley and ran parallel to the crane jib, determine the 'jib' and 'tie' forces respectively.

(5) A signboard is supported outside a wall in the manner indicated in Fig. 64. Assuming the upper hinge to be capable of exerting a horizontal reaction only, find (i) the direction of the reaction at the lower hinge, (ii) the magnitude of each reaction.

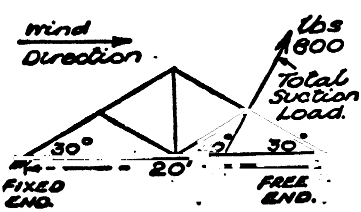


FIG. 65.

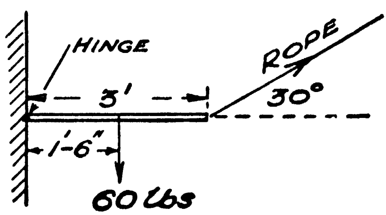


FIG. 66.

(6) Certain building regulations require the 'suction' effect on a roof, due to wind pressure, to be taken separately from the 'windward' loads. Taking the case given in Fig. 65, determine the support reactions for the truss. The reaction at the free end is vertical.

(7) A trap door is temporarily held in a horizontal position by a rope (Fig. 66). Assuming the particulars given, find the reaction at the hinge and the tension in the rope.

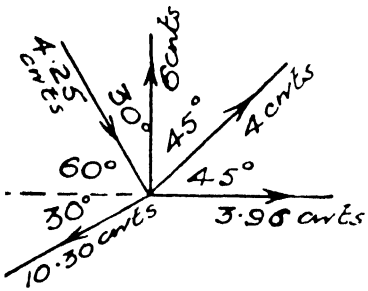


FIG. 67.

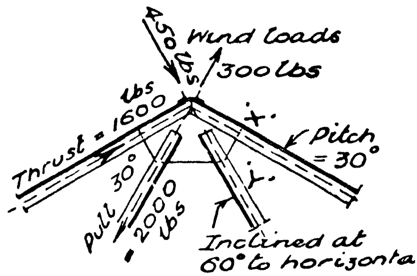


FIG. 68.

(8) Verify that the concurrent system of forces given in Fig. 67 is in equilibrium.

(9) Fig. 68 shows the apex joint in a steel roof truss. Find the magnitude and nature of the forces in the members marked 'X' and 'Y' respectively.

(10) The vertical load at a joint in a braced girder is 2 tons (Fig. 69). The forces in members 'AB' and 'BC' are 3 tons and 8 tons respectively, acting in the sense indicated by the arrow heads. Determine the forces in members 'DE' and 'EA' respectively. State whether the members are 'struts' or 'ties.'

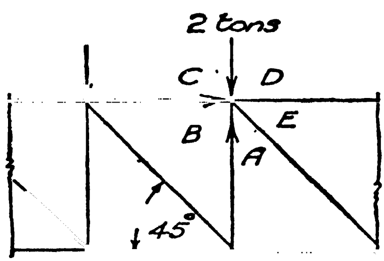


FIG. 69.

CHAPTER IV

MOMENTS. PRINCIPLE OF THE LEVER

THE previous chapters have dealt with the translational effect of forces, i.e. their tendency to move bodies from one position to another. A force may, however, have a rotational effect on a body, tending to turn it round some given point which acts as a hinge. In Fig. 70 (*a*) the force 'F' would turn the disc in a *clockwise* manner of rotation about the hinge 'O.' In Fig. 70 (*b*) the force would cause an *anticlockwise* rotation about 'O.' To express these turning effects correctly we would say that the disc in Fig. 70 (*a*) turns as indicated because it is acted upon by a '*clockwise moment.*' Similarly, an '*anticlockwise moment*' is the cause of the rotation indicated in Fig. 70 (*b*).

'Moments' are therefore concerned with rotational effects and, as we will see, play a very important part in structural calculations.

Moment of a Force

In considering the rotational effect of a force on a body we must have in mind some definite point about which the turning tendency is to be measured. The point concerned is usually referred to as the '*fulcrum.*' Thus 'O' is the 'fulcrum' in Fig. 70.

If we wish to open a door, i.e. subject it to a turning moment about the hinge, we must clearly exert a force on it. But a force,

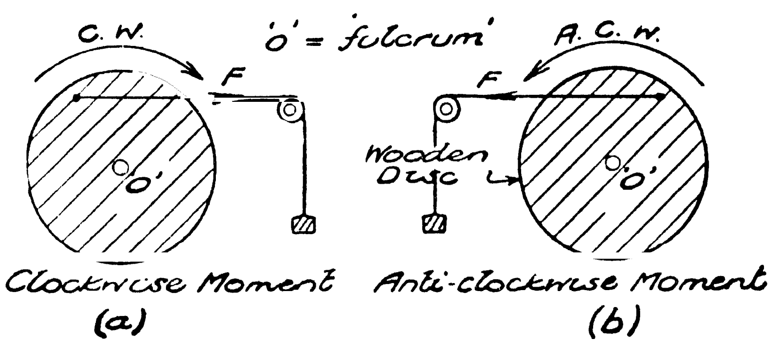


FIG. 70.—TYPES OF MOMENTS.

however big, applied at the hinge will be of no avail. We must apply the force at some distance from the hinge, and experience shows that the farther we get from the hinge the easier will be the operation of opening the door This, of course, accounts for the position of the door knob

Furthermore, it is when we pull at right angles to the door that we get the best turning effect Even if we do pull at the knob, there will be no tendency for the door to open if the line of action of the pull passes through the hinge (see Fig 71) The way in which 'distance' enters into the measurement of a 'moment' must therefore be carefully considered

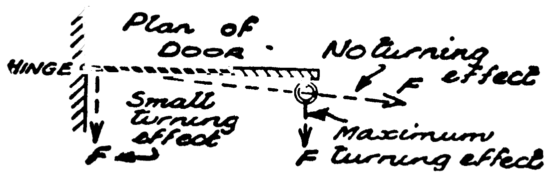


FIG 71

Measurement of a Moment.—Two quantities are involved in expressing the value of a '*moment*' or '*turning effect*';

(1) The magnitude of the applied force

(2) The *perpendicular* distance between the line of action of the force and the point about which turning is being considered, i e the fulcrum This distance is termed the '*arm of the moment*'

The value of the 'moment' increases directly as these two quantities increase, so that we multiply them together to obtain the actual moment.

Definition The moment of a force about a given point is the

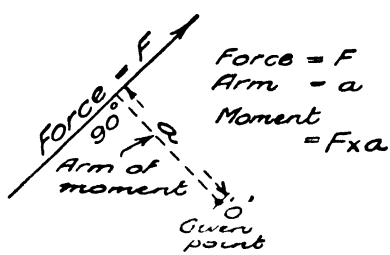


FIG. 72 —MEASUREMENT OF A MOMENT

product of the force and the perpendicular distance from the point to the line of action of the force.

Moment = Force × arm.

In Fig. 72 the moment of the force 'F' about the point 'O'

= Force × arm

= (F × *a*) units.

Examples of Moments (see Fig. 73)

In each of the examples given 'O' is the point about which the moment is taken. 'C.W.' will be used to denote a '*clockwise moment*' and 'A.C.W.' will denote an '*anticlockwise moment.*'

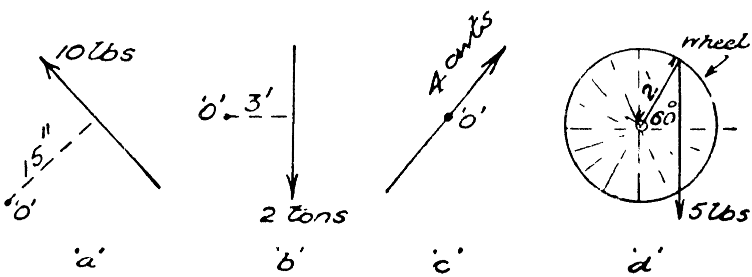


FIG. 73.

Example 'a'

Moment of 10 lb. force about 'O' = Force × arm

= 10 lb. × 15 ins.

= 150 lb. ins. (A.C.W.).

Note.—The unit used to express a 'moment' must contain both 'force' and 'distance' units. To determine whether a moment is 'C.W.' or 'A.C.W.,' imagine the '*arm*' of the moment to be a crank acted upon by the given force. If the 'crank' tends to turn like the hands of a clock the moment is 'clockwise.' Similarly, an 'anticlockwise' moment may be determined.

Example 'b'

Moment = Force × arm

= 2 tons × 3 ft.

= 6 tons ft. (C.W.).

If the value were required in '*lb. ft.*' units, the moment would be (6 × 2240) lb. ft. In '*lb. in.*' units, the value would be (6 × 2240 × 12) lb. ins.

Example 'c'

Moment = Force × arm
= 4 cwts. × 0
= 0.

The reader should remember that a force has no moment about a point on its own line of action.

Example 'd'

Moment of suspended weight about the axle of the wheel
= Force × arm
= 5 lb. × (2 cos 60°) ft.
= 5 lb. × 1 ft.
= 5 lb. ft. (C.W.).

Note how the '*arm*' is measured in this example.

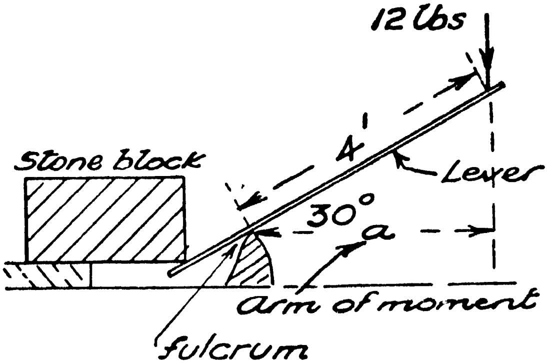


FIG. 74.

Example (see Fig. 74)

Moment of applied vertical force of 12 lb. about the given fulcrum = Force × arm
= 12 lb. × (4 cos 30°) ft.
= 41·57 lb. ft. (C.W.).

Example (see Fig. 75)

Moment of reaction at 'A' about fulcrum at 'B'
= Force × arm
= R_A tons × 5 ft. = (R_A × 5) tons ft. (C.W.).

Moment of load 'W_1' tons about 'B' = (W_1 × 4) tons ft. (A.C.W.).
" " " 'W_2' " " " = (W_2 × 2) tons ft. (A.C.W.).

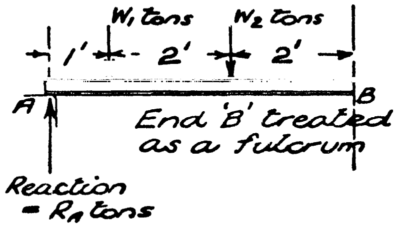


FIG. 75.—APPLICATION IN BEAMS.

Resultant Moment.—When a body is acted upon by several forces, each of which is trying to turn it about a given fulcrum, the net turning effect, i.e. the '*resultant moment*,' is the algebraic sum of all the separate or '*component*' moments. This simply means that we have to take the moment of each force about the fulcrum, add together all those moments which are clockwise and similarly all those which are anticlockwise and finally subtract the smaller of these totals from the larger.

EXAMPLE (i).—*A uniform rule is freely supported at its middle point* (Fig. 76). *Find the manner in which the rule will begin to*

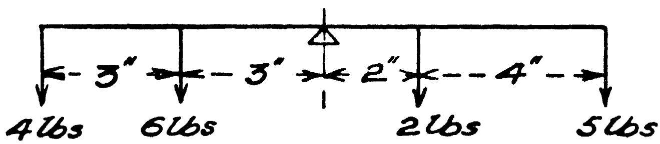


FIG. 76.

turn when loaded as indicated in the figure. Calculate the resultant moment about the fulcrum.

A.C.W. Moments	C.W. Moments
4 lb. × 6 ins. = 24 lb. ins.	2 lb. × 2 ins. = 4 lb. ins.
6 lb. × 3 ins. = 18 „ „	5 lb. × 6 ins. = 30 „ „
Total A.C.W. moment = 42 lb. ins.	Total C.W. moment = 34 lb. ins.

The totals indicate that the rule will turn in an anticlockwise manner. The resultant moment about the fulcrum

$$= (42 - 34) \text{ lb. ins.} = 8 \text{ lb. ins. (A.C.W.).}$$

EXAMPLE (ii).—*The moments disc in Fig.* 77A *is supported in*

such a way that there is no tendency to turn when unloaded. Calculate the resultant moment about the fulcrum when the disc is loaded in the manner indicated.

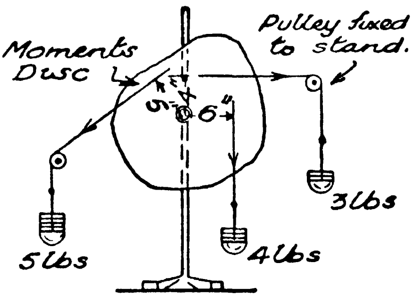


FIG 77A.

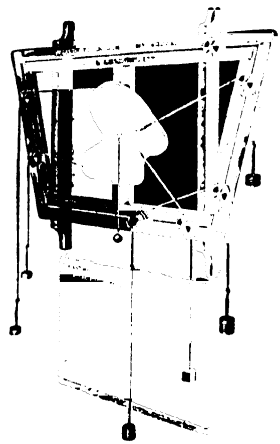


FIG 77B—FORCES FRAME WITH DETACHABLE BOARD
The load weights clear one another

The tabular method used in the last example is not usually adopted in the solution of 'moments problems.'

Total A.C.W. moment = 5 lb. × 5 ins. = 25 lb. ins.
Total C.W. moment = (3 lb. × 4 ins.) + (4 lb. × 6 ins.)
= (12 + 24) lb ins.
= 36 lb. ins.
Resultant moment about fulcrum = (36 − 25) lb. ins.
= 11 lb ins. (C.W.).

Fig. 77B shows a modern form of wall-board apparatus, made by Messrs. G. Cussons Ltd., which may be used in experiments illustrating the principles of 'rotation.' It is suitable for general use in experiments dealing with co-planar forces.

Principle of the Lever

Let us apply the methods used in the last two examples to the case given in Fig. 78.

Total A.C.W. moment $= [(6 \times 6) + (3 \times 2)]$ lb. ins.
$= (36 + 6)$ lb. ins. $= 42$ lb. ins.
Total C.W. moment $= [(4 \times 3) + (5 \times 6)]$ lb. ins.
$= (12 + 30)$ lb. ins. $= 42$ lb. ins.
Resultant moment $= (42 - 42)$ lb. ins. $=$ zero.

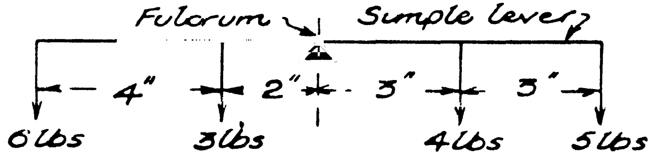


FIG. 78.

The result indicates that the lever will not tend to turn about the fulcrum. This example illustrates a very important law or '*principle*,' which for practical application may be stated as follows :

If a simple lever, capable of turning about a given hinge point or 'fulcrum,' be loaded in such a manner as to keep it in equilibrium, the sum of all the clockwise moments taken about the fulcrum will equal the sum of all the anticlockwise moments.

Just as a body cannot be in positional equilibrium if acted upon by a net resultant force, so it cannot be in rotational equilibrium if a resultant moment act on it.

Experimental Verification of the Principle of the Lever

Fig. 79 shows a typical arrangement of apparatus for verifying the principle in the case of a uniform rule with the fulcrum at the centre. The table on page 53 (Fig. 79A) gives detailed results illustrating the nature of the principle involved.

EXAMPLES :

In all the following examples, the weight of the 'lever' will be neglected. The method of allowing for the '*self-weight*' of members in such examples will be explained later.

(i) *Calculate the value of 'x' for equilibrium in the case given in Fig.* 80 (*a*).

$$\text{A.C.W.} = \text{C.W.}$$
$$(4 \times x) = (2 \times 10)$$
$$4x = 20$$
$$x = 5''.$$

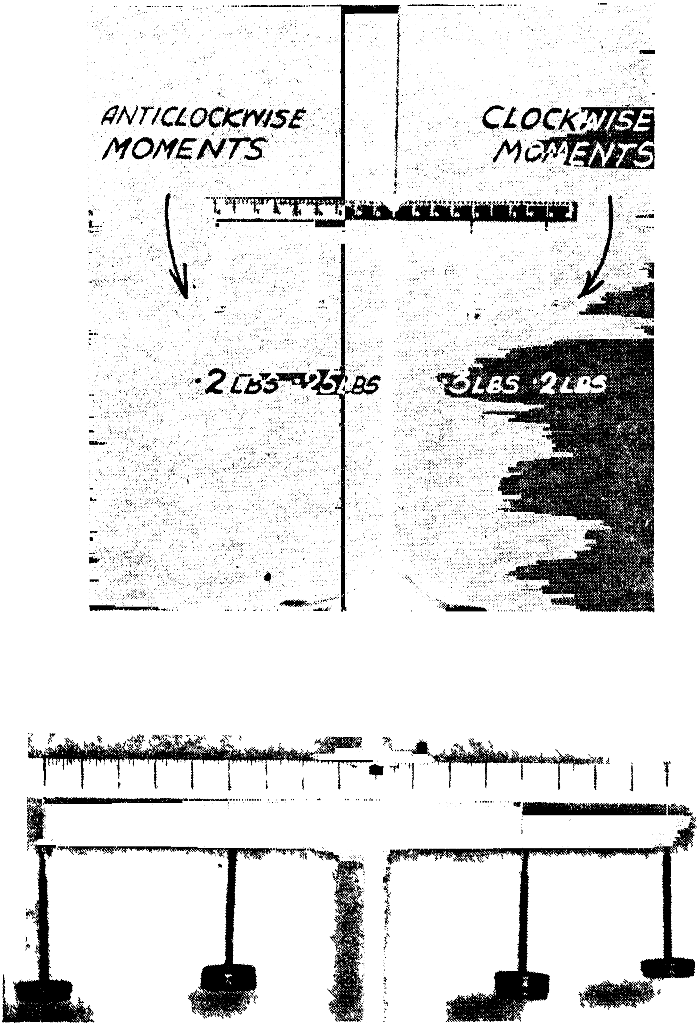


FIG 79.—PRINCIPLE OF THE LEVER

Expt	Anticlockwise Moments.				Clockwise Moments.			
	Force.	Arm.	Moment	Total Moment.	Force.	Arm.	Moment	Total Moment.
	lb	ins.	lb. ins	lb ins.	lb.	ins.	lb. ins.	lb ins
1	·4	3	1 2	1·2	·2	6	1·2	1 2
2	·5	2	1·0	1 0	·4	2·5	1·0	1 0
3	·2 ·4	3 4	0·6 1 6	— 2 2	— ·5	— 4·4	— 2·2	— 2·2
4	— ·3	— 6	— 1 8	— 1·8	·4 ·2	2 5	0·8 1·0	- 1 8
5 (See Fig 79)	·2 ·25	9 4	1 8 1 0	— 2 8	·3 ·2	4 8	1·2 1 6	— 2 8

FIG. 79A

Care must be exercised to have the '*force*' and '*length*' units consistent throughout. The units may then be omitted from the values in the '*equation of moments.*'

(ii) *Find 'W'* (Fig. 80 (*b*)) *so that the lever may remain in equilibrium.*

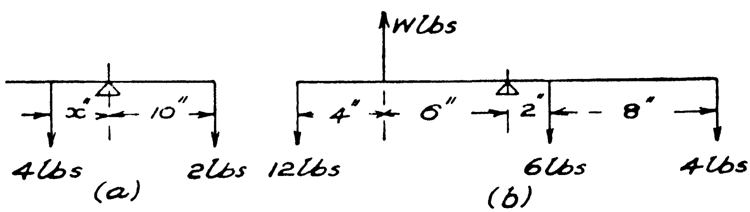


FIG. 80.

$$
\begin{aligned}
\text{A.C.W.} &= \text{C.W.} \\
12 \times 10 &= (6 \times 2) + (4 \times 10) + (W \times 6) \\
120 - 12 - 40 &= 6W \\
6W &= 68 \\
W &= 11\tfrac{1}{3} \text{ lb.}
\end{aligned}
$$

(iii) *Fig.* 81 (*a*) *shows a trap door freely hinged at one end and resting on a steel beam at the other. Find the reaction exerted by the steel beam for the given loads.*

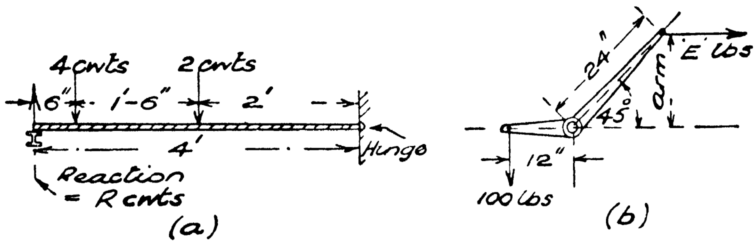


FIG. 81.

Taking moments about the hinge :

$$\text{C.W.} = \text{A.C.W.}$$
$$(\text{R} \times 4) = (4 \times 3{\cdot}5) + (2 \times 2)$$
$$4\text{R} = 14 + 4 = 18$$
$$\text{R} = 18/4 \text{ cwts.} = 4{\cdot}5 \text{ cwts.}$$

(iv) *Calculate the value of the effort ' E ' (Fig. 81 (b)) in order that the applied load of 100 lb. may be just balanced.*

In all problems of this type care must be taken to obtain the correct *' arm '* dimension for each moment.

Taking moments about the fulcrum of the cranked lever :

$$\text{C.W.} = \text{A.C.W.}$$
$$\text{E} \times 24 \sin 45^\circ = 100 \times 12$$
$$\text{E} \times (24 \times {\cdot}7071) = 1200$$
$$\text{E} = 1200/16{\cdot}97 = 70{\cdot}71 \text{ lb.}$$

If the effort had been applied at right angles to the lever, the *' moments equation '* would have been :

$$\text{E} \times 24 = 100 \times 12$$
$$\text{E} = 50 \text{ lb.}$$

Readers unfamiliar with trigonometry may obtain the necessary *' moment arms '* by direct measurement off a scale drawing of the given example.

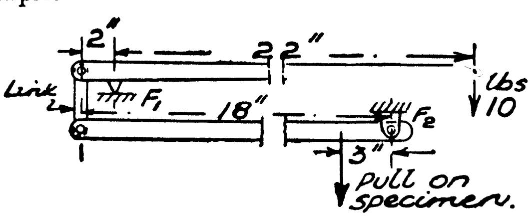


FIG. 82.—COMPOUND LEVER.

(v) *Fig.* 82 *gives details of a compound lever used in testing specimens in a cement briquette testing machine. Calculate the pull on the specimen corresponding to a test load of* 10 *lb. at the end of the lever arm.*

Step 1.—Find the pull in the link by taking moments about fulcrum 'F_1.'

Let 'L' lb. = pull in link.

$$L \times 2 = 10 \times 22$$
$$\therefore L = 110 \text{ lb.}$$

Step 2.—Take moments about fulcrum 'F_2,' using the value of 'L' found in Step 1.

Let 'P' lb. = pull exerted on specimen.

$$P \times 3 = L \times 18 \quad = 110 \times 18$$
$$\therefore P = \frac{110 \times 18}{3} = 660 \text{ lb.}$$

Calculation of Beam Support Reactions

An important application of 'moments' is in the calculation of the '*reactions*' which walls, columns, etc., exert on the beams they support—often the first step in the design of the beam.

Notation.—The evaluation of beam reactions occurs so frequently in structural calculations that it is necessary to adopt a definite system of symbols for the reaction forces.

If 'A' and 'B' be the left-end and right-end reaction points respectively of a beam 'AB,' the left-end reaction is conveniently represented by the symbol 'R_A.' Similarly, 'R_B' would stand for the '*reaction at B*.'

If, as in the case of continuous beams, there are, say, three support points 'A,' 'B' and 'C,' the reactions at these respective points would be 'R_A,' 'R_B' and 'R_C.' Sometimes in the case of loaded frameworks the letters 'A,' 'B,' etc., are used for other purposes. It may be more convenient in these cases to denote the left-end reaction by the symbol 'R_L,' and to use 'R_R' for the right-end reaction. This method is recommended by certain regulations as that to be adopted in all cases of simple beams, etc.

EXAMPLE.—*Calculate the reactions at 'A' and 'B' respectively for the simply supported beam given in Fig.* 83.

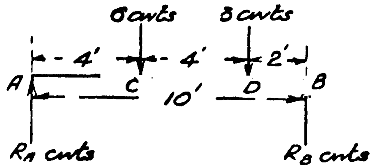


FIG. 83.

We first regard the beam as a simple lever with the fulcrum at 'B' and regard 'R_A' (the reaction at 'A') as an ordinary force acting upwards at 'A.'

Clockwise moment of 'R_A' about 'B' = ($R_A \times 10$) cwts. ft.

The sum of the anticlockwise moments of the loads on the beam about 'B' = [(6 × 6) + (3 × 2)] cwts. ft.

Equating clockwise and anticlockwise moments:

$$R_A \times 10 = (6 \times 6) + (3 \times 2)$$
$$10R_A = 36 + 6 = 42$$
$$R_A = 4{\cdot}2 \text{ cwts.}$$

The beam is now assumed to have its fulcrum at 'A,' so that 'R_A,' which passes through the fulcrum, will have no moment and therefore will be eliminated from the 'moments equation.'

Anticlockwise moment of 'R_B' about 'A' = ($R_B \times 10$) cwts. ft. The total clockwise moment of the beam loads about 'A' = [(3 × 8) + (6 × 4)] cwts. ft

$$\therefore R_B \times 10 = (3 \times 8) + (6 \times 4)$$
$$10R_B = 24 + 24 = 48$$
$$R_B = 4{\cdot}8 \text{ cwts.}$$

A check on the numerical working is provided by the addition of the two reaction values. Their sum should equal the total load on the beam.

$$R_A + R_B = 4{\cdot}2 \text{ cwts.} + 4{\cdot}8 \text{ cwts.} = 9 \text{ cwts.}$$
$$\text{Sum of loads} = 6 \text{ cwts.} + 3 \text{ cwts.} = 9 \text{ cwts.}$$

The reader is advised to work out the two reactions independently, as in this example, in order to take advantage of the check on the numerical working. The procedure of obtaining one reaction by subtracting the other from the 'load total' is not recommended to beginners. The method of allowing for the self-weight of a beam is given later.

Fig. 84 shows a form of apparatus devised to demonstrate 'beam support reactions.'

If a single central load be applied, each reaction will equal half the load value.

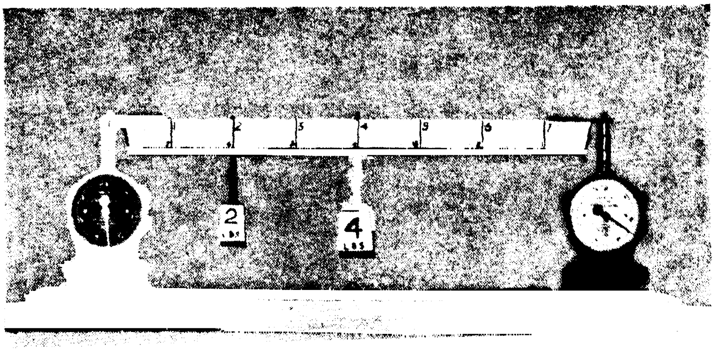


FIG. 84.

If a symmetrical system of loading be used, each reaction will equal half the total load value.

The compression spring balances, at the ends of the model beam, record the reactions in any given experimental test.

It will be noted from the readings of the spring balances that the left-end and right-end reactions are respectively $3\frac{1}{2}$ lb. and $2\frac{1}{2}$ lb. Checking these results by the calculation method we have:

$$R_L \times 8 = (2 \times 6) + (4 \times 4) = 12 + 16 = 28$$
$$\therefore R_L = 3{\cdot}5 \text{ lb.}$$
$$R_R \times 8 = (4 \times 4) + (2 \times 2) = 16 + 4 = 20$$
$$\therefore R_R = 2{\cdot}5 \text{ lb.}$$

Uniformly Distributed Loads.—Loads are often described in structural calculations as '*so much per unit run*,' e.g. '*2 cwts. per foot run of beam*,' or more simply '*2 cwts. per foot.*' In all problems of the present type, i.e. in which the equilibrium of a body is being considered, we may replace such a load system by one concentrated load of equal value at the middle point of the load

distribution. The self-weight of a beam may usually be regarded as a uniformly distributed load.

EXAMPLE.—*A 9″ × 4″ × 21 lb. B.S.B. is used to carry the loads given in Fig.* 85. *Calculate the support reactions.*

The total load per foot run = 21 lb. + 539 lb. = 560 lb. = ¼ ton.

The total uniformly distributed load on the beam = ¼ ton per foot × 8 ft. = 2 tons. We may regard this load as being (*for the present purpose*) concentrated at mid-point of beam.

Moments about B:

$$R_A \times 8 = (2 \times 6) + (2 \times 4) + (4 \times 2) + [2 \ (\textit{i.e. unif. dist. load}) \times 4]$$
$$= 12 + 8 + 8 + 8$$
$$= 36$$
$$R_A = 36/8 \text{ tons} = 4{\cdot}5 \text{ tons.}$$

Moments about A:

$$R_B \times 8 = (4 \times 6) + (2 \times 4) + (2 \times 2) + (2 \times 4)$$
$$= 24 + 8 + 4 + 8$$
$$= 44$$
$$R_B = 44/8 = 5{\cdot}5 \text{ tons.}$$

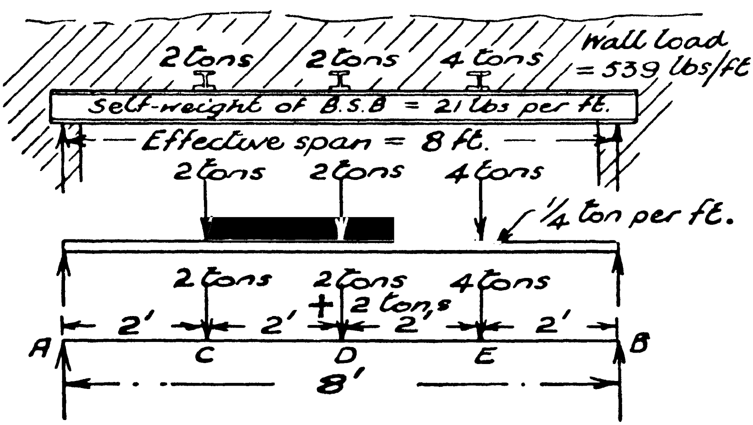


FIG. 85.

If the uniformly distributed load runs for the whole beam length, as in this case, we could find the reactions, in the first

instance, for the concentrated loads alone and then add to each reaction half the total uniform load.

EXAMPLE.—*Fig. 86 shows a roof truss carrying an unsymmetrical load system. Calculate the reactions at 'A' and 'B.'*

As has been emphasised previously, the 'arm' to be used in evaluating a moment is necessarily at right angles to the line of action of the force. We may therefore reduce this example to a simple beam problem as illustrated in Fig. 86.

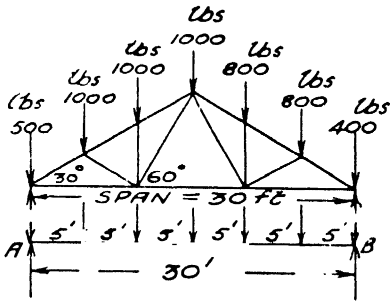


FIG. 86.—REACTIONS FOR A ROOF TRUSS.

There will be no need, ordinarily, to actually draw the equivalent beam in such examples.

Moments about B:

$$R_A \times 30 = (500 \times 30) + (1000 \times 25) + (1000 \times 20) + (1000 \times 15) + (800 \times 10) + (800 \times 5) + (400 \times 0)$$

$$30R_A = 15000 + 25000 + 20000 + 15000 + 8000 + 4000 = 87000$$

$$R_A = 2900 \text{ lb.}$$

Moments about A:

$$R_B \times 30 = (400 \times 30) + (800 \times 25) + (800 \times 20) + (1000 \times 15) + (1000 \times 10) + (1000 \times 5) + (500 \times 0)$$

$$= 12000 + 20000 + 16000 + 15000 + 10000 + 5000$$

$$30R_B = 78000$$

$$R_B = 2600 \text{ lb.}$$

$R_A + R_B = 5500$ lb. = total load on truss.

The self-weight of a truss is proportioned between the joint loads. (See Chapter VIII.)

EXAMPLE.—*Fig. 87 illustrates a beam overhanging its supports. Calculate the support reactions for the loads given.*

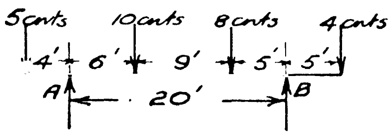


FIG. 87.

In this example we do not

take moments about the ends of the beam but about the support points 'A' and 'B' respectively. Considering the fulcrum to be at 'B,' it will be observed that the '4 cwts. load' has a clockwise moment, i.e. the same type of moment as 'R_A' has. These two moments will therefore have to be added.

Moments about B:

$$(R_A \times 20) + (4 \times 5) = (5 \times 24) + (10 \times 14) + (8 \times 5)$$
$$20R_A + 20 = 120 + 140 + 40$$
$$20R_A + 20 = 300$$
$$20R_A = 300 - 20 = 280$$
$$R_A = 14 \text{ cwts.}$$

Moments about A:

$$(R_B \times 20) + (5 \times 4) = (4 \times 25) + (8 \times 15) + (10 \times 6)$$
$$20R_B + 20 = 100 + 120 + 60$$
$$20R_B + 20 = 280$$
$$20R_B = 280 - 20 = 260$$
$$R_B = 13 \text{ cwts.}$$

$R_A + R_B = (14 + 13)$ cwt. $= 27$ cwts. $=$ total load on beam.

EXAMPLE.—*Find the reactions at the supports in the case of the beam shown in Fig.* 88.

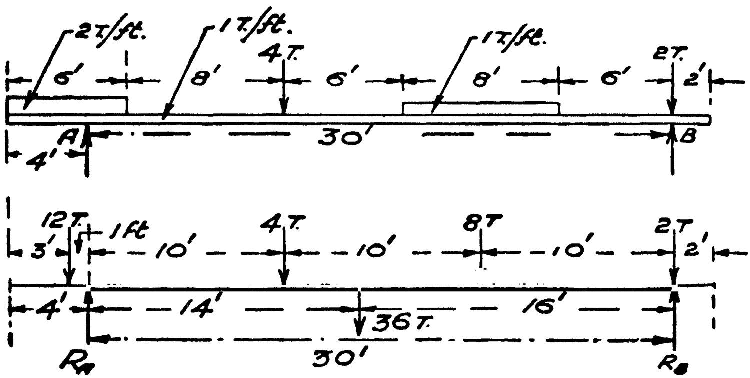


FIG. 88.

Using the rule for replacing uniformly distributed loads by concentrated loads of equal magnitude *for the purpose of beam reaction calculations*, the original load system is reduced to a much simpler form as shown in the lower diagram. When the method

is thoroughly understood there should be no actual need to draw a separate equivalent beam diagram.

The '2 ton per foot load,' 6 ft. long, is partly *to the left* and partly *to the right* of the support 'A.' When replaced by an equivalent load of 12 tons, the load appears at 1 ft. *to the left* of 'A.' There need be no hesitancy in accepting this. If we treated this load as consisting of two separate portions, one in and one out of the centre span, the net result would be exactly as indicated above.

Moments about B:

$$
\begin{aligned}
R_A \times 30 &= (12 \times 31) + (4 \times 20) + (36 \times 16) + (8 \times 10) + (2 \times 0) \\
&= 372 + 80 + 576 + 80 \\
&= 1108 \\
R_A &= 36{\cdot}93 \text{ tons.}
\end{aligned}
$$

Moments about A:

$$
\begin{aligned}
(R_B \times 30) + (12 \times 1) &= (2 \times 30) + (8 \times 20) + (36 \times 14) + (4 \times 10) \\
30R_B + 12 &= 60 + 160 + 504 + 40 \\
30R_B + 12 &= 764 \\
30R_B &= 764 - 12 = 752 \\
R_B &= 25{\cdot}07 \text{ tons.}
\end{aligned}
$$

$$R_A + R_B = (36{\cdot}93 + 25{\cdot}07) \text{ tons} = 62 \text{ tons} = \text{total load on beam.}$$

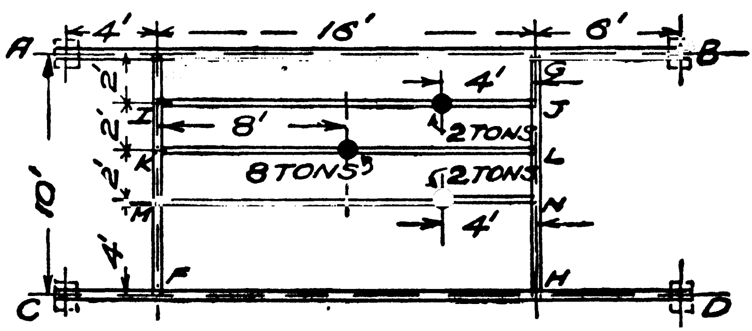


FIG. 89.—COLUMN LOADS.

EXAMPLE.—*Calculate the column loads at 'A,' 'B,' 'C' and 'D' (due to the concentrated loads indicated) in the example of the steel-framed floor given in Fig.* 89.

In such problems the calculated reactions for the secondary beams become the load values for the beams which carry them.

Beam MN: $R_M \times 16 = 2 \times 4 \quad \therefore R_M = 8/16 \text{ ton} \quad = \frac{1}{2} \text{ ton.}$
$R_N \times 16 = 2 \times 12 \therefore R_N = 24/16 \text{ tons} = 1\frac{1}{2} \text{ tons.}$

Beam KL: $R_K = R_L = 4$ tons.

Beam IJ: $R_I = R_M = \frac{1}{2}$ ton (under similar conditions).
$R_J = R_N = 1\frac{1}{2}$ tons ,, ,, ,,

Beam 'EF' takes the left-end reactions from the beams above (see Fig. 90).

Beam EF:
$$R_E \times 10 = (\tfrac{1}{2} \times 8) + (4 \times 6) + (\tfrac{1}{2} \times 4)$$
$$= 4 + 24 + 2 = 30$$
$$R_E = 3 \text{ tons.}$$
$$R_F \times 10 = (\tfrac{1}{2} \times 6) + (4 \times 4) + (\tfrac{1}{2} \times 2)$$
$$= 3 + 16 + 1 = 20$$
$$R_F = 2 \text{ tons.}$$

Beam 'GH' takes the right-end reactions of the beams it supports.

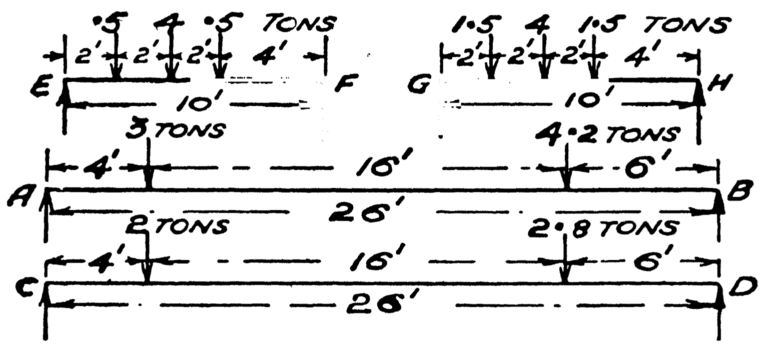


FIG. 90.—BEAM LOAD DIAGRAMS (see Fig. 89).

Beam GH:
$$R_G \times 10 = (1\tfrac{1}{2} \times 8) + (4 \times 6) + (1\tfrac{1}{2} \times 4)$$
$$= 12 + 24 + 6 = 42$$
$$R_G = 4{\cdot}2 \text{ tons.}$$
$$R_H \times 10 = (1\tfrac{1}{2} \times 2) + (4 \times 4) + (1\tfrac{1}{2} \times 6)$$
$$= 3 + 16 + 9 = 28$$
$$R_H = 2{\cdot}8 \text{ tons.}$$

Beams 'AB' and 'CD,' which are supported by the columns, carry the reaction loads transmitted by beams 'EF' and 'GH.'

Beam AB: $R_A \times 26 = (3 \times 22) + (4{\cdot}2 \times 6)$
$= 66 + 25{\cdot}2 = 91{\cdot}2$
$R_A = 3{\cdot}51$ tons.
$R_B \times 26 = (4{\cdot}2 \times 20) + (3 \times 4)$
$= 84 + 12 = 96$
$R_B = 3{\cdot}69$ tons.

Beam CD: $R_C \times 26 = (2 \times 22) + (2{\cdot}8 \times 6)$
$= 44 + 16{\cdot}8 = 60{\cdot}8$
$R_C = 2{\cdot}34$ tons.
$R_D \times 26 = (2{\cdot}8 \times 20) + (2 \times 4)$
$= 56 + 8 = 64$
$R_D = 2{\cdot}46$ tons.

Summary:

$R_A = 3{\cdot}51$ tons
$R_B = 3{\cdot}69$,,
$R_C = 2{\cdot}34$,,
$R_D = 2{\cdot}46$,,
TOTAL $= 12{\cdot}00$ tons

Total load on frame = 12 tons.

Further examples on 'reactions' appear throughout the book.

Moment of a Resultant Force

Theorem: The resultant moment of a system of forces about any given point in the plane of the forces is equal to the moment of the resultant of the system about the same point.

In Fig. 91 there is shown a system of forces 'F_1,' 'F_2,' etc., with 'R,' the resultant of the system, represented in its correct position in space. The point 'O' is any selected point in the plane of the forces.

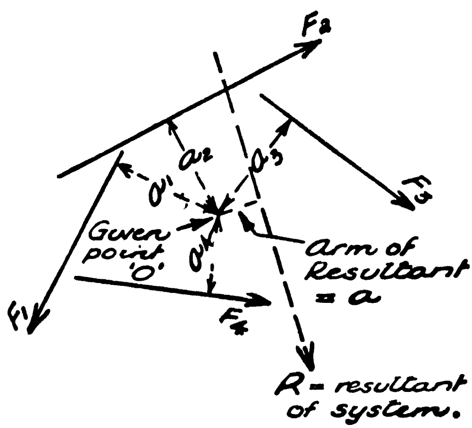


FIG. 91.

In this case 'R' has a clockwise moment about the point 'O.' Regarding clockwise moments as 'positive,' the theorem expressed in the form of an equation of moments becomes:

$$R \times a = (F_2 \times a_2) + (F_3 \times a_3) - (F_1 \times a_1) - (F_4 \times a_4).$$

This theorem is of great assistance in the solution of structural problems.

EXAMPLE.—*Verify the theorem above for the case of two intersecting forces, 7 lb. and 8 lb. respectively, whose lines of action contain an angle of* 60° (see Fig. 92).

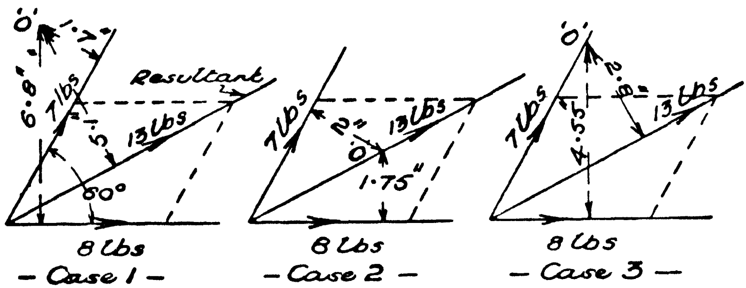


FIG. 92

The reader should carry through this exercise to a fairly large scale. In addition to verifying the theorem, it provides excellent practice in '*taking moments*.'

Three positions are taken for the point 'O.' The parallelogram of forces gives the resultant in its correct position in space. The 'arms' of the various moments are measured in actual inches on the drawing paper. The following results illustrate the significance of the theorem.

Case 1.—*Moments about 'O'*

7-lb. force: Force × arm = 7 lb. × 1·7 ins. = 11·9 lb. ins. A.C.W.
8-lb. force: Force × arm = 8 lb. × 6·8 ins. = 54·4 lb. ins. A.C.W.

Resultant moment (by addition) = 66·3 lb. ins. A.C.W.

13-*lb. force* (*Resultant*):

Force × arm = 13 lb. × 5·1 ins. = 66·3 lb. ins. A.C.W.

∴ **Sum of component moments = moment of resultant.**

Case 2.—Moments about 'O'

7-lb. force: Force × arm = 7 lb × 2 ins. = 14 lb. ins. C.W.
8-lb. force: Force × arm = 8 lb. × 1·75 ins. = 14 lb. ins. A.C.W.

Resultant moment (by algebraic addition) = 0

13-lb. force (Resultant):

Force × arm = 13 lb. × 0 in. = 0 lb. ins.

∴ **Resultant moment = moment of resultant.**

Case 3.—Moments about 'O'

7-lb. force: Force × arm = 7 lb. × 0 ins. = 0 lb. ins.
8-lb. force: Force × arm = 8 lb. × 4·55 ins. = 36·4 lb. ins. A.C.W.

Resultant moment (by addition) = 36·4 lb. ins. A.C.W.

13-lb. force (Resultant):

Force × arm = 13 lb. × 2·8 ins. = 36·4 lb. ins A C.W.

∴ **Resultant moment = moment of resultant.**

In each case, therefore, the theorem has been verified. Exercise 12 at the end of the chapter involves the verification of the theorem for a system of four concurrent forces.

The theorem is extensively employed in Chapter VI.

EXAMPLE. — *Fig.* 93 *shows a masonry pier weighing* 9000 *lb. A horizontal force 'P lb.' acts at* 3 *ft. from the base of the pier. It is desired that the resultant of 'P' and the weight* (9000 *lb.*) *shall pass through a given point 'A,'* 6 *ins. from the centre of the base. Find the corresponding value of 'P.'*

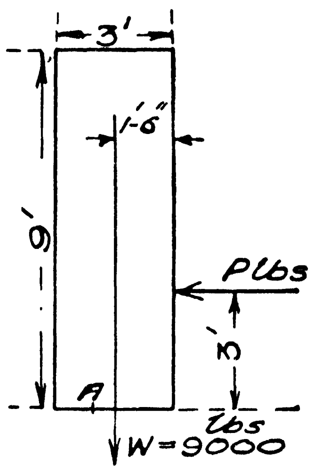


FIG 93

If the resultant of 'P' and 'W' passes through 'A,' it will have no moment about 'A.' Hence, by the theorem above, the resultant moment of 'P' and 'W' will be zero about 'A,' i.e. the moment of 'P' must be *equal and opposite to that of 'W.'*

$$\therefore P \times 3 \text{ (A.C.W.)} = W \times \cdot 5 \text{ (C.W.)}$$
$$P \times 3 = 9000 \times \cdot 5$$
$$\therefore P = 1500 \text{ lb.}$$

This example illustrates a typical 'retaining wall problem.' Retaining walls are considered in detail in Chapter XV.

Exercises 4

(1) Write down the moment of the given force about the fulcrum 'O' in each of the cases shown in Fig. 94.

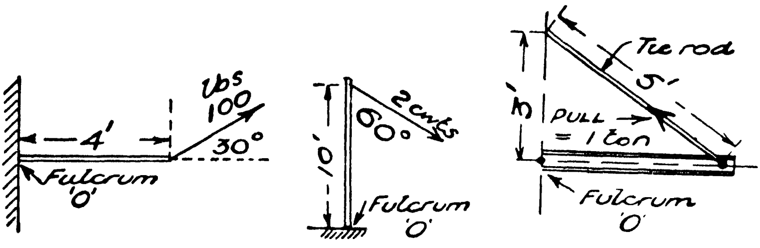


Fig. 94.

(2) A nut requires an applied moment of 120 lb. ins. in order to turn it. Calculate the necessary effort if applied as in (*a*) and (*b*) respectively, in Fig. 95.

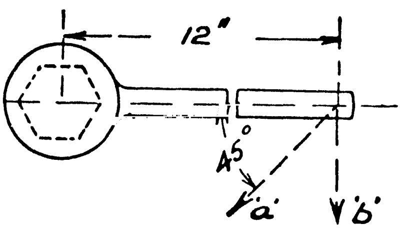


Fig. 95.

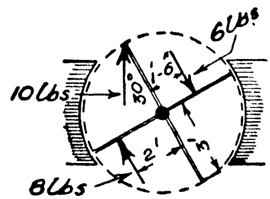


Fig. 96.

(3) A swing door is pushed by three people in the manner indicated in the diagram shown in Fig. 96. Calculate the resultant turning moment on the door.

(4) Fig. 97 gives a diagram of the lever arm of a timber beam-testing machine. Assuming the lever arm to be balanced by the compensating weight when the travelling weight is at the zero of the scale, calculate the length in inches of the scale graduations, which represent 1-cwt. pull on specimen.

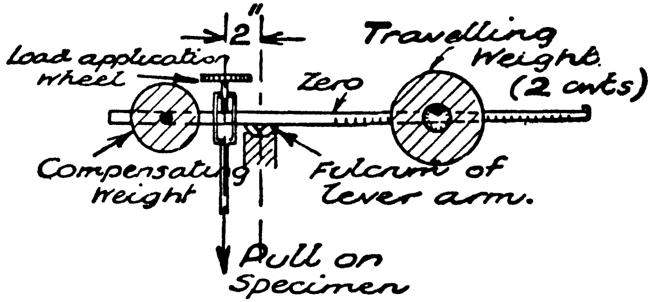


FIG. 97.

(5) A plank, resting on two scaffold poles, is lashed to one and is free at the other (Fig. 98). Calculate the pull on the lashing when the plank carries the loads indicated. Neglect self-weight of plank.

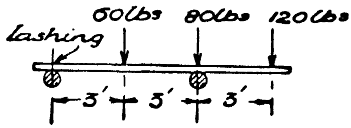


FIG. 98.

(6) Fig. 99 shows a north-light roof truss carrying wind loads. Find the value of the vertical reaction at the left end of the truss. (Take moments about right end of truss.)

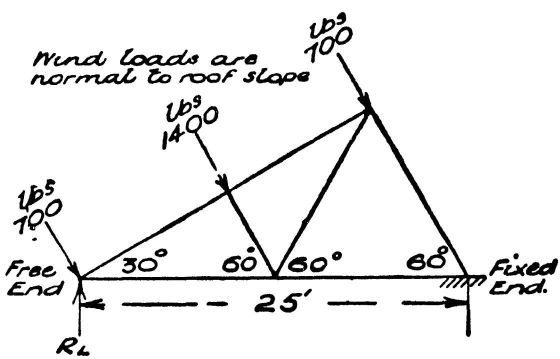


FIG. 99

(7) Calculate the support reactions (for the loads indicated) for each of the beams shown in Fig. 100.

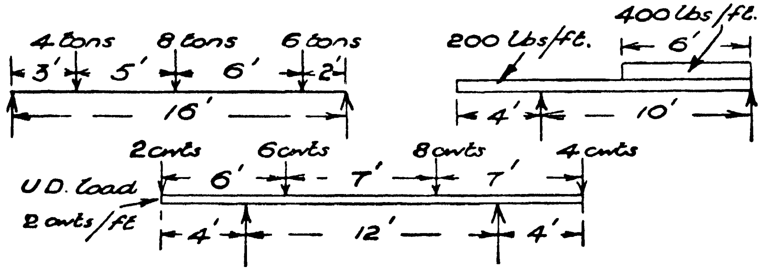


FIG. 100.

(8) Find the support reactions for the steel beam given in Fig. 101. The density of the brickwork is 1 cwt. per cu. foot. The wall is 9 ins. thick. Neglect the self-weight of beam.

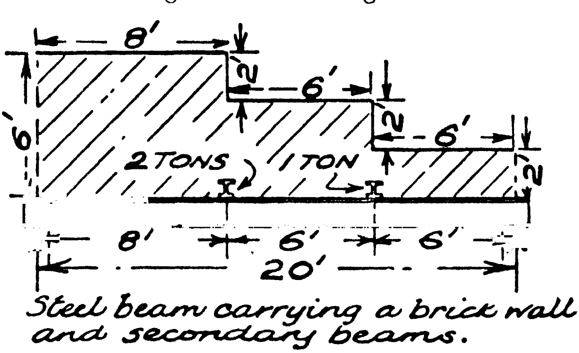


FIG. 101.

(9) A roof truss carries rafter and tie loads as shown in Fig. 102. Calculate the reactions at the supports.

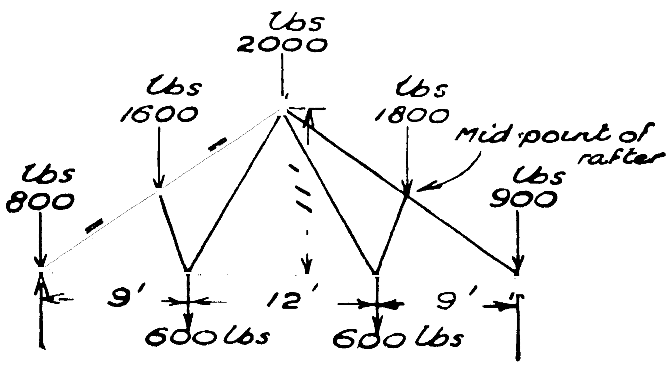


FIG. 102.

(10) Fig. 103 shows a girder 'AB' along which a weight of 2 tons is able to travel. The subsoil under the concrete at end 'A' must not (due to the travelling weight alone) be subjected to a higher pressure than 280 lb. per sq. foot. Calculate the minimum permissible distance between the weight and the centre line of the column.

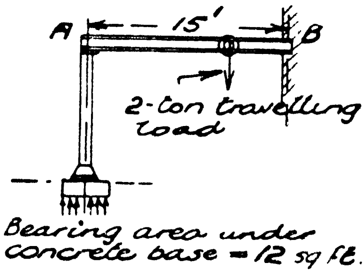


FIG. 103

(11) Find the resultant moment *about the point 'P'* of all the forces which act on the beam (Fig. 104) to the left of 'P.' Similarly find the resultant moment of all the forces which act to the right of 'P.' In which respect do these two resultant moments (i) agree, (ii) differ?

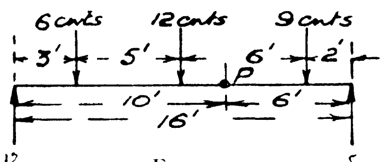


FIG 104.

Repeat the exercise, taking any other point on the beam.

(See page 198 for an application of this exercise.)

(12) Show that the resultant of the co-planar system of concurrent forces given in Fig. 105 will have the same moment about point 'A' as the algebraic sum of the moments of the four forces (taken separately) about point 'A.' Select a suitable position for point 'A,' in the approximate position shown in figure. Repeat, taking point 'A' on one of the forces.

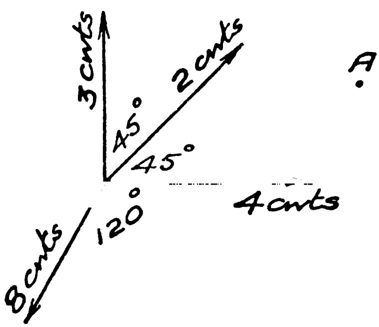


FIG. 105.

(13) Calculate the load carried by each of the columns at 'A,' *'B,' 'C,' and 'D' respectively (Fig. 106), due to the given point load of 6 tons on the secondary beam.*

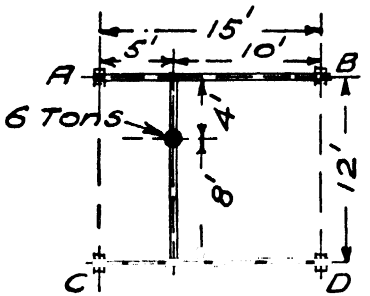


FIG. 106.

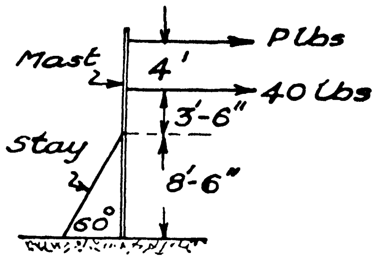


FIG. 107.

(14) Fig. 107 shows a mast supporting two horizontal pulls. The mast is kept vertical by a back stay. Assuming the maximum safe load for the stay to be 400 lb., find the highest permissible value of 'P.'

The bottom of the mast may be considered to be a pin-jointed connection.

CHAPTER V

PARALLEL FORCE SYSTEMS. COUPLES. GENERAL CONDITIONS OF EQUILIBRIUM

PARALLEL force systems may be divided into two classes:

(i) *Like parallel systems*, in which all the forces act in the same direction.

(ii) *Unlike parallel systems*, in which some of the forces act in the opposite direction to the others. (See Fig. 108.)

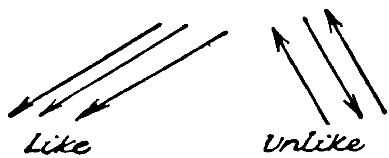


FIG 108.—PARALLEL FORCE SYSTEMS.

Like parallel systems are the more common in structural problems, but there is one case of an unlike parallel system which is of especial importance. The theorem given on page 63 with respect to resultant moments is also true for parallel force systems. This may be verified by means of the experimental apparatus shown in Fig. 109. (See also Fig. 136.)

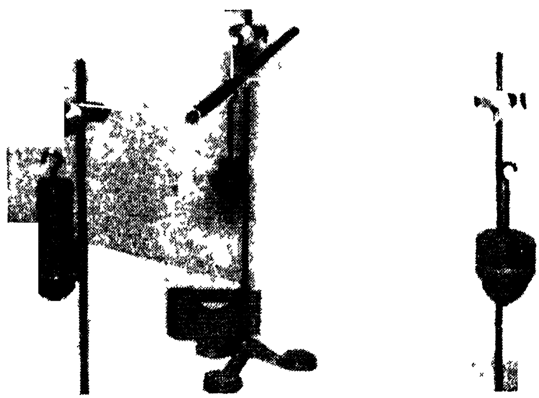


FIG 109.—PARALLEL FORCES.

If we reverse the direction of the equilibrant of the system, as found in the experiment, we have a parallel force system with its resultant. The resultant will be found to be equal in magnitude to the sum of the forces, act in a direction parallel to the forces,

and have its line of action in such a position as to comply with the provisions of the theorem referred to

Like Parallel Systems

Two Like Parallel Forces

EXAMPLE —*Find the magnitude, direction and position of the resultant of the two like parallel forces given in Fig 110*

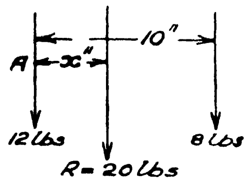


FIG 110

Magnitude of resultant

= sum of forces
= 12 lb + 8 lb = 20 lb

Direction Vertically downwards

Position Let 'x' ins = distance from point 'A' (on the line of action of the '12-lb' force)

Moments about 'A'

Moment of resultant = 20 lb × x ins = 20x lb ins
Sum of moments of components = (12 × 0) + (8 lb × 10 ins)
= (0 + 80) lb ins = 80 lb ins

$$\therefore\ 20x = 80$$
$$x = 4 \text{ ins}$$

It will be noted that the line of action of the resultant force divides the distance between the forces *inversely* as the forces

Thus $\dfrac{4 \text{ ins}}{6 \text{ ins}} = \dfrac{8 \text{ lb}}{12 \text{ lb}}$

This rule is used in the graphical problem explained on page 106

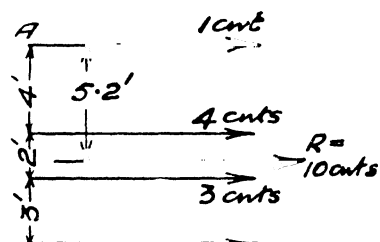


FIG 111

Several Like Parallel Forces

EXAMPLE —*Find the resultant of the like parallel system shown in Fig.* 111

Magnitude of resultant = (2 + 3 + 4 + 1) cwts
= 10 cwts

Direction The resultant will act horizontally towards the right

Position: Let 'x' ft. = distance from point 'A' on the '1-cwt.' force.

Moments about 'A':

$$\begin{aligned} 10 \times x &= (2 \times 9) + (3 \times 6) + (4 \times 4) \\ &= 18 + 18 + 16 = 52 \\ x &= 5{\cdot}2 \text{ ft.} \end{aligned}$$

Note that in writing down the equations we are not equating clockwise to anticlockwise moments, as in the principle of the lever, but merely expressing the equality of resultant moments, obtained in two different ways.

EXAMPLE.—*Fig.* 112 *shows a roof truss carrying wind loads on the windward side. Find the resultant wind thrust on the roof slope.*

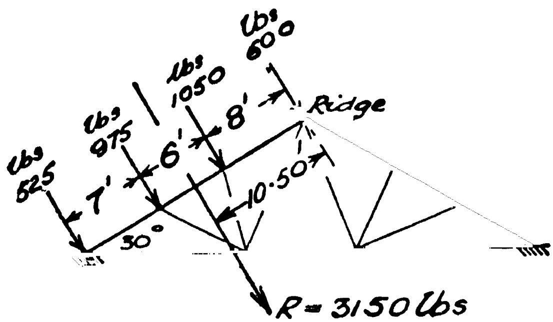


FIG 112

Magnitude of resultant = (525 + 975 + 1050 + 600) lb.
= 3150 lb.

Direction: Normal to roof slope, acting inwards.

Position: Let 'x' ft. = distance, from the ridge, at which the resultant acts.

Taking moments about the ridge:

$$\begin{aligned} 3150x &= (600 \times 0) + (1050 \times 8) + (975 \times 14) + (525 \times 21) \\ &= 0 + 8400 + 13650 + 11025 \\ &= 33075 \\ x &= \frac{33075}{3150} \text{ ft.} = 10{\cdot}50 \text{ ft.} \end{aligned}$$

Unlike Parallel Systems

Two Unlike Parallel Forces

EXAMPLE.—*A dock gate, with water on both sides, is subject to the two horizontal thrusts shown in Fig.* 113. *Find the resultant water thrust on the gate.*

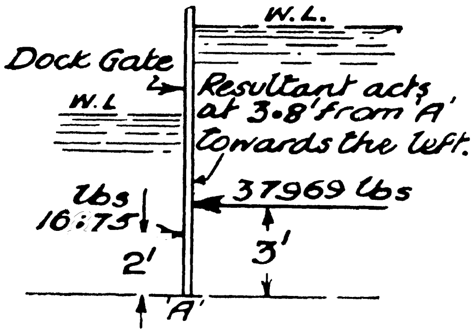


FIG 113.

Magnitude of resultant = the 'algebraic' sum of the forces
= (37969 − 16875) lb.
= 21094 lb.

Direction: The resultant acts towards the left, at right angles to the gate.

Position: Let 'x' ft. = distance from bottom of gate ('A').

$$\begin{array}{cccc} 21094x = & [(37969 \times 3) & - & (16875 \times 2)] \\ \text{A.C.W.} = & \text{A.C.W.} & - & \text{C.W.} \end{array}$$

Note that the kind of moment the resultant produces (A.C.W. in this case) is given the positive sign.

$$21094x = (113907 - 33750) = 80157$$
$$x = 3{\cdot}8 \text{ ft.}$$

In the case of two unlike and unequal parallel forces, the resultant does not act between the forces, but beyond the greater force.

Several Unlike Parallel Forces

EXAMPLE.—*Find the resultant of the unlike parallel system given in Fig.* 114.

Magnitude of resultant = algebraic sum of forces

$= (5 + 6 - 2 - 4 - 1)$ cwts.

$= 4$ cwts.

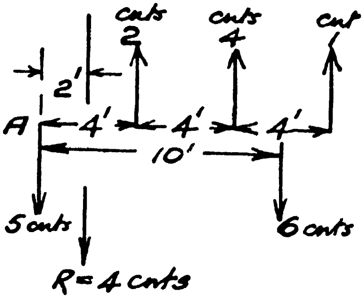


FIG. 114.

Direction: Vertically downwards.

Position: Let 'x' ft. = distance from the point 'A' (on the '5 cwts.' force) measured towards the right.

$$(4 \times x) = (6 \times 10) - (2 \times 4) - (4 \times 8) - (1 \times 12)$$
$$= 60 - 8 - 32 - 12$$
$$4x = 8$$
$$x = 2 \text{ ft.}$$

Couples

If, in such an example as the one solved above, *the sum of the forces acting upwards equalled the sum of those acting downwards,* two possibilities would ensue :

(i) If the upward and downward resultant forces acted in the same vertical line, the system would be in equilibrium.

(ii) If they acted out of line the net effect of the system would be one of *rotation.* A system of forces, such as that suggested in (ii), is known as a '*couple.*'

Definition: A couple consists of two equal, unlike parallel forces, acting out of line.

The effect of a couple on a body is one of pure rotation.

Moment of a Couple.—The 'moment of a couple' measures the actual turning tendency the couple has on the body to which it is applied. It is obtained by multiplying one of the forces by the perpendicular distance between them (Fig. 115).

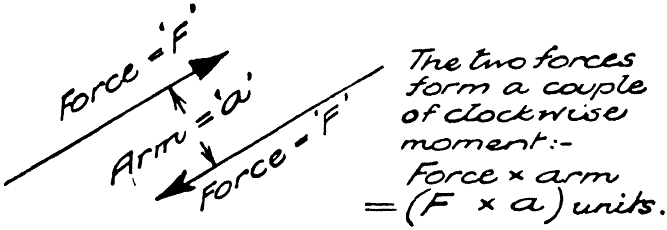


FIG. 115.

Moment of couple = Force × arm of couple.

A couple has a '*clockwise*' or an '*anticlockwise*' moment according to the manner in which it tends to cause rotation (Fig 116).

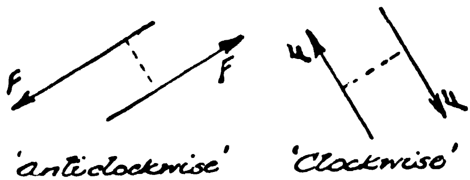


FIG. 116.—TYPES OF COUPLES.

Practical Examples of Couples.—Whenever we measure the turning effect a force has on a body, as in the simple lever problems in Chapter IV, we are really finding the moment of a couple. The reaction at the point of support of the body, i.e. the fulcrum, provides the 'equal and unlike parallel force' to form the couple with the net applied force.

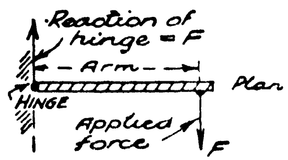


FIG. 117.

Example (i).—Fig. 117 shows a force applied to the handle of a door. The hinge provides a reaction which forms a couple with the applied force. The door opens under the moment of this couple.

Example (ii).—A very important application of couples occurs in beam problems (see Chapter XII). When a beam deflects, the fibres of the beam material become stretched or compressed according to their position in the beam. These fibres act like tiny springs, and they resist deformation. At a beam section, therefore, we have, acting across the section, a series of thrusts and pulls. The resultant

thrust and the resultant pull form a couple which resists the bending of the beam at the section (see Fig. 118). The moment of this couple is termed the '*moment of resistance*' at the section (see page 240).

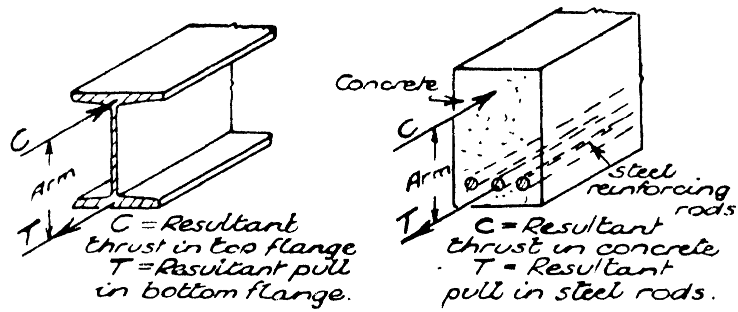


FIG. 118.—COUPLES IN BEAMS.

Important Properties of Couples.—There are a number of 'theorems' enunciated with respect to the subject of '*couples*.' These have applications in beam problems and in column problems (e.g. in '*eccentric loading*'), and will be referred to, as necessary, later. The reader will at this stage be able to appreciate the following facts about the nature of 'couples.'

(i) A couple cannot be balanced by a single force. To bring to equilibrium a body acted upon by a couple, it is necessary to apply to the body another couple of *equal* and *opposite* moment (see page 240).

(ii) A couple has the same moment about any point in the plane of the forces. The value of the moment is that of the couple itself.

Consider the cases shown in Fig. 119. The resultant turning effect or 'moment' about the point 'O' in each case equals the

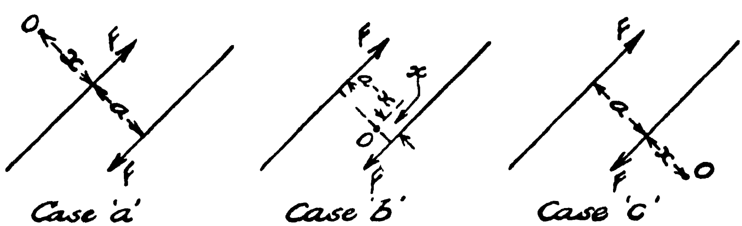


FIG. 119.

algebraic sum of the two moments of the respective forces 'F' forming the couple.

Case (*a*). Resultant moment about 'O' $= F \times (a + x) - F \times x$
$$= Fa + Fx - Fx$$
$= Fa$ (clockwise) $=$ *moment of couple.*

Case (*b*). Resultant moment about 'O' $= Fx + F(a - x)$
$$= Fx + Fa - Fx$$
$= Fa$ (clockwise) $=$ *moment of couple.*

Case (*c*). Resultant moment about 'O' $= F(a + x) - Fx$
$$= Fa + Fx - Fx$$
$= Fa$ (clockwise) $=$ *moment of couple.*

An important application of this theorem occurs later in this chapter in connection with the general conditions of equilibrium.

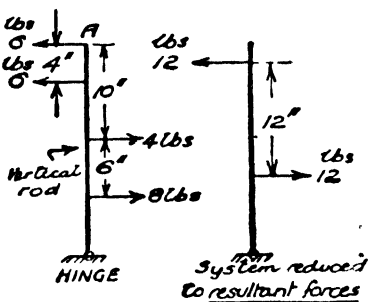


FIG. 120.

EXAMPLE.—*A vertical rod is hinged at the bottom (Fig.* 120). *Calculate the turning moment on the rod (about the hinge) for the given system of loads.*

Apparently the dimensions given are insufficient as the position of the hinge is not defined. The force system, however, reduces to a couple, and all we require to find is the moment of the couple (*theorem* ii, page 77).

To illustrate the principles of parallel force systems this problem will be solved in two ways.

(i) *By finding the 'force' and 'arm' of the couple to which the system reduces.*

The resultant force to the left is (6 + 6) lb. = 12 lb., acting in position shown.

The resultant force to the right is (8 + 4) lb. = 12 lb., acting in position shown.

The arm of the couple is the distance between the lines of action of these two forces = 12 ins. (The reader should check this figure.)

Moment of couple $=$ Force $\times$ arm
$= 12$ lb. $\times$ 12 ins.
$= 144$ lb. ins. (A.C.W.).

(ii) *By taking moments of the forces in the system about 'any' convenient point.*

The point 'A' (see Fig. 120) is chosen.

Resultant moment about 'A'

$$= [(4 \times 10) + (8 \times 16) - (6 \times 4)] \text{ lb. ins.}$$
$$= (40 + 128 - 24) \text{ lb. ins.}$$
$$= 144 \text{ lb. ins. (A.C.W.).}$$

The latter method is clearly the easier to employ.

The moment of the couple is therefore 144 lb. ins. (A.C.W.), and this is the required turning effect of the force system about the hinge.

EXAMPLE (ii).—*A rigid T-shaped member is hinged at the point 'O' (Fig. 121). Assuming a load of* 10 *cwts. to be applied as shown, calculate the necessary values of the forces 'C' and 'T' respectively in order to maintain the member in equilibrium. (Neglect the self-weight of the lever.)*

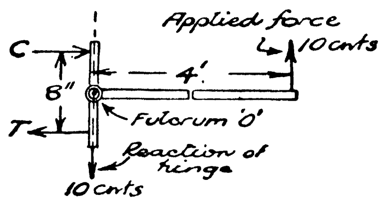


FIG. 121.

For vertical equilibrium it is necessary for the hinge at 'O' to exert a force of '10 cwts.' on the member, vertically downwards. The two forces of '10 cwts.' constitute a couple of anticlockwise moment: 'force $\times$ arm' $= 10$ cwts. $\times$ 4 ft. $= 40$ cwts. ft.

To maintain equilibrium, 'C' and 'T' must form a couple of clockwise moment $= 40$ cwts. ft.

The forces 'C' and 'T' are therefore equal and

$$C \text{ (or T)} \times 8 \text{ ins.} = (40 \times 12) \text{ cwts. ins.}$$

$$\therefore C = T = \frac{40 \times 12}{8} \text{ cwts.} = 60 \text{ cwts.}$$

The reader should compare this problem with the beam problem discussed on page 243.

Conditions of Equilibrium for a Non-concurrent Force System

Consider the force system shown in Fig. 122. If we resolve both forces, horizontally, by the cosine rule, we get 'F cos θ' acting towards the left and 'F cos θ' acting towards the right. There is no net horizontal force, therefore the system is in '*horizontal equilibrium.*'

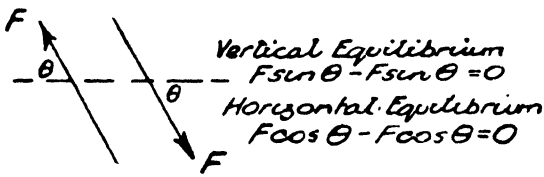


FIG. 122.

Resolving vertically we get 'F sin θ' up and 'F sin θ' down. The system is therefore also in '*vertical equilibrium.*'

We can see that the given system is not in '*complete equilibrium*' as it forms a couple, and would cause rotation.

If we were given a more complicated system of non-concurrent forces we could not decide so easily whether it reduced to a 'resultant force,' or to a 'couple' or was, in fact, 'in equilibrium.' To show that a non-concurrent system of forces is in equilibrium we have to resolve the forces into their vertical and horizontal components and show that the total horizontal component equals zero and that the total vertical component equals zero. If such were the case the system would not reduce to a single resultant force. But we still have to show that the system does not reduce to a couple. To do this, we may select *any* convenient point in the plane of the forces, and having taken the moments of all the forces about this point, verify that the total clockwise moment equals the total anticlockwise moment.

The position of the point about which moments are taken is immaterial because a couple has the same moment about any point in the plane of the forces. If, therefore, we get zero moment about any convenient selected point, there is no possibility of the system reducing to a couple.

For a co-planar system of forces to maintain a body in equilibrium

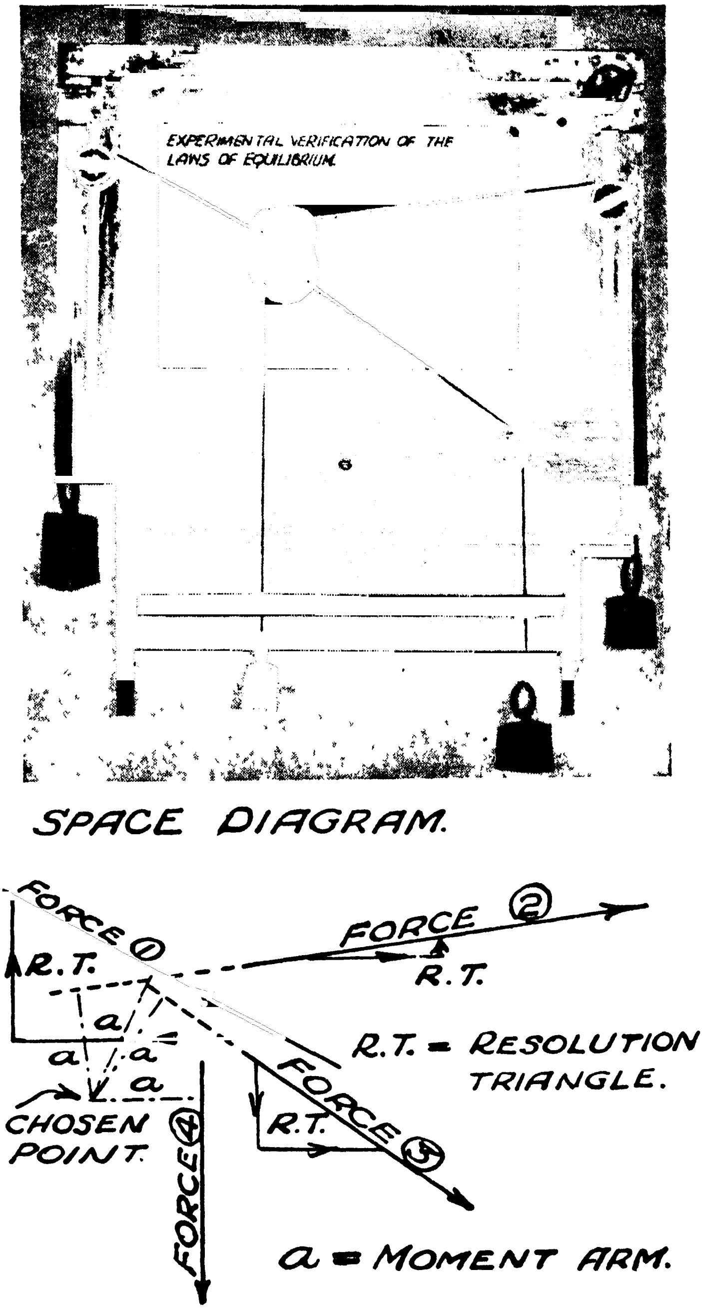


FIG 123 (see also Fig. 123A).—LAWS OF EQUILIBRIUM.

there must be (i) no resultant force acting on the body and (ii) no resultant moment about any point in the plane of the forces.

Experimental Verification of the Laws of Equilibrium

A suitable form of apparatus is shown in Fig. 123. The weights suspended over the pulleys should be fairly large in order that the weight of the thin disc of cardboard may be neglected. A space diagram of four non-concurrent forces in equilibrium is obtained. Each force is resolved into its respective vertical and horizontal components. These are then tabulated in the manner indicated in Fig. 123A. The convention of signs adopted for force direction is indicated.

A suitable point is chosen and the moment of each force is taken about it. Moments are deemed to be 'plus' or 'minus' according as to whether they are 'clockwise' or 'anticlockwise'

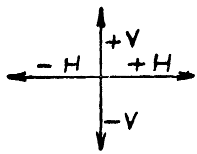


TABLE OF COMPONENTS.

	FORCE	HORIZONTAL COMP.		VERTICAL COMP		ARM	MOMENT.	
	LBS	+ LBS	− LBS	+ LBS	− LBS.	INS.	+ LBS INS.	− LBS. INS.
1	4		3·50	1·90		3·80		15·20
2	2	1·95		0·26		2·80	5·60	
3	2	1·55			1·16	3·30	6·60	
4	1				1·00	3·00	3·00	
	TOTALS	3·50	3·50	2·16	2·16		15·20	15·20

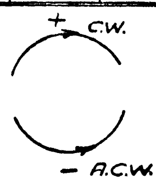


FIG. 123A (see Fig. 123).

respectively. The moment values are tabulated as shown in the figure. The totals given at the foot of the table verify the conditions of equilibrium.

Principle of Moments

In the problems we have hitherto taken, illustrating the '*principle of the lever*,' the structural unit concerned had a definite fulcrum about which turning was considered. The 'principle of moments' is much wider in its application. We may now imagine the fulcrum to be *anywhere* in space in the plane of the forces, not necessarily on the member concerned. The choice of a fulcrum about which to take moments is just a question as to which is the best position from the point of view of assisting in the solution of the problem in hand. If several of the unknown forces in a question pass through one common point, this point is usually a suitable one about which to take moments, as these unknown forces will then not appear in the '*moments equation*.' We may have to choose several points before solving all the unknowns, but, of course, the other conditions of equilibrium are also utilised as desirable. Care must be taken to ensure that all the forces in the problem are considered in taking moments, including any reactions whose lines of action do not pass through the chosen '*fulcrum*.'

Summary of 'Equilibrium Conditions'

Concurrent Forces

(i) The algebraic sum of all the horizontal components of the forces must equal zero, i.e. the total sum to the *left* must equal the total sum to the *right*.

(ii) The algebraic sum of all the vertical components of the forces must equal zero, i.e. the total sum *upwards* must equal the total sum *downwards*.

Non-concurrent Forces

The two conditions given for concurrent forces must be satisfied and, in addition, the algebraic sum of all the moments taken about any point in the plane of the forces must equal zero, i.e. the sum of all the clockwise moments must equal the sum of all the anticlockwise moments.

Examples on 'Conditions of Equilibrium'

(i) *A bracket (the self-weight of which may be neglected) is hinged in the manner indicated in Fig.* 124. *Calculate the value of the*

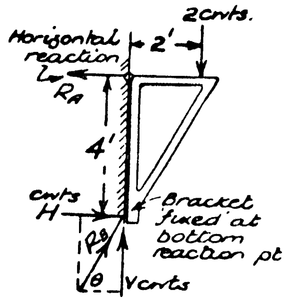


FIG. 124.

horizontal reaction at the upper hinge and the magnitude and direction of the reaction at the lower hinge.

Let 'V' cwts. and 'H' cwts. be the vertical and horizontal components respectively of the reaction at the lower hinge, and let 'R_A' = horizontal reaction at the upper hinge.

Expressing the condition for *vertical* equilibrium :

$$V \text{ cwts. (up)} = 2 \text{ cwts. (down)}$$
$$\therefore V = 2 \text{ cwts.}$$

The condition for *horizontal* equilibrium gives :

$$R_A \text{ (left)} = H \text{ (right)}$$

Taking moments about the lower hinge point—chosen because it gets rid of the completely unknown lower hinge reaction from the 'moments equation' :

$$(2 \text{ cwts.} \times 2 \text{ ft.}) \text{ C.W.} = (R_A \times 4 \text{ ft.}) \text{ A.C.W.}$$
$$\therefore R_A = 1 \text{ cwt.}$$

But $H = R_A$. $\qquad \therefore H = 1$ cwt.

Having found 'H' and 'V,' we can find 'R_B,' the total reaction at the lower hinge, by the parallelogram of forces :

$$R_B^2 = H^2 + V^2$$
$$= 1^2 + 2^2 = 5$$
$$\therefore R_B = \sqrt{5} \text{ cwts.} = 2{\cdot}24 \text{ cwts.}$$

If 'θ' be the angle made by the lower reaction with the horizontal,

$$\tan\theta = V/H = 2/1 = 2$$
$$\therefore \theta = 63^\circ\ 26'.$$

(ii) *Calculate the left-end and right-end reactions respectively for the roof truss (given in Fig* 125), *due to the* 600-*lb. resultant wind load. The left-end reaction may be assumed to be vertical.*

This problem will be solved in various ways in order to illustrate different methods of attack in such questions.

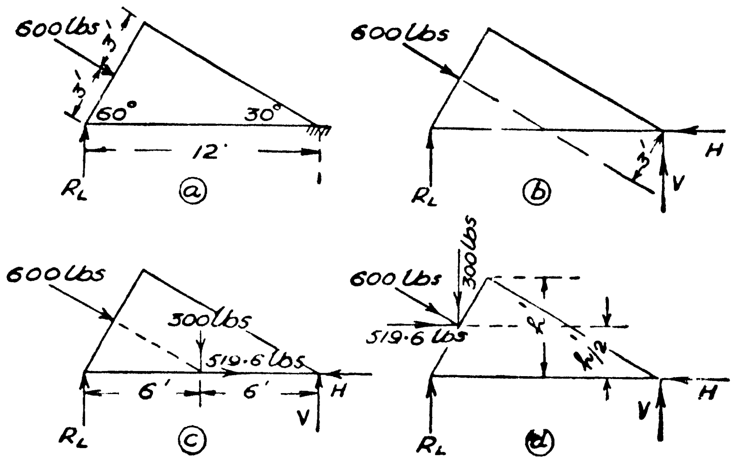


FIG 125 – REACTIONS BY VARIOUS METHODS

First Method (Figs. '*a*' and '*b*').

Taking moments about the right-end reaction point :

$$R_L \times 12 = 600 \times 3$$
$$\therefore R_L = 150 \text{ lb.}$$

Replace the right-end reaction by components 'V' and 'H.'

The vertical component of the 600-lb. load = 600 cos 60° = 300 lb.

For vertical equilibrium: $R_L + V = 300$.

$\therefore 150 + V = 300. \quad \therefore V = 150$ lb.

For horizontal equilibrium: H = 600 cos 30°.

$\therefore H = 600 \times \cdot 866 = 519.6$ lb.

$$R_R = \sqrt{150^2 + 519{\cdot}6^2} = 540{\cdot}8 \text{ lb.}$$

Let 'θ' = angle 'R_R' makes with the horizontal.

$$\tan \theta = \frac{150}{519{\cdot}6} = \cdot 2887$$
$$\theta = 16^\circ\ 6'.$$

Second Method (Fig. '*c*')

The 600-lb. force may be resolved into its vertical and horizontal components at any convenient point in its line of action. A simple solution of the problem may be effected by resolving

the force at the point where its line of action cuts the bottom tie.

$R_L = V = \frac{300 \text{ lb.}}{2} = 150$ lb., as the vertical component '300 lb.' acts at the mid-point of the tie.

For horizontal equilibrium, $H = 600 \times \cos 30° = 519{\cdot}6$ lb.

R_R is found, as in first method, from its components 'V' and 'H.'

Third Method (Fig. '*d*')

Resolve the '600-lb.' load into its horizontal and vertical components at the point of application of the load on the rafter.

Let 'h' ft. = height of truss

$$h = 6 \sin 60° = 6 \times {\cdot}866 = 5{\cdot}196.$$

Taking moments about right-end reaction point:

$$(R_L \times 12) + \left(519{\cdot}6 \times \frac{5{\cdot}196}{2}\right) = 300 \times 10{\cdot}5$$

$$12R_L = 3150 - 1350 = 1800$$

$$R_L = 150 \text{ lb.}$$

V (right-end reaction) $= 300 - R_L = 300 - 150 = 150$ lb.

$H = 519{\cdot}6$ lb. (horizontal equilibrium). The right-end reaction is compounded of 'H' and 'V' as before.

The results by all the three methods agree. The best method to employ in any given case will depend upon the particular type of problem.

Method 2 gave the quickest solution in this example, because the line of action of the '600-lb.' load cut the bottom tie in a very convenient point, i.e. at centre of span.

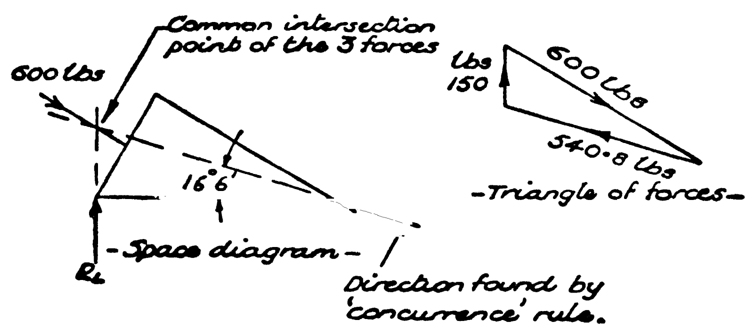


FIG. 126.

A final check of the problem will be made by the graphical method shown in Fig. 126.

Further examples on the application of the conditions of equilibrium will be found throughout the book.

EXERCISES 5

(1) Find the magnitude, direction and position of the resultant, in each of the two cases of *like* parallel force systems shown in Fig. 127.

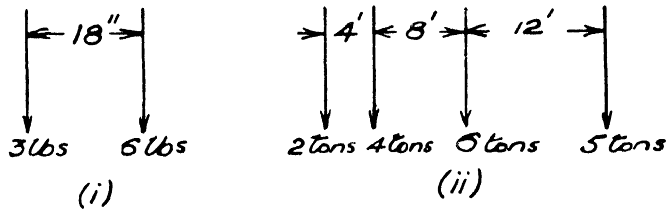


FIG 127.

(2) In Fig. 128 are given two *unlike* parallel force systems. Find, completely, the resultant of each of these respective systems.

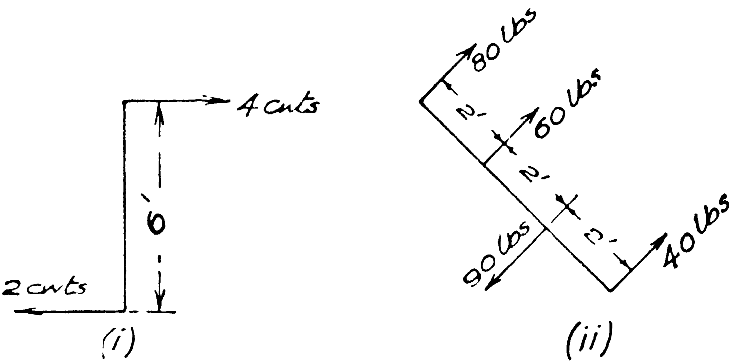


FIG 128.

(3) Find the resultant of the vertical load system carried by the simply supported beam shown in Fig. 129.

Calculate the support reactions for the beam (i) by taking the loads as given, (ii) by assuming the beam to carry the single resultant load calculated.

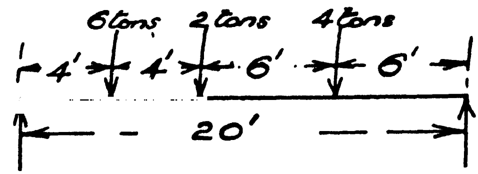


FIG. 129.

(4) A roof truss is subjected to the system of wind '*suction*' loads shown in Fig. 130. Find the resultant wind force on the truss due to suction effect.

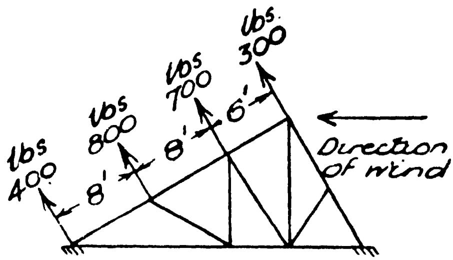


FIG 130.

(5) In designing a compound grillage to support several columns in line, it is desirable to have the resultant column load at the centre of length of the grillage. Assuming the grillage to extend 2 ft. beyond the centre line of the left-hand column (Fig. 131), find the necessary overall length of the grillage.

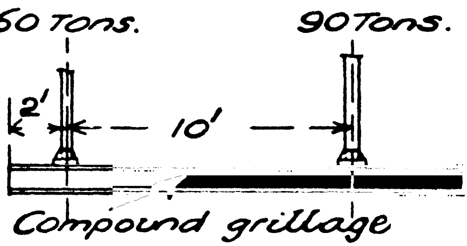


FIG 131.

(6) A system of moving loads (Fig. 132) crosses a beam of 20-ft. span. How far is the resultant of the load system from the right end of the beam when the left-hand wheel of the system is 5 ft. from the left end of the beam ?

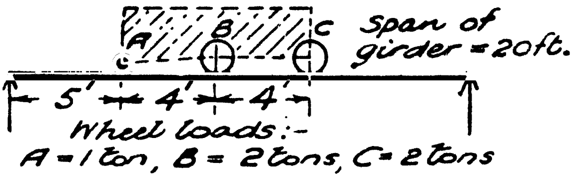


FIG. 132

(7) Find the moment of the couple to which the system of forces given in Fig. 133 reduces (i) by finding the *force* and *arm* of the couple, (ii) by taking moments of the given forces about any convenient point.

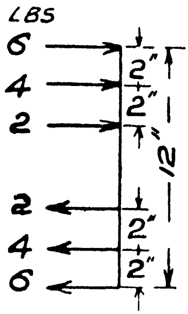


FIG 133.

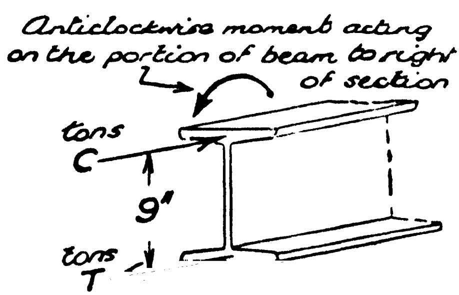


FIG 134.

(8) If the beam section (Fig. 134) is subjected to a couple of anticlockwise moment 27 tons ins., calculate the value of 'C' (the thrust in the top flange) and 'T' (the pull in the bottom flange).

(9) Obtain the two reactions in the ladder example given on page 39, by calculation method.

(10) A rod, whose weight may be neglected, is held in a horizontal position by a string in the manner indicated in Fig. 135.

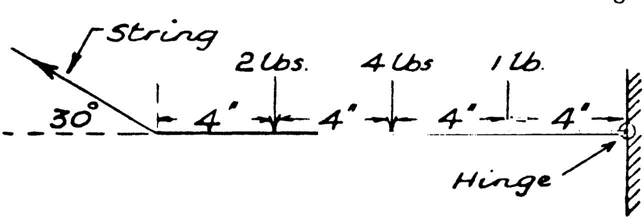


FIG. 135.

(i) *Calculate* the pull in the string and also the direction and magnitude of the hinge reaction. (ii) Check the calculated values by a graphical method.

(11) Find the values of 'W' and 'x' respectively (Fig. 136) for the light rod shown to remain in equilibrium.

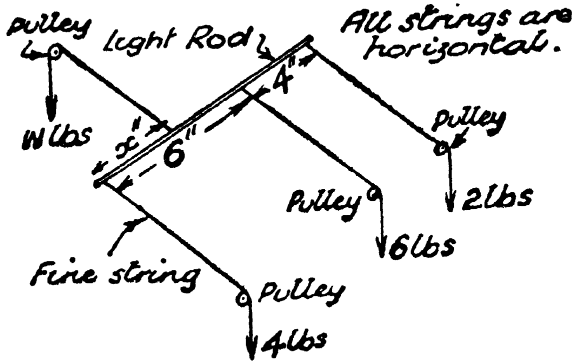


FIG. 136.

(12) A vertical steel frame is subjected to the load system given in Fig. 137. Calculate the reactions at 'A' and 'B' respectively.

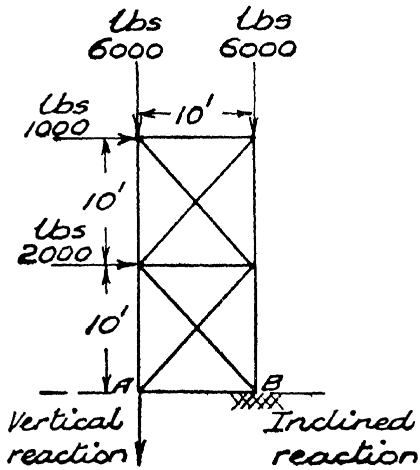


FIG. 137.

CHAPTER VI

CENTRE OF GRAVITY

FIG. 138 shows a body divided up into very small portions. Each of these portions is subjected to a vertical pull due to the attractive force of gravity. All the pulls together constitute a *like parallel system* of forces. This system will have a resultant, equal in magnitude to the sum of the forces, acting vertically downwards in a definite line of action. In Fig. 138 the same body is shown turned from one position to another. The component

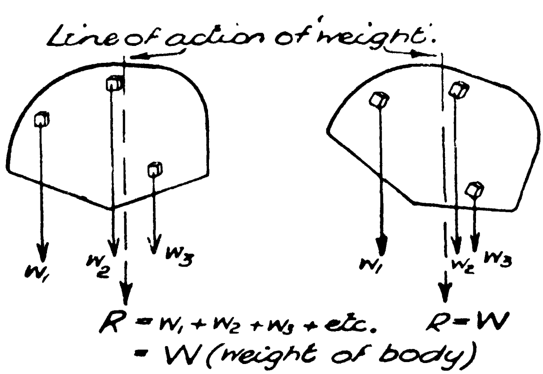


FIG. 138—CENTRE OF GRAVITY.

pulls in the two cases will be in a relatively different arrangement amongst themselves, so that while their resultant will remain the same in 'magnitude' it will act along a new 'line of action.' Although we could obtain many such lines of action by successively turning the body round, we would find that they would all pass through one common point in, or near, the body. To this point is given the name '*centre of gravity*.' Each body has, therefore, one centre of gravity.

Definition: The centre of gravity (C.G.) of a body is that point in space through which the resultant pull of the earth, i.e. the 'weight' of the body, acts, for all possible positions of the body.

The C.G. of a body is not necessarily in the material of which the body is composed.

Position of Centre of Gravity

Regular Geometrical Solids.—If a solid has a *geometrical centre*, this point will be the C.G., assuming the solid to be of uniform density throughout. If a solid has a central axis, the C.G. will lie somewhere along this axis. The exact position in the case of

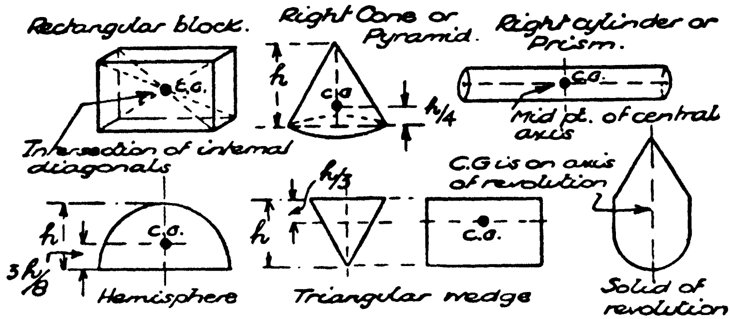


FIG. 139.—STANDARD CASES.

certain solids, such as a right cone, may be found by a formula. The proofs of some of these formulæ are rather beyond the scope of the book and they will be assumed. Fig. 139 indicates the C.G. positions for a few simple bodies.

Two Equal Masses

It should be noted that in the case of a body composed of *two* simpler bodies (whose C.G. positions we know), the C.G. will be somewhere along the line joining the C.G.'s of the two respective

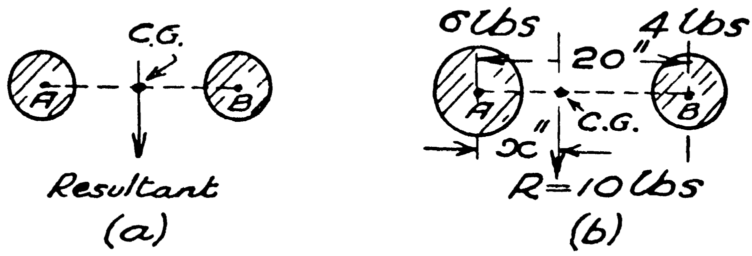


FIG. 140.—TWO MASSES.

component bodies. If we imagine the body in such a position that the C.G. of one of the component bodies is vertically above that of the other, it will be clear that, as the earth's pull will pass

through the two C.G.'s, the C.G. of the body must be on the line joining them.

In Fig. 140 (*a*) we have to find the position of the resultant of two equal parallel forces. The C.G. is the mid-point of 'AB.'

Two Unequal Masses.—In Fig. 140 (*b*) we have to find the point in which the resultant 'weight' cuts the line 'AB.'

Resultant of system = W = 6 lb. + 4 lb. = 10 lb.

Let 'x' = distance of resultant from 'A.'

Moments about 'A':

$$(10 \times x) = (6 \times 0) + (4 \times 20)$$
$$= 0 + 80 = 80$$
$$\therefore x = 8 \text{ ins.}$$

The C.G. of the two masses is 8 ins. from 'A'

Rule.—In the case of two masses, to obtain the C.G., divide the distance between their respective C.G.'s *inversely* as the masses.

In the example, 4 lb./6 lb. $= \dfrac{8''}{(20-8)''} = \dfrac{8''}{12''} = \dfrac{2}{3}$.

The rule is applied in the example on page 106. In working examples, the 'moments' method given previously is the easier.

Several Unequal Masses.—The masses shown in Fig. 141 have their C.G.'s in one straight line. The methods of parallel force systems are again applied.

The symbol '$\bar{x}$' ('x *bar*') is usually employed to denote the *horizontal* distance of a C.G. from an axis of reference.

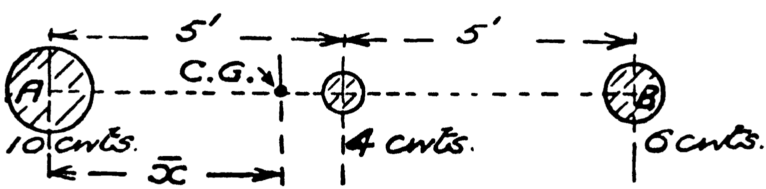


FIG. 141.—SEVERAL UNEQUAL MASSES.

$$W = (10 + 4 + 6) \text{ cwts.} = 20 \text{ cwts.}$$

Moments about 'A': Let $\bar{x}$ = C.G. distance from 'A.'

$$20 \times \bar{x} = (10 \times 0) + (4 \times 5) + (6 \times 10)$$
$$20\bar{x} = 80$$
$$\bar{x} = 4 \text{ ft.}$$

The C.G. is on the line 'AB,' at a point 4 ft. from 'A.'

Several Masses whose C.G.'s are not in One Straight Line

The C.G.'s of the three masses shown in Fig. 142 are assumed to be in one plane. Suitable axes 'OX' and 'OY' must be chosen to which to relate the C.G. of the system.

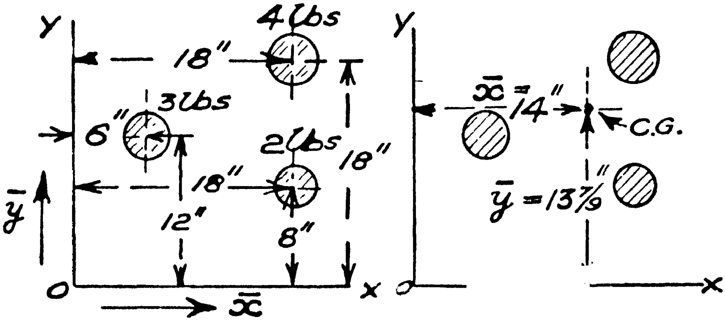


FIG 142.—GENERAL CASE OF UNEQUAL MASSES.

Let $\bar{x}$ = distance of C.G. *from* axis 'OY.'
$\bar{y}$ = „ „ „ „ 'OX.'
Total mass = $(3 + 4 + 2)$ lb. = 9 lb.

Moments about axis 'OY' to find $\bar{x}$.
In this case we multiply each mass by its distance from 'OY.'

$$9\bar{x} = (3 \times 6) + (4 \times 18) + (2 \times 18)$$
$$= 18 + 72 + 36 = 126$$
$$\bar{x} = 14 \text{ ins.}$$

Moments about axis 'OX' to find $\bar{y}$.

$$9\bar{y} = (3 \times 12) + (4 \times 18) + (2 \times 8)$$
$$= 36 + 72 + 16 = 124$$
$$\bar{y} = 13\tfrac{7}{9} \text{ ins.}$$

The C.G. is shown in position in Fig. 142.

C.G. of Composite Bodies

Provided a body be of uniform density, *volumes* may be used instead of *masses* in calculating C.G. positions. If the terms in the previously given '*moments equations*' were divided by the density of the material each term would represent a '*volume moment.*'

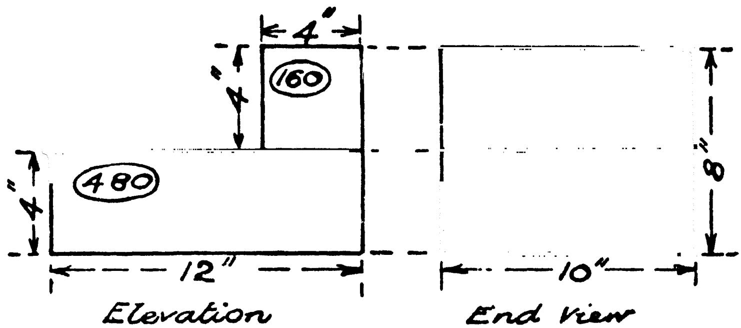


FIG. 143 —COMPOSITE SOLID

EXAMPLE.—*Calculate the position of the C G of the given solid* (*Fig* 143).

Total volume of solid =

$[(4 \times 12 \times 10) + (4 \times 4 \times 10)]$ cu. ins. $= (480 + 160)$ cu. ins.
$= 640$ cu. ins.

It is sometimes helpful to indicate, on each component part, the volume of the particular portion, so as to avoid error in taking moments.

Let $\bar{x}$ = distance of C.G. from left face of solid.
$\bar{y}$ = ,, ,, ,, bottom face of solid.

$640\ \bar{x}$ = the total sum of '*each volume multiplied by the distance of its own C.G. from the left face*'

$$= (480 \times 6) + (160 \times 10)$$
$$= 2880 + 1600$$
$$= 4480$$
$$\bar{x} = \frac{4480}{640} = 7 \text{ ins.}$$

$640\ \bar{y}$ = the total sum of '*each volume multiplied by the distance of its own C.G. from the bottom face*'

$$= (480 \times 2) + (160 \times 6)$$
$$= 960 + 960$$
$$= 1920$$
$$\bar{y} = \frac{1920}{640} = 3 \text{ ins.}$$

The solid is of uniform thickness (= 10 ins.), hence the C.G. is 5 ins. from the front face. The distance from front (or back)

face in any given case could be found by taking moments about that face, using the procedure indicated above.

EXAMPLE.—*A masonry pillar has the detail shown in Fig.* 144. *Find the position of the C.G.*

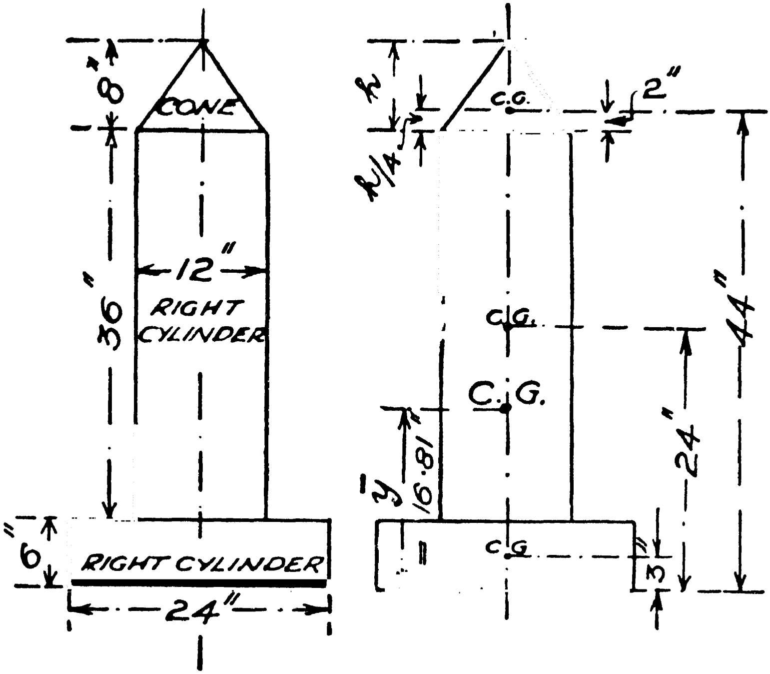


FIG. 144.—COMPOSITE REGULAR SOLID.

The C.G. will lie somewhere along the vertical central axis of the pillar.

Let $\bar{y}$ = height of C.G. above base.

Volume of cone $= \dfrac{\pi r^2 h}{3} = \dfrac{\pi \times 6^2 \times 8}{3}$ cu. ins. $= 96\pi$ cu. ins.

Volume of right cylinder (upper) $= \pi r^2 h = \pi \times 6^2 \times 36$ cu. ins.
$= 1296\,\pi$ cu. ins.

" " " (lower) $= \pi r^2 h = \pi \times 12^2 \times 6$ cu. ins.
$= 864\pi$ cu. ins.

Total volume $= (96\pi + 1296\pi + 864\pi)$ cu. ins.
$= 2256\pi$ cu. ins.

$$2256\pi \times \bar{y} = (96\pi \times 44) + (1296\pi \times 24) + (864\pi \times 3)$$
$$2256\bar{y} = 4224 + 31104 + 2592$$
$$= 37920$$
$$\bar{y} = 16{\cdot}81 \text{ ins.}$$

C.G. is 16·81 ins. above bottom of pillar.

C.G. of a Plane Figure

A figure, having no mass, will not be affected by gravitational force, so that the term '*centre of gravity*' is not strictly applicable in this case. In order to get a physical interpretation of 'C.G.' in the case of '*areas*' and '*sections*,' we may imagine the figure to represent the outline of an extremely thin slice of material, so thin that the C.G. may be regarded as being practically on the surface. We may therefore employ the general methods used in the case of solids, and take '*moments of areas*' just as we took '*moments of volumes*.' The term '*centroid*' is sometimes used instead of 'centre of gravity.'

The determination of the C.G. of an area is frequently required in structural calculations The methods of solution given should be thoroughly mastered by the reader.

Figures with Axes of Symmetry.—If a figure has an axis of symmetry, the C.G. will be somewhere in that axis (Fig. 145). If there are two axes of symmetry, their point of intersection will be the required C.G. Symmetrical axes may be used to find the C.G. of a rectangle, but it is easier to find the point by drawing-in the diagonals (see Fig. 146 (*a*)).

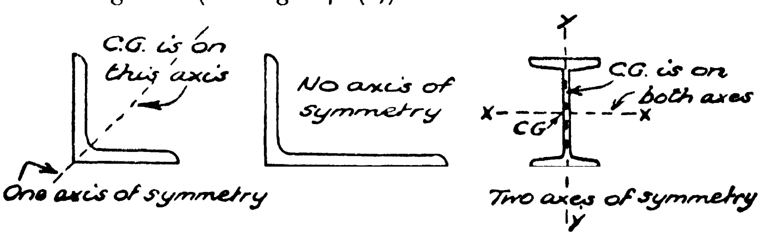


FIG. 145—STRUCTURAL SECTIONS

Parallelogram.—In the case of the parallelogram (Fig. 146 (*b*)) the thin strips indicated may be regarded as being rectangles. The line 'AB' which passes through the respective C.G.'s of all the horizontal strips will clearly pass through the C.G. of the

parallelogram. Similarly the C.G. must be on the line 'C.D.' The quickest method again in this case will be to draw-in the diagonals of the parallelogram.

Triangle.—Consider thin strips of the triangle (Fig. 146 (*c*)), parallel to the base 'BC.' The mid-point of each strip will lie on a straight line joining the apex 'A' to the mid-point of 'BC.'

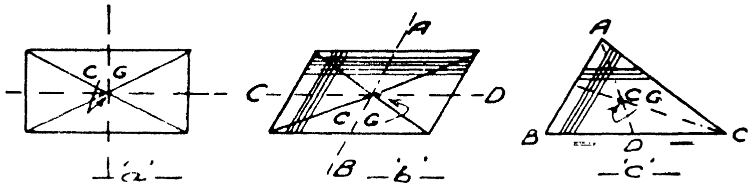


FIG 146—PLANE SECTIONS

This is a geometrical theorem, the line 'AD' being termed a '*median*' of the triangle 'ABC.' The median 'AD' passes through the C.G. of each strip and will therefore contain the C.G. of the triangle. Similarly the other two medians of the triangle must pass through its C.G. We may define the C.G. of a triangle as being the intersection point of any two of its medians.

Right-angled Triangle.—It is shown in geometry that the common intersection point of the three medians of *any* triangle divides each median into two parts having a ratio of 1 to 2 (see Fig. 147). This fact is very useful in dealing with right-angled triangles, and

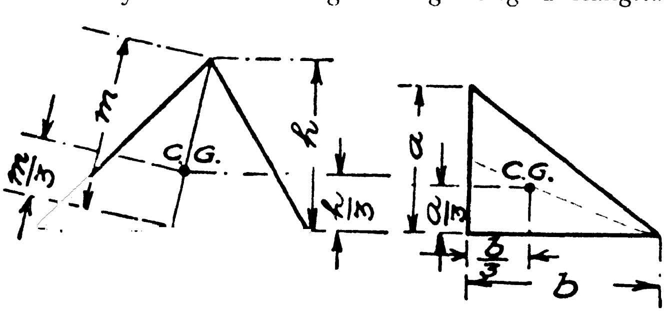


FIG 147.—C G OF A TRIANGLE.

is especially applicable to structural problems when extended, by the principle of similar triangles, to the form shown in Fig. 147.

Other Geometrical Figures.—Structural calculations sometimes

involve the consideration of the C.G. position in such figures as *semi-circles*, *parabolæ*, etc. The reader is referred to structural handbooks for a complete list of such cases. Fig. 148 indicates a few important C.G. positions.

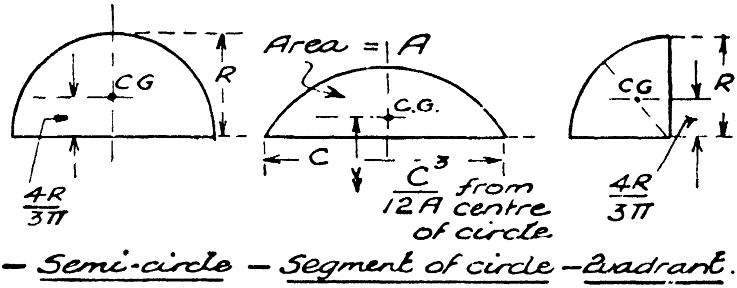


FIG 148 —STANDARD CASES

Centre of Gravity of Composite Figures

The general method of treatment of practical structural sections is exemplified in the diagram given in Fig. 149.

The areas 'a_1, 'a_2,' 'a_3' are assumed to represent component parts of a given figure or structural section. 'OX' and 'OY' are two convenient axes of reference. In a practical example it would be usual to regard the bottom edge of the figure and the left edge as forming these respective axes.

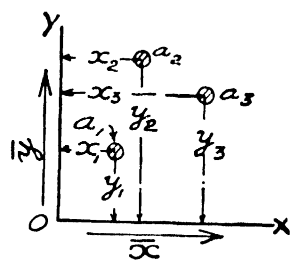


FIG 149 GENERAL CASE.

Let 'A' = total area of the component parts.

$$A = a_1 + a_2 + a_3.$$

Moments about axis 'OY'

$$A\bar{x} = a_1 x_1 + a_2x_2 + a_3x_3$$

$= \Sigma ax$ (i.e. *the sum of all such quantities as* '$a \times x$')

$$\therefore \bar{x} = \frac{\Sigma ax}{A}.$$

Moments about axis ' OX ':

$$A\bar{y} = a_1y_1 + a_2y_2 + a_3y_3$$
$$= \Sigma ay$$
$$\therefore \bar{y} = \frac{\Sigma ay}{A}.$$

In simple words the procedure is as follows:

Divide up the given figure into suitable simple figures. Take each of these component areas and multiply by its own C.G. distance from the selected axis of reference. Add together all such ' *area moments.*' Equate the sum to the product ' *total area of figure multiplied by the required C.G. distance from the given axis.*' Hence determine the C.G. distance.

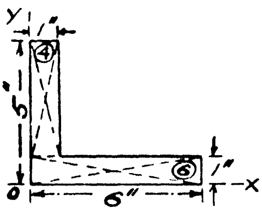


FIG. 150.—UNEQUAL ANGLE

EXAMPLE (i).— *Find the C.G. position in the case of the unequal angle section shown in Fig.* 150.

In this case we divide the figure into two rectangles.

Component areas: upper rectangle $= 4'' \times 1'' = 4$ sq. ins.
lower rectangle $= 6'' \times 1'' = 6$ sq. ins.

Total area $= 10$ sq. ins.

Axis ' OX ' is taken to coincide with the bottom edge of the section and ' OY ' is assumed to coincide with the left edge. It is not necessary to draw-in these axes provided the meanings of ' $\bar{x}$ ' and ' $\bar{y}$ ' respectively are clearly stated.

Moments about ' O Y ' (left edge of section):

Moment of total area = sum of moments of component areas

$$10\bar{x} = (4 \times \tfrac{1}{2}) + (6 \times 3)$$
$$= 2 + 18 = 20$$
$$\bar{x} = 20/10 = 2 \text{ ins.}$$

Moments about ' OX ' (bottom of section):

Moment of total area = sum of moments of component areas

$$10\bar{y} = (4 \times 3) + (6 \times \tfrac{1}{2})$$
$$= 12 + 3 = 15$$
$$\bar{y} = 15/10 = 1{\cdot}5 \text{ ins.}$$

C.G. is 2 in. from left edge of figure and 1·5 ins. from the bottom.

(ii) *Determine the position of the C.G. of the given Z-section (Fig. 151).*

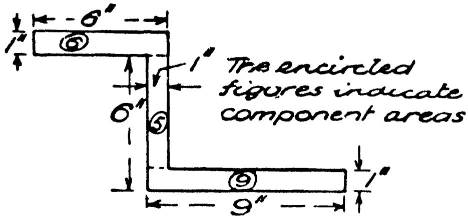


FIG 151 —Z-SECTION

Let $\bar{x}$ = distance of C.G. from left edge of section.

$\bar{y}$ = distance of C.G. from bottom edge of section.

Total area of section = [(6 × 1) + (5 × 1) + (9 × 1)] sq. ins.
= (6 + 5 + 9) sq. ins. = 20 sq. ins.

$$20\bar{x} = (6 \times 3) + (5 \times 5{\cdot}5) + (9 \times 9{\cdot}5)$$
$$= 18 + 27{\cdot}5 + 85{\cdot}5 = 131$$
$$\bar{x} = \frac{131}{20} \text{ ins.} = 6{\cdot}55 \text{ ins.}$$
$$20\bar{y} = (6 \times 6{\cdot}5) + (5 \times 3{\cdot}5) + (9 \times {\cdot}5)$$
$$= 39 + 17{\cdot}5 + 4{\cdot}5 = 61$$
$$\bar{y} = \frac{61}{20} \text{ ins.} = 3{\cdot}05 \text{ ins.}$$

(iii) *Fig. 152 shows the section of a retaining wall. Calculate the distances of the C.G. from the base and from the vertical back of the wall respectively.*

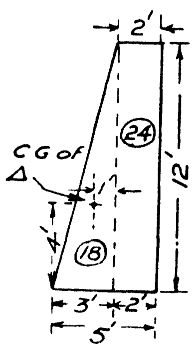


FIG. 152 —RETAINING-WALL SECTION.

Divide the section into a rectangle and a triangle.

Total area of section = [(12 × 2) + (½ × 3 × 12)] sq. ft.
= (24 + 18) sq. ft. = 42 sq. ft.

Let $\bar{x}$ = distance of C.G. from back of wall.

$\bar{y}$ = distance of C.G. from base of wall.

Moments about back of wall:

$$42\bar{x} = (24 \times 1) + (18 \times 3).$$

[The C.G. of the triangle is ($\frac{1}{3} \times 3'$) from the right-angle corner, so that, from the back of the wall, it is ($\frac{1}{3} \times 3'$) $+ 2' = 3$ ft.]

$$\therefore 42\bar{x} = 24 + 54 = 78$$

$$\bar{x} = \frac{78}{42} \text{ ft.} = 1\tfrac{6}{7} \text{ ft.}$$

Moments about bottom of wall:

$$42\bar{y} = (24 \times 6) + (18 \times \tfrac{12}{3})$$

$$= 144 + 72 = 216$$

$$\bar{y} = \frac{216}{42} \text{ ft.} = 5\tfrac{1}{7} \text{ ft.}$$

Allowance for Rivet Holes in Structural Sections

In cases in which rivet holes have to be deducted, it is convenient to treat the section, first of all, as being '*gross*' (i.e. having no holes), and then to assume the rivet holes to be negative superimposed areas.

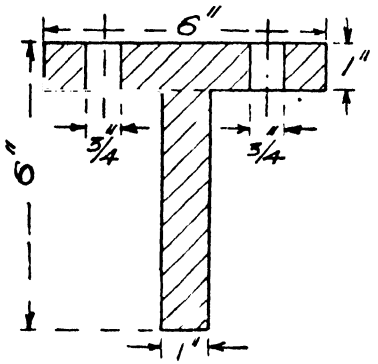


FIG 153—RIVET HOLES IN SECTION

In the example shown in Fig 153, the gross section is symmetrical about a vertical axis. The rivet holes are also symmetrical about this axis, so that the C.G. must lie somewhere in this axis.

Let '$\bar{y}$' be depth of C.G. below top of section.

Total gross area $= [(6 \times 1) + (5 \times 1)]$ sq. ins.

Area of rivet holes $= (2 \times \frac{3}{4} \times 1)$ sq. ins.

Net area $= (6 + 5 - 1{\cdot}5)$ sq. ins.

Moments about top edge of section:

$$(6 + 5 - 1{\cdot}5)\bar{y} = (6 \times {\cdot}5) + (5 \times 3{\cdot}5) - (1{\cdot}5 \times {\cdot}5)$$

$$9{\cdot}5\bar{y} = 3 + 17{\cdot}5 - {\cdot}75$$

$$= 19{\cdot}75$$

$$\bar{y} = 2{\cdot}08 \text{ ins.}$$

EXAMPLE.—*Find the centre of gravity of the rivet group given in Fig.* 154.

The determination of the C.G. in such a case is the first step in the calculation of the strength of the connection.

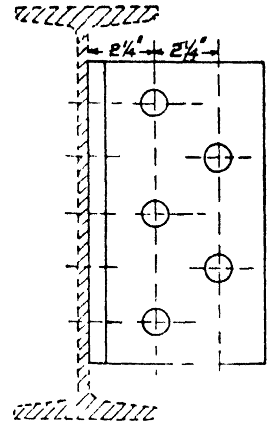


FIG 154 – RIVET GROUP.

The C.G. will lie on a horizontal line through the centre of the middle rivet in the first row (principle of symmetry).

Let $\bar{x}$ = distance of C.G. from left edge of cleat.

Let 'A' sq. ins. = area of one rivet.

Total area $= 5A$ sq. ins.

$$\therefore 5A\bar{x} = (3A \times 2{\cdot}25) + (2A \times 4{\cdot}5)$$

$$5A\bar{x} = 6{\cdot}75A + 9A$$

$$= 15{\cdot}75A$$

$$\therefore \bar{x} = \frac{15{\cdot}75}{5} = 3{\cdot}15 \text{ ins.}$$

EXAMPLE.—*Determine the depth, beneath the top of section, of the centre of gravity of the given girder section (Fig. 155). Make no allowance for rivet holes.*

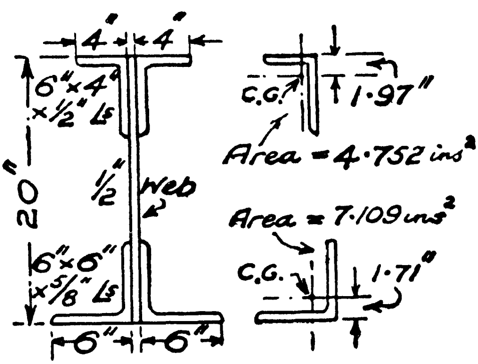


FIG 155 —BUILT-UP GIRDER SECTION.

To solve this problem we require to know certain 'properties' of the 'British Standard Sections' involved. We must know the 'sectional area' and the position of the C.G. of the 'angle' sections given. These properties are shown in Fig. 155, having been found by reference to 'section property tables.' Such

tables are to be found in any of the handbooks issued by steel firms.

Total area of girder section

$$= [(2 \times 4{\cdot}752) + (2 \times 7{\cdot}109) + (20 \times \tfrac{1}{2})] \text{ sq. ins.}$$
$$= (9{\cdot}504 + 14{\cdot}218 + 10) \text{ sq. ins.}$$
$$= 33{\cdot}722 \text{ sq. ins.}$$

Let '$\bar{y}$' = distance of C.G. below top of section.

Moments about top of section:

$$33{\cdot}722\bar{y} = (9{\cdot}504 \times 1{\cdot}97) + (14{\cdot}218 \times 18{\cdot}29) + (10 \times 10)$$
$$= 18{\cdot}723 + 260{\cdot}05 + 100$$
$$= 378{\cdot}77$$
$$\bar{y} = 11{\cdot}23 \text{ ins.}$$

EXAMPLE.—*The diagram given in Fig.* 156 *represents the end of a building which is subjected to a uniform wind pressure of* 15 *lb. per square foot of area. Calculate the overturning moment, due to the wind, about the base ' A B.'*

The wind pressure being uniform, the resultant wind force will act at the 'centre of gravity' of the area.

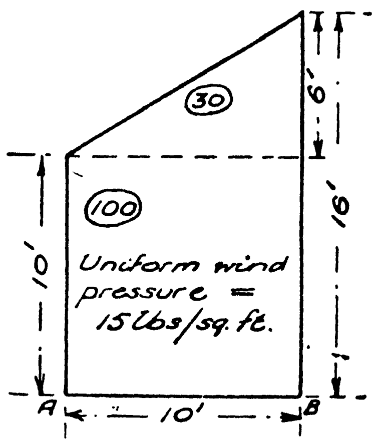


FIG. 156.—UNIFORM WIND LOAD.

$$\text{Total area} = \left[(10 \times 10) + \left(\frac{10 \times 6}{2}\right)\right] \text{ sq. ft.}$$
$$= (100 + 30) \text{ sq. ft.} = 130 \text{ sq. ft.}$$

Let '$\bar{y}$' be height of C.G. above base 'AB.'

$$130\bar{y} = (100 \times 5) + (30 \times 12)$$
$$= (500 + 360)$$
$$= 860$$
$$\bar{y} = 860/130 \text{ ft.} = 6\tfrac{8}{13} \text{ ft.}$$

The resultant wind load $= 130$ sq. ft. $\times$ 15 lb./sq. ft.
$= 1950$ lb.

Overturning moment about base $=$ Force $\times$ arm
$= 1950 \text{ lb.} \times 6\tfrac{8}{13}$ ft.
$= 12900$ lb. ft.

Note: We could have dealt with each component area separately.

Graphical Methods for Determining the C.G. of a Plane Figure

A graphical method applicable to any plane figure is explained in the next chapter. Certain figures may be dealt with by special forms of graphical solution, based upon principles already considered.

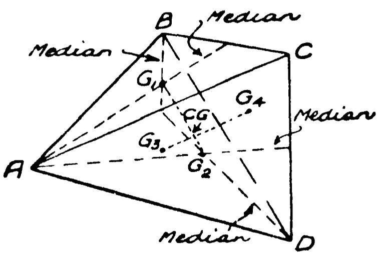


FIG. 157.—QUADRILATERAL.

Quadrilateral

Join 'A' to 'C,' forming two triangles 'ABC' and 'ADC,' whose respective C.G.'s are 'G_1' and 'G_2.' Now join 'B' to 'D,' forming the two triangles 'BAD' and 'BCD.' The C.G.'s of these two triangles are 'G_3' and 'G_4' respectively (the medians used to fix these two points are omitted from the diagram shown, in order to avoid confusion of lines). Join 'G_3G_4.' The C.G. of the quadrilateral will be the intersection point of 'G_1G_2' and 'G_3G_4,' as it must lie on both these lines.

Fig. 158 illustrates the application of the same principle of

'*subdividing a given figure into two simpler figures in two different ways.*'

Alternative Method.—Divide the given figure into two simpler figures and find 'G_1' and 'G_2,' their centres of gravity. Join 'G_1G_2.' Just as in the case of two solids (page 93), we must find the point which divides 'G_1G_2' *inversely* as the respective

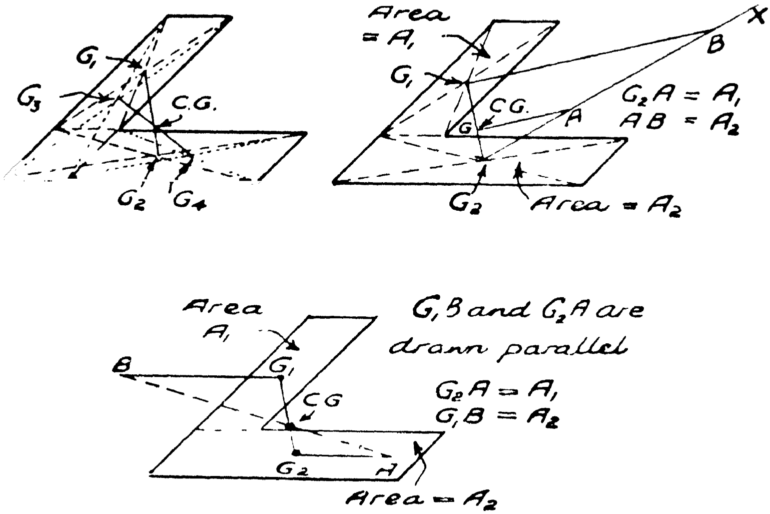


FIG. 158.—ALTERNATIVE GRAPHICAL METHODS.

areas. In the example of the acute-angle section given in Fig. 158, 'G_2X' is set off in any suitable direction. Along 'G_2X,' 'G_2A' is marked off to a convenient area scale to represent area 'A_1' and 'AB' is similarly marked off to represent area 'A_2.' 'B' is joined to 'G_1' and 'AG' is drawn parallel to 'BG_1.' By principles of geometry, 'G' divides G_1G_2 in the required ratio, and will be, therefore, the C.G. of the given section.

The third method shown is based upon the same principles as the second method.

Retaining-wall Section.—The figure given in Fig. 159 represents the type section for a 'gravity' retaining wall. To determine the stability of such a wall it is necessary to find the C.G. of the section (see Chapter XV). The methods already explained may be employed, but for a four-sided figure with a pair of opposite sides parallel, a simpler procedure is usually adopted.

Draw AA′ and BB′ each equal to the base DC, and CC′ and DD′ each equal to the top AB. Join A′C′ and B′D′. The point of intersection of these two transversals will be the C.G. of the given wall section. The C.G. will also be in the line 'MN,' where 'M' and 'N' are the mid-points of the top and base respectively.

Alternatively, if 'a' = length of base and 'b' = length of top, we may find 'G' by the formula:

$$NG = \frac{a + 2b}{3(a + b)} \times NM.$$

Proof.—The proof of the foregoing methods is an interesting application of the general principles of C.G. determination The figure is divided into two triangles.

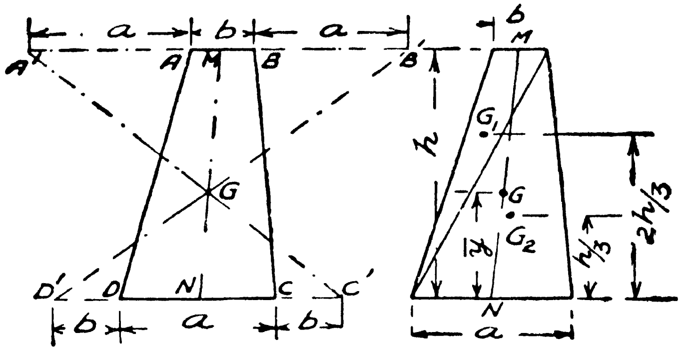


FIG 159 —RETAINING-WALL SECTION.

Let $\bar{y}$ = *height* of C.G. above base (Fig. 159).

$$\text{Total area} = \left(\frac{ah}{2} + \frac{bh}{2}\right) = \frac{h}{2}(a + b).$$

Taking moments about the base:

$$\frac{h}{2}(a + b) \times \bar{y} = \left(\frac{ah}{2} \times \frac{h}{3}\right) + \left(\frac{bh}{2} \times \frac{2h}{3}\right)$$

$$= h^2\left(\frac{a}{6} + \frac{2b}{6}\right)$$

$$\therefore \bar{y} = \frac{2h}{a + b}\left(\frac{a + 2b}{6}\right) = \frac{a + 2b}{3(a + b)} \times h.$$

But $\bar{y} : h = NG : NM$ by the principle of similar triangles.

$$\therefore NG = \frac{a + 2b}{3(a + b)} \times NM.$$

Further, considering the graphical construction, $\triangle$s GND′ and GMB′ are similar.

$$\therefore \quad \frac{NG}{GM} = \frac{ND'}{MB'} = \frac{b + a/2}{a + b/2} = \frac{2b + a}{2a + b}$$

$$\therefore \frac{NG}{GM + NG} = \frac{2b + a}{(2a + b) + (2b + a)} = \frac{a + 2b}{3(a + b)}$$

$$\therefore \quad \frac{NG}{NM} = \frac{a + 2b}{3(a + b)}.$$

The geometrical construction used therefore gives the correct position for the C.G.

Beam-reaction Problems Involving 'C.G.'

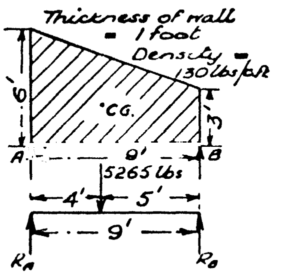


FIG 160.—BEAM REACTIONS

EXAMPLE (i).—*Calculate the support reactions for the beam shown in Fig.* 160. *The wall is uniform in thickness. The self-weight of the beam may be neglected.*

The beam is regarded as having a single-point load acting through the C.G. of the wall and equal in magnitude to the total weight of the wall.

If a scale drawing is made, the C.G. of the wall may be found graphically.

By formula: distance of C.G. of wall from support at 'A'

$$= \frac{a + 2b}{3(a + b)} \times l = \frac{6 + (2 \times 3)}{3(6 + 3)} \times 9 \text{ ft.}$$

$$= \frac{12}{27} \times 9 = 4 \text{ ft.}$$

$$\text{Total weight of wall} = \text{volume} \times \text{density}$$
$$= [\tfrac{1}{2}(6 + 3) \times 9] \times 1 \times 130 \text{ lb.}$$
$$= 5265 \text{ lb.}$$
$$R_A \times 9 = 5265 \times 5 = 26325$$
$$\therefore R_A = 2925 \text{ lb.}$$
$$R_B \times 9 = 5265 \times 4 = 21060$$
$$\therefore R_B = 2340 \text{ lb.}$$
$$R_A + R_B = (2925 + 2340) \text{ lb.} = 5265 \text{ lb.}$$

We could have treated the wall section as being composed of a rectangle (9′ × 3′) plus a triangle, and had two equivalent beam loads.

EXAMPLE (ii).—*Fig.* 161 *shows a beam carrying a wall of varying*

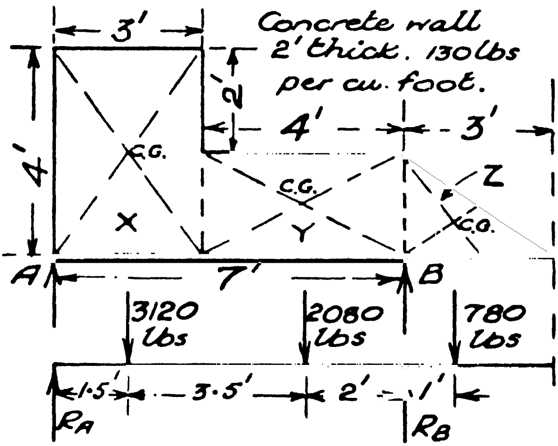


FIG 161.—OVERHANGING BEAM.

height but of uniform thickness 2 ft. Taking the particulars given, obtain the support reactions due to this load.

We could, as in the last example, find the C.G. of the whole wall and reduce the problem to that of an overhanging beam carrying a single-point load. It is easier, in this case, to divide the wall up into convenient portions.

$$\begin{aligned} \text{Load } X &= (4 \times 3 \times 2 \times 130) \text{ lb.} = 3120 \text{ lb.} \\ Y &= (2 \times 4 \times 2 \times 130) \text{ lb.} = 2080 \text{ lb.} \\ Z &= (1 \times 3 \times 2 \times 130) \text{ lb.} = \underline{780 \text{ lb.}} \\ &\qquad\qquad \text{Total load} = \underline{\underline{5980 \text{ lb.}}} \end{aligned}$$

These loads are regarded as acting through their respective C.G.'s.

$$(R_A \times 7) + (780 \times 1) = (3120 \times 5{\cdot}5) + (2080 \times 2)$$

$$\therefore R_A \times 7 = 17160 + 4160 - 780 = 20540$$

$$R_A = 2934\tfrac{2}{7} \text{ lb.}$$

$$\begin{aligned} R_B \times 7 &= (3120 \times 1{\cdot}5) + (2080 \times 5) + (780 \times 8) \\ &= 4680 + 10400 + 6240 \\ &= 21320 \end{aligned}$$

$$\therefore R_B = 3045\tfrac{5}{7} \text{ lb.}$$
$$R_A + R_B = (2934\tfrac{2}{7} + 3045\tfrac{5}{7}) \text{ lb.}$$
$$= 5980 \text{ lb.}$$

Eccentrically Loaded Columns.—Loads which do not lie on the symmetrical axes 'XX' and 'YY' of a column section are termed '*eccentric loads.*' One method of dealing with an eccentric load system (in certain circumstances) is to reduce the system to one resultant load acting at the C.G. of the system.

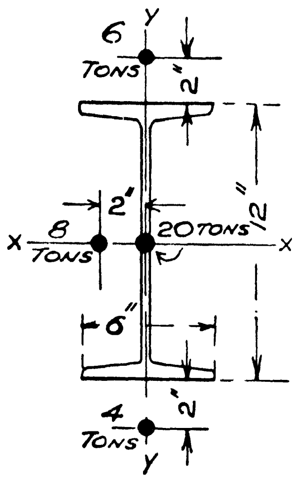


FIG 162.—ECCENTRICALLY LOADED COLUMN

EXAMPLE.—*Calculate the eccentricities, with respect to the 'principal' axes 'XX' and 'YY,' of the resultant column load for the case shown in Fig.* 162.

We require to find the position of the C.G. of the four given loads expressing '$\bar{x}$' and '$\bar{y}$' with respect to axes 'YY' and 'XX' respectively, as axes of reference.

Moments about YY (distances to left, positive):

$$(6 + 20 + 4 + 8)\bar{x} = (6 \times 0) + (20 \times 0) + (4 \times 0) + (8 \times 2)$$
$$38\bar{x} = 16$$
$$\bar{x} = 16/38 \text{ ins.} = \cdot 421 \text{ in.}$$

Moments about XX (distances upwards, positive):

$$38\bar{y} = (6 \times 8) + (20 \times 0) - (4 \times 8) + (8 \times 0)$$
$$= 48 - 32 = 16$$
$$\bar{y} = 16/38 \text{ ins.} = \cdot 421 \text{ in.}$$

The resultant load is ·421 in. eccentric with respect to both the principal axes of the section. In practical problems the eccentricity with respect to the 'XX' axis is sometimes denoted by the symbol '*ex.*' Similarly '*ey*' would represent the eccentricity with respect to the 'YY' axis. Thus $\bar{x} = ey$ and $\bar{y} = ex$ in the foregoing example.

Experimental Determination of C.G. Position

If a section has a very irregular outline, made up of portions of curves and straight lines, it may be difficult to determine the C G by calculation or by graphics If the C G has been fixed by either of these methods, it may be useful to check the accuracy of the result by an experimental method The given section should be drawn either full size or to a convenient scale, on a piece of stiff cardboard, or sheet metal, of uniform thickness Three small holes must be made near the edge of the template from which, in turn, the template is suspended on a smooth pin (see Fig 163) At each suspension the direction of the plumb

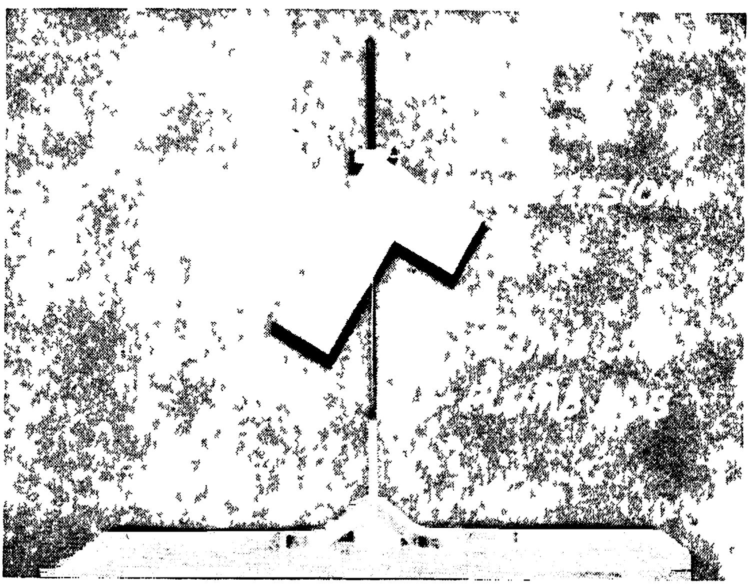


FIG 163 EXPERIMENTAL DETERMINATION OF CENTRE OF GRAVITY

line is marked on the template The intersection of two of these markings gives the required C G position, the third line provides a check on accuracy The reader should use this simple method for verifying the C G position in the case of a 'triangle' or a 'semi-circle,' etc

Application of C.G. in Reinforced Concrete Beam Section

Readers interested in reinforced concrete theory should note the example illustrated in Fig. 164. To reduce the composite

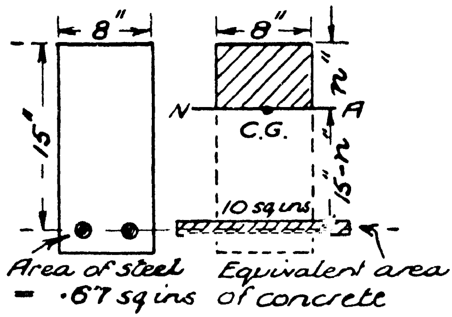


FIG 164—REINFORCED CONCRETE BEAM SECTION.

concrete and steel section to an equivalent '*all-concrete*' section, the steel area must be multiplied by a constant usually taken as '15.' The ·67 sq. in. of steel thus becomes (·67 × 15) sq. ins. = 10 sq. ins. of equivalent concrete. In order to calculate the strength in bending of a beam of the given section it is first of all necessary to find an axis 'NA' (see page 241) such that the C.G. of the concrete area above it, taken in conjunction with the '*equivalent concrete area*' below it, shall lie on the axis. All other concrete below 'NA' is ignored.

Let n'' = depth of the axis 'NA' from top of section. Moment of area above 'NA' must equal moment of area below 'NA,' both moments being taken about 'NA.'

$$\therefore (8 \times n \times n/2) = 10 \times (15 - n)$$
$$\therefore 4n^2 = 150 - 10n$$
$$n^2 + 2{\cdot}5n - 37{\cdot}5 = 0.$$

$n = 5$ satisfies this quadratic equation. The C.G. of the '*equivalent all-concrete section*' is therefore 5" below the top of the section. The position of 'NA' is thus determined.

EXERCISES 6

(1) Calculate the position of the centre of gravity of the four masses shown in Fig. 165.

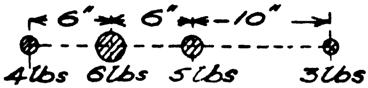


FIG. 165.

(2) A solid of uniform density is composed of three rectangular blocks (Fig. 166). Calculate the distance of the C.G. of the solid from (*a*) the left face, (*b*) the bottom.

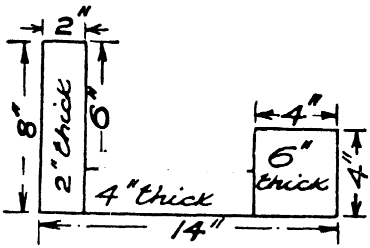


FIG. 166.

(3) Find the C.G. of the equal-angle section given in Fig. 167. Check the position by a graphical method.

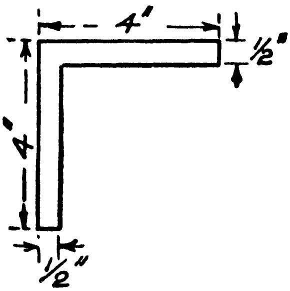


FIG. 167.

(4) Determine the centre of gravity of each of the structural sections given in Fig. 168.

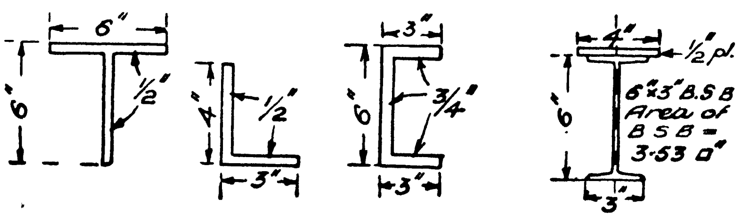


FIG. 168.

(5) The girder section (Fig. 169) has two 1-in. diameter bolt holes in the bottom flange. Find the C.G. of the net section.

(6) A retaining-wall section has a vertical back. The widths at the top and bottom of the wall are, respectively, 3 ft. and 9 ft. The height of the wall is 20 ft. Calculate the distance of the C.G. of the wall section from the back of the wall. Check the distance by a graphical method.

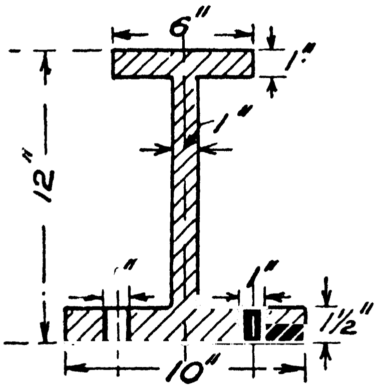


FIG. 169.

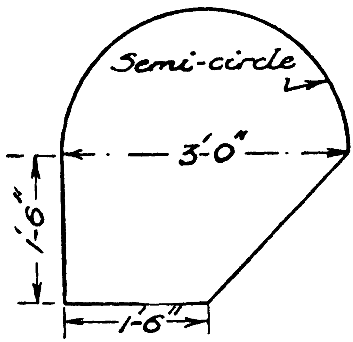


FIG. 170.

(7) Fig. 170 gives the outline of a masonry slab of uniform thickness, which has to be hoisted by a crane. Find the best position at which to fix a bolt for the crane chain in order that the slab shall remain horizontal during lifting.

(8) Find the beam-support reactions in each of the cases given in Fig. 171. Neglect the self-weight of the beams.

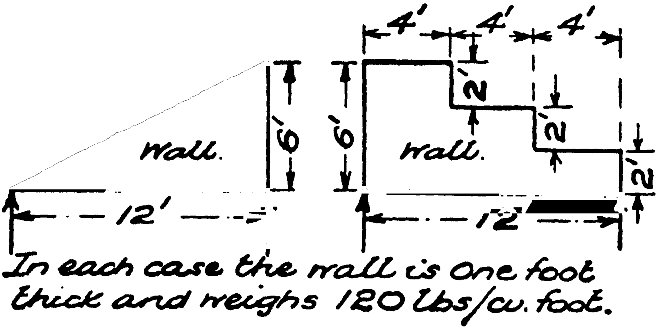


FIG. 171.

(9) Determine the centre of gravity of the three column loads indicated in Fig. 172.

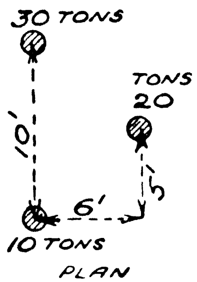


FIG 172.

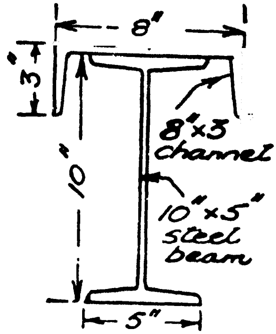


FIG 173.

(10) Find the centre of gravity of the structural section shown in Fig. 173. The following *properties* are taken from section tables:

Sectional area of 10" × 5" B.S.B. = 8·85 sq. ins.

Sectional area of 8" × 3" B.S.C. = 4·69 sq. ins.

Thickness of web of channel = ·28 in.

Distance of C.G. of channel section from back of channel web = ·83 in.

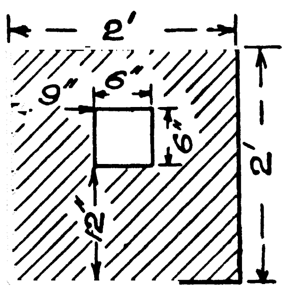


FIG. 174.

(11) A casting has a square hole in the position indicated in Fig. 174.

The material is of uniform thickness and density. Find the C.G. of the casting.

(12) Fig. 175 shows a reinforced concrete retaining wall with the retained earth resting on the base. Calculate the distance, from the left edge of base, of the centre of gravity of the combined earth and wall. The average wall density = 144 lb. per cu. foot, and the earth weighs 90 lb. per cu. foot. (Take 1-ft. length of wall, in plan, and reduce to ' weights ' before taking moments.)

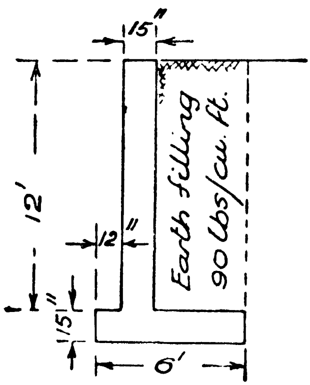


FIG. 175.

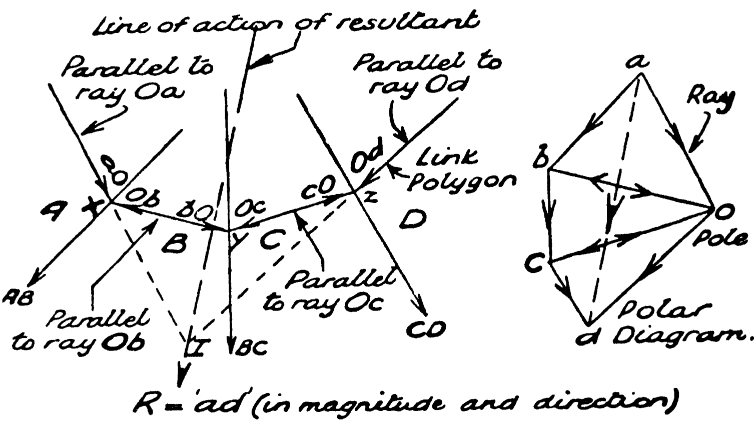
CHAPTER VII

THE LINK POLYGON

The '*link polygon*' construction is used for the solution of a number of types of structural problems. These range from simple beam-reaction problems to problems involving bending moments and beam deflections. An important application occurs in masonry arch design, the line of thrust in the arch ring being determined by a link polygon. It is clear, therefore, that the principles underlying this graphical construction should be thoroughly understood.

Resultant of a Co-planar Force System

Consider the three forces 'AB,' 'BC' and 'CD,' shown in Fig. 176. If we represent these forces by the vector lines '*ab*,' '*bc*' and '*cd*' respectively, the vector line '*ad*' will represent the resultant of the three forces in *magnitude and direction*. We do not know, however, where it acts in space in relationship to the forces in the system. To ascertain this it is necessary to proceed as indicated on page 118.



176.—Resultant of Co-planar System.

Procedure

Choose *any* point ' O ' adjacent to the load line '***abcd***' and draw in the '***rays***' ' O*a*,' ' O*b*,' ' O*c* ' and ' O*d*.' At any convenient point ' X ' in the line of action of force ' AB ' and in the space ' A,' draw a '***link***' parallel to ' O*a*.' Draw link ' XY ' (in space B) parallel to the ray ' O*b*,' and so on as indicated in Fig. 176. Produce the links in the spaces ' A ' and ' D ' to intersect in the point ' I.' The resultant of the given force system will pass through the point ' I.'

Proof.—The triangles formed by the '***rays***' in the '*polar diagram*' are really ' resolution triangles ' for the forces in the system. Because all the triangles have a common apex point ' O ' (the '*pole*') they achieve the ' resolution ' in such a manner as to lead to the rule indicated above for the position of point ' I.' At the point ' X,' the force ' AB ' is resolved into two components ' *a*O ' and ' O*b* ' by means of the resolution triangle ' *ab*O.' At the point ' Y,' by resolution triangle ' *bc*O,' the force ' BC ' is replaced by its two components ' *b*O ' and ' O*c*.' Similarly at ' Z,' the components ' *c*O ' and ' O*d* ' are substituted for force ' CD.' But the components in space ' B ' cancel out each other (they are equal in *magnitude* and *opposite* in direction). Similarly in space ' C ' the components balance each other. The original system is therefore reduced to two forces, viz. force ' *a*O ' in space ' A ' and force ' O*d* ' in space ' D.' The resultant of these two forces passes through ' I,' their point of intersection. The resultant of forces ' AB,' ' BC ' and ' CD ' must therefore pass through ' I.'

In the practical employment of the link polygon the arrow heads and the letters, as shown on the ' links ' in Fig. 176, should be omitted. The arrow heads on the ' rays ' are also omitted.

EXAMPLE (i).—*Find the resultant of the like parallel system of forces given in Fig. 177, by the link polygon method. Check the position of the resultant by the usual method of moments.*

The graphical solution is shown in Fig. 177. The reader should draw out the diagrams to the following scales : 1" = 2 ft. and 1" = 4 cwts.

Calculation of ***Resultant.***—The magnitude of the resultant = (4 + 8 + 6 + 2) cwts. = 20 cwts.

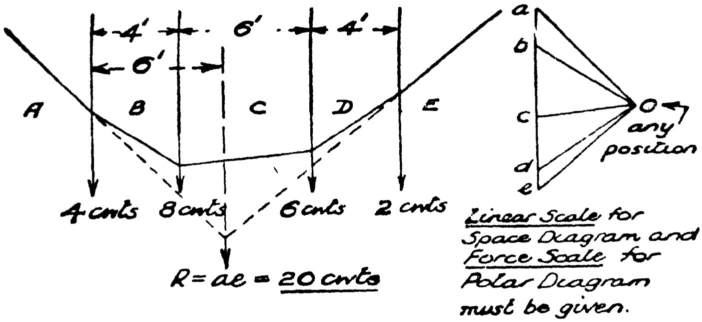


FIG 177—RESULTANT OF A PARALLEL SYSTEM

Let 'x' ft. = distance at which the resultant acts from the 4-cwt. force.

Moment of resultant = sum of moments of components.

$$20 \times x = (4 \times 0) + (8 \times 4) + (6 \times 10) + (2 \times 14)$$
$$= 0 + 32 + 60 + 28$$
$$= 120$$
$$x = 6 \text{ ft. (as in graphical method).}$$

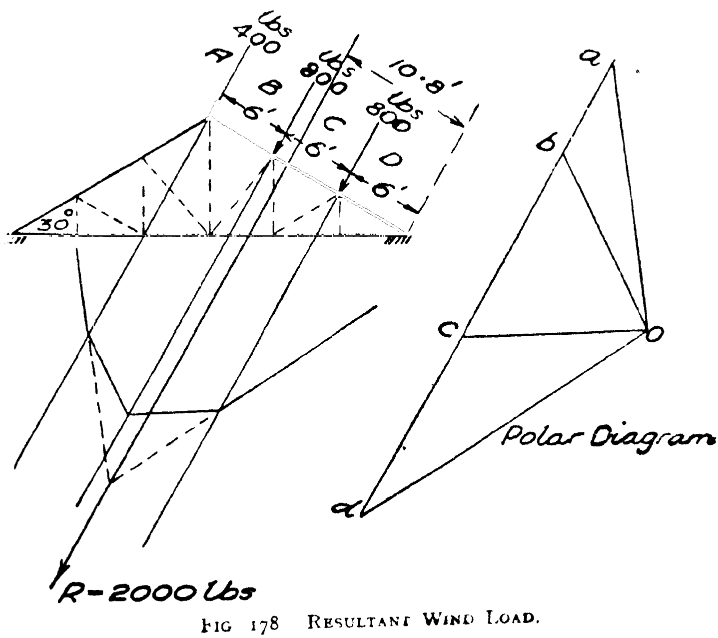


FIG 178 RESULTANT WIND LOAD.

EXAMPLE (ii).—*The loads shown acting on the roof truss in Fig. 178 are due to wind acting from the right. Find, by link polygon construction, the position of the resultant wind thrust on the truss.*

The load line is drawn, to a suitable force scale, *parallel to the wind loads.* The pole ' O ' is chosen in any convenient position and the link polygon constructed in the usual way. The resultant wind load is a force of 2000 lb., acting at 10·8 ft. from the right-end reaction point and in a direction normal to the roof slope.

Experimental Demonstration of the Link Polygon

The apparatus shown in Fig. 179 may be used to illustrate the nature of the link polygon. The fine string is suitably loaded so as to hang in segments in front of the wall board. The string

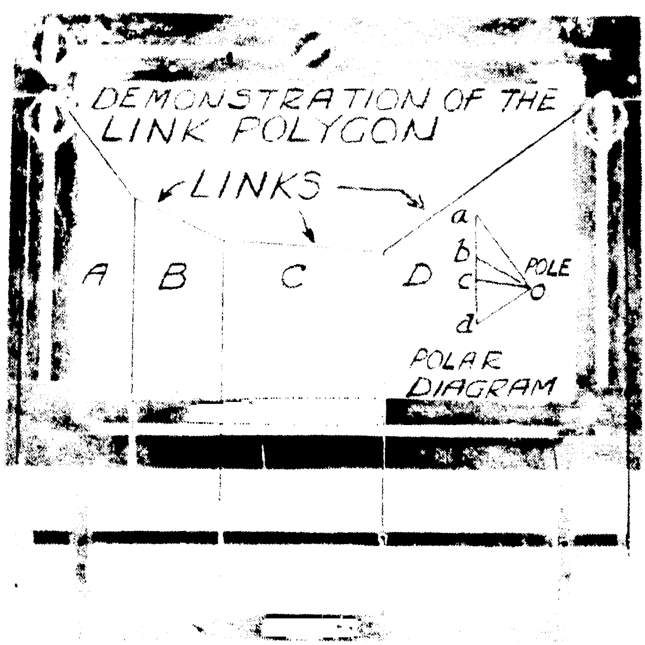


FIG. 179.—THE LINK POLYGON.

directions are transferred to the board and the magnitudes of the weights noted. 'AB,' 'BC' and 'CD' form the load system.

If these three loads be set out as a '*load line*' and lines be drawn parallel to the string segments, from the appropriate points, the lines will all intersect in one common point—the '*pole*' in the usual construction. The lengths of the top and bottom rays in the built-up '*polar diagram*' will be found to represent the magnitudes of the suspended weights at the left and right ends of the string respectively. If spring balances were inserted in the string segments, their readings would correspond to the force values given by the corresponding '*rays*' in the polar diagram.

The experimental apparatus clearly demonstrates the 'vector diagram' nature of the 'polar diagram,' and the fact that the '*links*' represent lines of action of component forces.

Graphical Solution of Centre of Gravity Problems

When a body or an 'area' is composed of several component parts the link polygon construction is a convenient method of graphical solution. We may imagine the component parts as being subjected to a system of parallel 'pulls,' acting in any

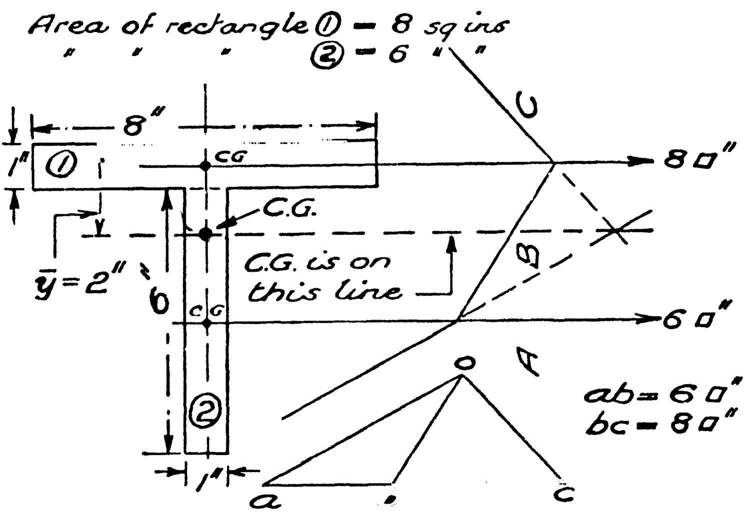


FIG. 180—C.G. BY LINK POLYGON.

desired direction. Using the procedure previously described we can then fix the line of action of the resultant 'pull' for the chosen direction. The C.G. will lie somewhere along this line of action. By selecting another direction and repeating the method for this case, another line will be obtained along which the C.G. must lie. The C.G. will be the intersection point of any two such lines. The horizontal and vertical directions are usually the most convenient to take.

EXAMPLE.—*Find, by graphical construction, the height above base of the C.G. of the T-section given in Fig.* 180 (page 121).

The horizontal direction is chosen for the 'pulls' in this case. Draw the section to a scale of $\frac{3}{4}'' = 1''$. Divide up into two rectangles, marked (1) and (2) respectively in Fig. 180.

Area of rectangle (1) = $8'' \times 1'' = 8$ sq. ins.
,, ,, ,, (2) = $6'' \times 1'' = 6$,, ,,

Set off '*ab*' to represent 'AB' (= 6 sq. ins.) and '*bc*' to represent 'BC' (= 8 sq. ins.). The area scale should be about $\frac{3}{8}'' =$ 1 sq. in. Choose any pole 'O' (above or below the line '*ac*') and construct the polar and link polygons.

The C.G. is on the vertical axis of symmetry of the section and is 2 ins. below the top of the section.

EXAMPLE.—*By means of the link polygon method, determine the C.G. of the Z-section given in Fig.* 181.

The section is divided into three rectangles:

Area of rectangle (1) = $6'' \times 2'' = 12$ sq. ins.
,, ,, ,, (2) = $8'' \times 2'' = 16$,, ,,
,, ,, ,, (3) = $10'' \times 2'' = 20$,, ,,

The complete graphical solution is given in Fig. 181. The C.G. is $6\frac{1}{8}$ ins. from the left edge of the section and $5\frac{1}{8}$ ins. from the bottom.

Recommended scales: Linear: $\frac{3}{8}'' = 1$ in.
Area: $1'' = 10$ sq. ins.

Centre of Gravity of an Irregular Figure

As shown in Fig. 182, the link polygon construction may be employed to determine the C.G. of an irregular figure. The area is divided up into thin strips, the number of strips taken being

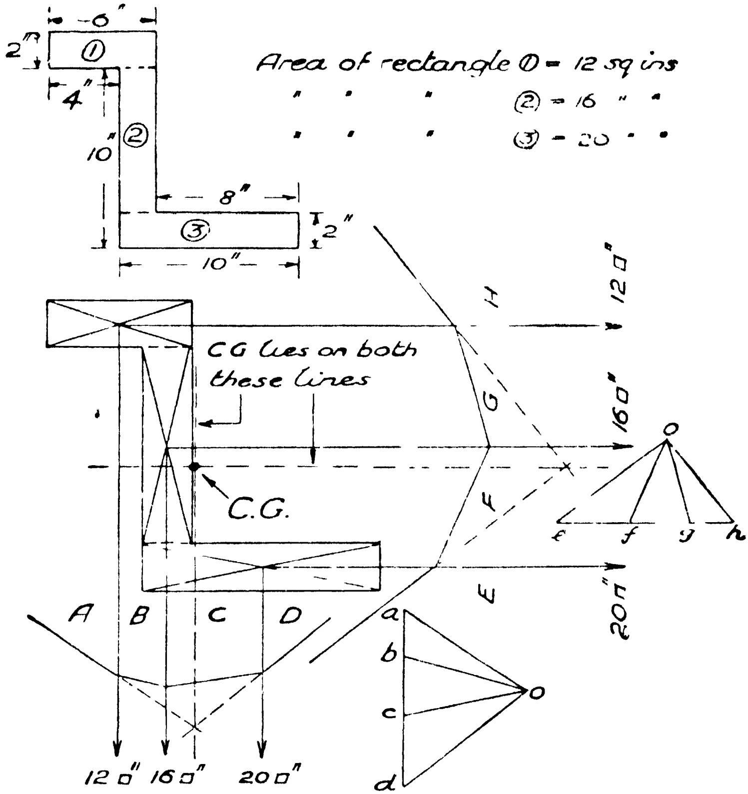


FIG 181.—C G OF STRUCTURAL SECTION.

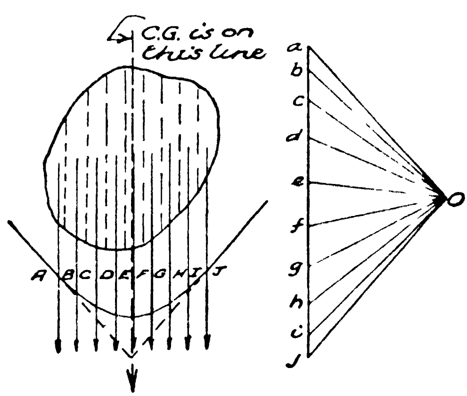
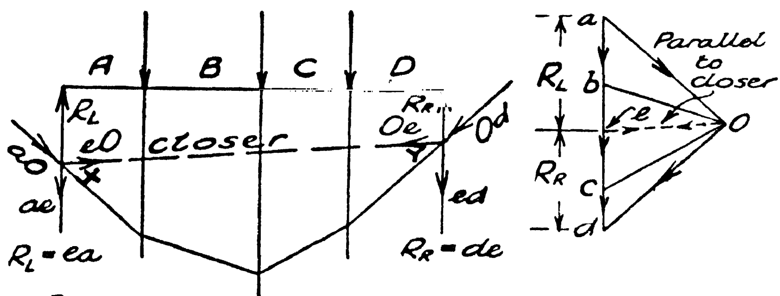


FIG 182—IRREGULAR FIGURE.

governed by the nature of the outline of the figure. The area of each strip is computed and assumed to 'act' vertically downwards through the C.G. of the strip, as indicated. The C.G. of a strip may, in the usual case, be taken to be at mid-width of the strip. Having obtained one line, along which the C.G. must lie, the construction is repeated for another direction, e.g. horizontal. The second line obtained will intersect the first line at the required C.G.

Beam Support Reactions by Link Polygon

Procedure (see Fig. 183).—Draw the beam to a convenient scale and indicate the lines of action of the respective loads. If the beam should carry a distributed load system, the load must be divided up into a number of portions and the weight of each portion regarded as acting through its own centre of gravity. The number of portions chosen will be decided by the nature of the loading.



R_L = left end reaction. R_R = right end reaction

FIG. 183.—BEAM-SUPPORT REACTIONS.

A polar diagram is drawn for the loads and the link polygon constructed therefrom exactly in the manner previously described. The two outer links (in spaces 'A' and 'D') are not produced in this case to intersect. The points 'X' and 'Y' are found, these being the points in which the two outer links cut the reaction lines, respectively at the left end and right end of the beam. Points 'X' and 'Y' are joined, forming a line to which the name '*closer*' is given.

From the pole 'O' in the polar diagram a line 'O*e*' is drawn, parallel to the 'closer.' The vector line '*ea*' will represent the

left-end reaction and '*de*' will represent the *right-end reaction*, both, of course, to the force scale of the polar diagram.

Proof.—As previously shown on page 118, the force system 'AB,' 'BC' and 'CD' is equivalent to the two forces '*a*O' (in space 'A') and 'O*d*' (in space 'D').

At point 'X,' by means of the resolution triangle '*a*O*e*' resolve force '*a*O' into force '*ae*' (vertically downwards) and force '*e*O' (along the closer). Similarly at point 'Y,' by means of resolution triangle 'O*de*,' resolve force 'O*d*' into force '*ed*' (vertically downwards) and force 'O*e*' along the closer. The two forces along the closer, being '*equal and opposite*,' cancel out and we are left with force '*ae*' (at the left-end reaction point) and force '*ed*' (at the right-end reaction point). To balance these forces the support at the left end must exert an upward force equal to '*ea*' and, at the right end, the support must provide a reaction equal to '*de*.' The reader will note that in the preliminary drawing-in of the links in the appropriate spaces the reaction lines are completely ignored. The reaction lines do not form the boundary lines of a '*space*,' and enter into the construction only at the point when their intersection with the outer links is being considered.

EXAMPLE.—*Find the support reactions for the beam given in Fig.* 184 (i) *by a graphical method,* (ii) *by calculation.*

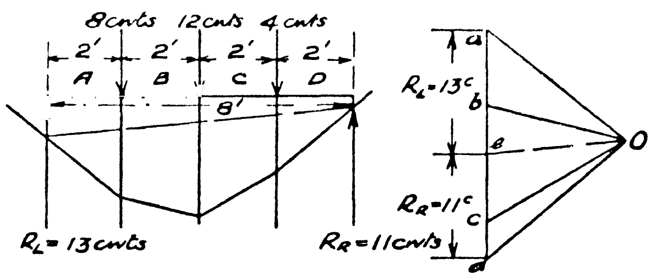


FIG. 184.—EXAMPLE ON REACTIONS.

In Fig. 184 the load line '*abcd*' is drawn and any pole 'O' chosen. The link polygon is constructed, the closer line drawn in and a line 'O*e*' is drawn parallel to the closer from the pole 'O.' In this case the line '*de*' scales 11 cwts. (= R_R) and '*ea*' scales 13 cwts. (= R_L).

It does not matter at what point in space 'A' the link polygon is commenced; and if the link in the space 'D' should come above the beam line, produce the right-end reaction line upwards to obtain the point of intersection for the closer.

Suggested scales : Linear, $\frac{3}{4}'' = 1$ ft.
Force, $1'' = 4$ cwts.

Calculation of reactions :

$$\begin{aligned} R_L \times 8 &= (8 \times 6) + (12 \times 4) + (4 \times 2) \\ &= 48 + 48 + 8 = 104 \\ R_L &= 104/8 \text{ cwts.} = 13 \text{ cwts.} \\ R_R \times 8 &= (4 \times 6) + (12 \times 4) + (8 \times 2) \\ &= 24 + 48 + 16 = 88 \\ R_R &= 88/8 \text{ cwts.} = 11 \text{ cwts.} \end{aligned}$$

EXAMPLE.—*Apply the link polygon construction to the example shown in Fig. 185, in which the support reactions are required for a beam which overhangs at both ends.*

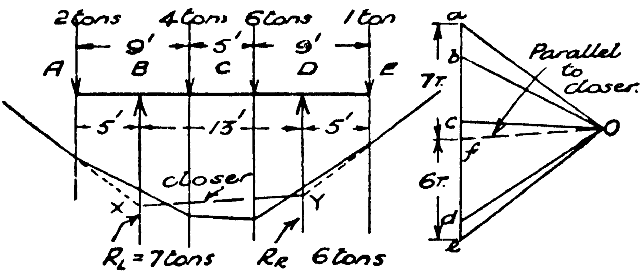


FIG. 185.—OVERHANGING BEAM.

Proceed as in the last example. When drawing the link in space 'B,' ignore altogether the reaction line of 'R_L.' Similarly, disregard the reaction line for 'R_R' when drawing the link in space 'D.' Produce the links in spaces 'A' and 'E' until they cut their respective reaction lines (at 'X' and 'Y'). Join 'XY' to form the 'closer' and proceed as in last example.

$$R_L = fa = 7 \text{ tons.}$$
$$R_R = ef = 6 \text{ tons.}$$

Suggested scales : Linear : $\frac{1}{4}'' = 1$ ft.
Force : $1'' = 2$ tons.

The reader should check these reactions by the method shown on page 60.

Reactions for a Roof Truss

When a frame, such as a roof truss, carries loads inclined to the vertical, at least one reaction must be inclined. The practical treatment of reactions for trusses with inclined loads is considered in Chapter VIII.

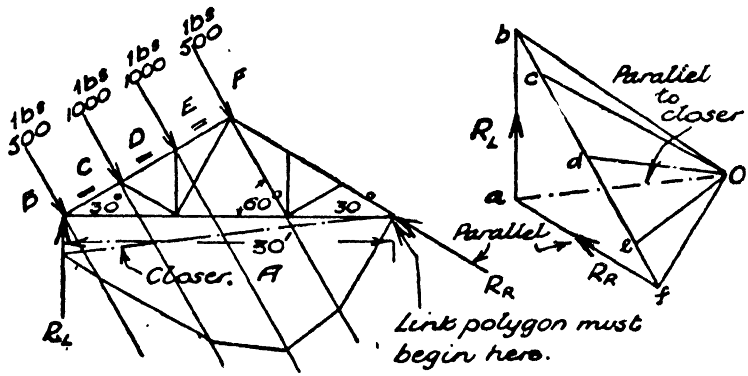


FIG 186.—REACTIONS FOR WIND LOADS.

In the example given in Fig. 186, the left-end reaction (R_L) is assumed to be vertical. With this assumption the value of 'R_L' and the direction and value of 'R_R' may be determined.

EXAMPLE.—*Find, by link polygon method, the support reactions for the roof truss shown in Fig. 186. The truss carries wind loads acting inwards, normally to the roof slope.*

Having drawn the load line '*bcdef*,' a pole 'O' is chosen in any position and the polar diagram completed in the usual manner. (The letter 'A' is placed in the space between the support reactions to correspond with the procedure adopted in Chapter VIII.) As the direction of 'R_R' is unknown, it is *necessary to begin the link polygon at the right-end reaction point.* If this were not done it would be impossible to ascertain the point in which the link in space 'F' (i.e. the link parallel to '*Of*') would cut the reaction line, as the direction of the reaction cannot at this stage be drawn-in. The only point we are certain will be on the reaction line is the *right-end reaction point*, hence

we arrange the link in space 'F' to go through this point and construct the link polygon by working towards the left. The intersection of the link in space 'B' with the left-end reaction is determinable, hence the 'closer' may be constructed.

The left-end reaction being vertical in this case, a vertical line is drawn downwards from '*b*' in the polar diagram, to cut a line drawn from 'O,' parallel to the closer, in the point '*a*' Point '*f*,' in the load line, is joined to '*a*.'

'*fa*' will represent the right-end reaction The left-end reaction will be represented by the vector line '*ab*' The proof is similar to that given on page 125

The magnitudes of both reactions will be found to be 1732 lb

Suggested scales 1" = 4 ft. and 1" = 500 lb

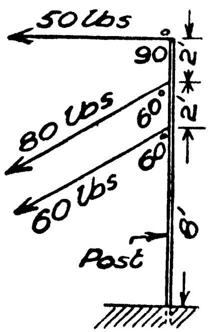


FIG 187

Further examples illustrating the employment of the link polygon occur throughout the book

EXERCISES 7

(1) A vertical post 12 ft high is pulled by three wires as shown in Fig 187 Find, by means of a link polygon, the resultant pull on the post.

(2) The wheel loads given in Fig 188 are transmitted to a girder by the carriage of a travelling crane. Find (i) by link polygon, (ii) by calculation method, the distance, from left-hand wheel, at which the resultant wheel load acts.

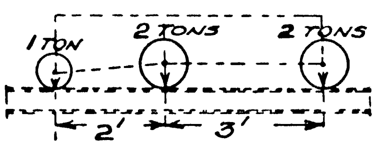


FIG 188

(3) Find, by link polygon construction, the centre of gravity of the unequal angle section given in Fig 189.

(4) Verify, by means of a link polygon, that the C G. of the girder section shown in Fig. 190 is 4" above the bottom of the section.

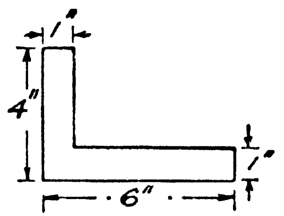


FIG. 189.

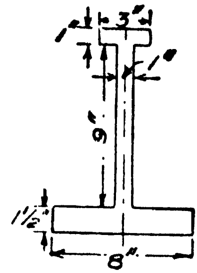


FIG. 190.

(5) In order to test the stability of the retaining wall represented by the section given in Fig. 191, it is necessary to determine the distance of the C.G. of the section from the vertical back of the wall. Find this distance by the link polygon method.

(If a '*line of action*' through the centre of gravity of one component area be coincident with that through another, the two

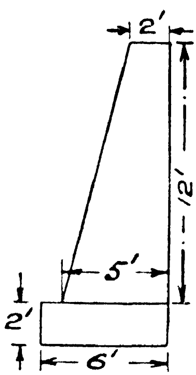


FIG. 191.

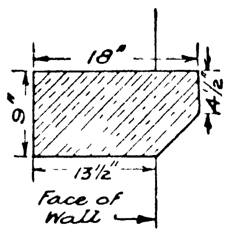


FIG. 192.

areas may be regarded as being one combined area for purposes of the link polygon construction.)

(6) Fig. 192 shows the section of a stone corbel built into a brick wall. Show, by link polygon construction, that the centre of gravity of the corbel section is 5" from the face of the wall.

(Divide up the section into simple component figures, e.g. two rectangles and a triangle.)

(7) Find the beam support reactions for each of the beam examples given in Fig. 193 (i) by link polygon, (ii) by calculation.

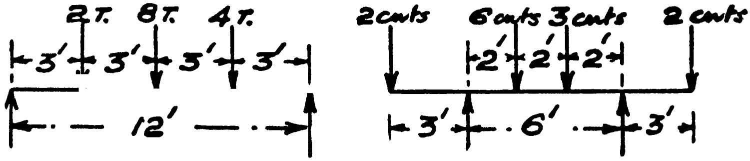


FIG. 193.

(8) Obtain, graphically, the support reactions for the given braced girder (Fig. 194).

(The ' panel ' loads shown may be regarded as acting (in their respective vertical lines of action) on a simple beam 30 ft. long.)

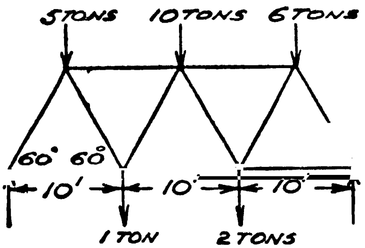


FIG. 194.

(9) The roof truss given in Fig. 195 carries four inclined wind loads (acting at 90° to roof slope). Find the value of each support reaction by means of a link polygon.

(Commence the link polygon at the fixed end, i.e. the left-end reaction point. Carefully watch the correspondence between ' *space* ' letters and polar diagram ' *ray* ' letters. The ray corresponding to the space letter to the left of the left-hand rafter will not be required to be

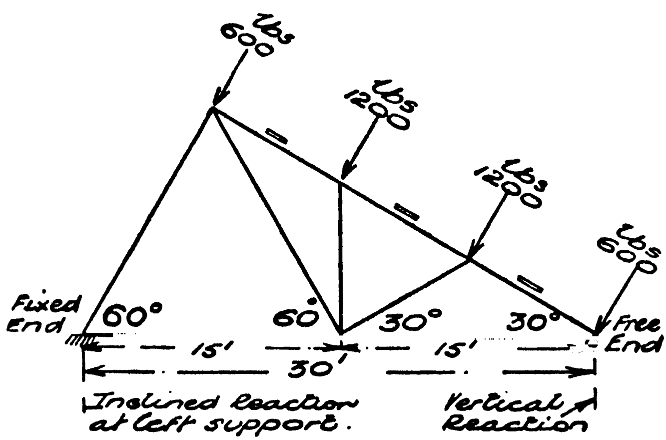


FIG. 195.

considered, as the link to be drawn parallel to it passes through the reaction point and need not actually be drawn in.)

(10) A law of equilibrium states '*the link polygon for a system of non-concurrent forces in equilibrium is a closed diagram.*'

Fig. 196 shows a non-concurrent system of four forces in equilibrium, viz. 4 lb., 2 lb., 2 lb. and 2 lb. respectively, with the corresponding force polygon. Draw out a space diagram according to the data given. Taking a '*pole*' in the approximate position shown, construct a link polygon and verify the law given above.

(An example of such a link polygon is shown in broken lines.)

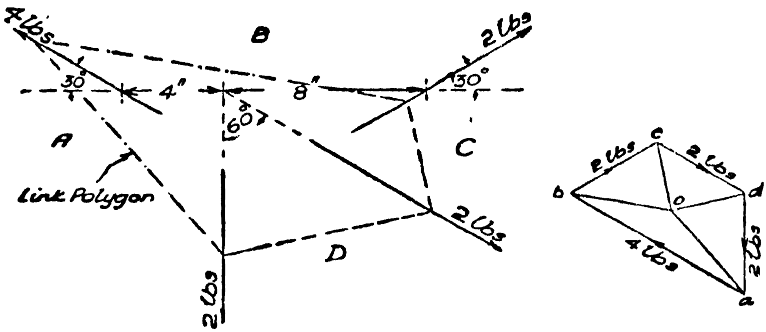


FIG. 196.—NON-CONCURRENT SYSTEM IN EQUILIBRIUM.

(11) Find the support reactions for the beam given in Fig. 197 (i) by link polygon, (ii) by calculation method.

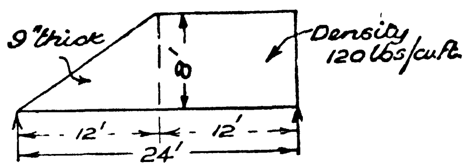


FIG. 197.

(12) Check the position of the resultant wind load for the example given in Fig. 112 by means of a link polygon.

CHAPTER VIII

CONSTRUCTION OF A STRESS OR FORCE DIAGRAM FOR A LOADED FRAME

Introduction.—Fig. 198 shows an experimental roof truss. When a load is placed on the hook at the apex of the truss it will be observed that the various members forming the frame alter in length—the inclined members become shorter and the horizontal string member is extended.

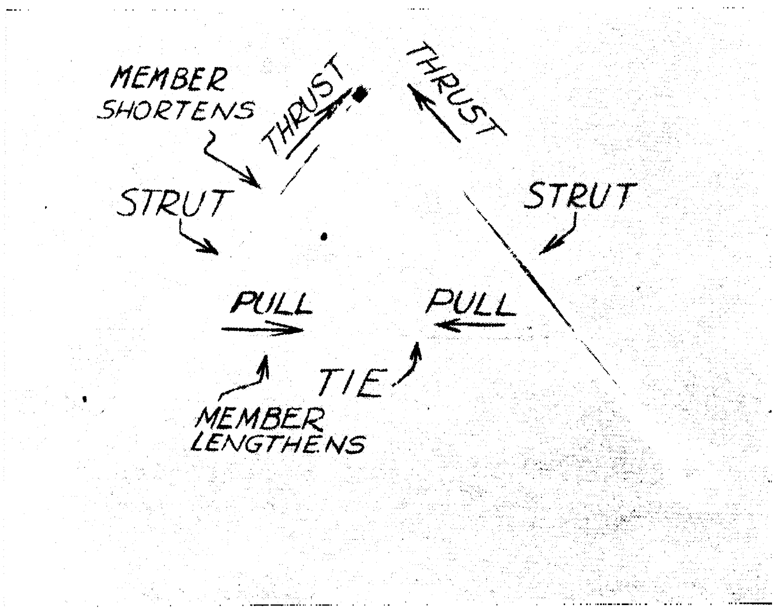


FIG 198.—MODEL ROOF TRUSS.

Alteration in length is just what members of a practical frame, such as a steel or timber roof truss or a lattice girder, undergo when external load is applied to the frame. We may regard the members of a frame as being very strong springs, and as such they tend to resist any attempt at deformation. Members which are extended are said to be in *tension.* They exert pulls on their end connections and are termed '*ties.*' Those members which are

shortened are in '*compression*' and are termed '*struts.*' A 'strut' exerts a thrust at both its ends (Fig. 199).

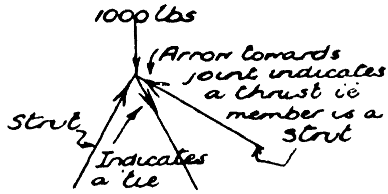


FIG. 199.—STRUTS AND TIES.

The purpose of drawing a *stress diagram* is to determine which members of a loaded frame are 'struts' and which are 'ties,' and also to ascertain the magnitude of the force in each member. A better name for a 'stress diagram' would be '*force diagram.*' This name is sometimes applied, but 'stress diagram' is distinctive of the type of force diagram now under consideration.

Assumptions made.—It is assumed that each member pulls or pushes in the direction of its length. If we draw a diagram showing the centre lines of the members forming the frame (the '*frame*' or '*space*' diagram) we may regard these lines as indicating the lines of action of the various member forces. For the typical practical frame, in which the joints are not constructed with a view to possessing special rigidity, this is a reasonable assumption.

It is also assumed that the loading may be reduced to a system of point loads, acting at the joints or '*nodes*' of the frame. In the case of a roof truss the 'purlins,' which actually transmit the roof load to the truss, are situated at, or near, the joints. Loads applied between joints tend to cause bending in the members, and special calculations may have to be made to allow for this. For the purpose of stress-diagram construction such loads are proportioned between the adjacent joints in the manner of beam support reaction calculations.

Calculation of Joint Loads for a Roof Truss

In examination problems the joint loads are usually given. The examples following illustrate the methods of obtaining these loads. It is assumed that the purlins transmit the roof load to the truss at the joints.

EXAMPLE (i).—*Calculate the joint loads for the roof truss given in Fig.* 200. *The roof covering is composed of the material indicated in the following list of weights:*

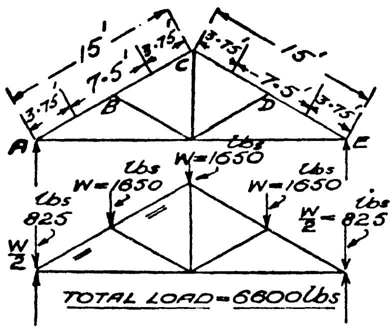


FIG. 200.—JOINT LOADS FOR TRUSS.

Purlins and ridge:	1 *lb. per*	*sq. foot*	*of*	*roof*	*area.*
Common rafters:	$1\frac{5}{8}$,,	,,		,,	,,
Roof boarding:	3 ,,	,,		,,	,,
Felt:	1 ,,	,,		,,	,,
Slating laths:	$\frac{5}{8}$,,	,,		,,	,,
Slates:	9 ,,	,,		,,	,,
Possible snow (say):	3 ,,	,,		,,	,,
	$19\frac{1}{4}$ *lb. per*	*sq. foot*	*of*	*roof*	*area.*

The trusses are spaced at 10-*ft. centres.*

Each truss has to support a roof area of $(15 + 15)$ ft. $\times$ 10 ft. $=300$ sq. ft. The total roof load for one truss is therefore $(300 \times 19{\cdot}25)$ lb. $= 5775$ lb., say, 6000 lb. The self-weight of the truss is added in to the roof-covering load, and for this a provisional estimate must be made. Experience of previous similar calculations will indicate the allowance to be made.

The reader is referred to text-books on building construction, etc.,* for formulæ and graphs which will also assist in arriving at a reasonable figure for the self-weight of a particular type of truss.

The truss will be assumed to weigh 600 lb. in the present example.

Total load carried by one truss $= (6000 + 600)$ lb.
$= 6600$ lb.

* *Experimental Building Science*, Vol. II. Manson and Drury.

We have now to divide up this load proportionately between the joints.

Area of roof for joint 'A' = $3{\cdot}75' \times 10' = 37{\cdot}5$ sq. ft.
" " " 'B' = $7{\cdot}5' \times 10' = 75{\cdot}0$ " "

The loads at 'B,' 'C' and 'D' will be equal and each will be twice that at 'A' or 'E.'

Let 'W' lb. = load at 'B,' 'C' or 'D.'
$\therefore$ W/2 lb. = load at 'A' or 'E.'
$\therefore 3W + 2(W/2) = 6600.$
$\therefore W = 1650$ lb.

EXAMPLE (ii).—*Assuming the data given in Example (i), calculate the joint loads for the truss given in Fig.* 201. *The trusses are at* 10-*ft. centres.*

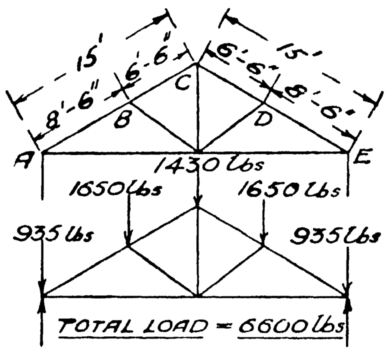


FIG. 201.—JOINT LOADS.

As in previous example, the total load carried by the truss = 6600 lb.

Area of roof for joint	'A' = $4{\cdot}25' \times 10'$		= $42{\cdot}5$ sq. ft.
" " "	'B' = $(4{\cdot}25' + 3{\cdot}25') \times 10'$		= $75{\cdot}0$ " "
" " "	'C' = $(3{\cdot}25' + 3{\cdot}25') \times 10'$		= $65{\cdot}0$ " "
" " "	'D' = (as 'B')		= $75{\cdot}0$ " "
" " "	'E' = (as 'A')		= $42{\cdot}5$ " "
			$300{\cdot}0$ sq. ft.

The load at any particular joint will be proportional to the area of roof associated with it.

$\therefore$ Joint load at 'A' = joint load at 'E'

$$= \frac{42 \cdot 5}{300} \times 6600 \text{ lb.} = 935 \text{ lb.}$$

$\therefore$ Joint load at 'B' = joint load at 'D'

$$= \frac{75}{300} \times 6600 \text{ lb.} = 1650 \text{ lb.}$$

$$\text{Joint load at 'C'} = \frac{65}{300} \times 6600 \text{ lb.} = 1430 \text{ lb.}$$

$$\text{Total load} = [(2 \times 935) + (2 \times 1650) + 1430] \text{ lb.} = 6600 \text{ lb.}$$

Wind Loads on Roof Trusses.—The magnitude and treatment of wind pressure in the design of a structure depend upon the nature of the structure. In the case of roof trusses the following points should be noted :

(i) The wind may blow from the left or from the right on to the truss. The two cases may have to be considered independently in order to arrive at the worst case for any particular member of the truss.

(ii) It is the wind load acting at right angles to the roof slope which has to be considered. Empirical formulæ are sometimes used in order to derive a practical value for this component pressure from a given horizontal wind pressure. Some building regulations give the necessary normal wind pressure directly.

(iii) The windward side of the roof truss will have '*positive wind loads*,' i.e. loads acting inwards towards the roof slope. The suction effect produced on the leeward side of the truss requires '*negative wind loads*' to be taken. These act outwards at right angles to the leeward slope. Some regulations give 15 lb. per sq. foot of windward roof area for the total positive wind load, and 10 lb. per sq. foot of leeward roof area for the total negative wind load.

(iv) As in the case of dead loads, all wind loads are actually transmitted to the roof truss through the purlins. The usual example will therefore have both the dead loads and the wind loads acting at the joints of the truss.

An example illustrating the calculation of wind loads will be found on page 153.

Nature of a Stress Diagram

A stress diagram is an amalgamation into one diagram of a number of force polygons. Consider the example of the north-light roof truss given in Fig. 202. For the loading given, the support reactions are: 1050 lb. (left end) and 1150 lb. (right end).

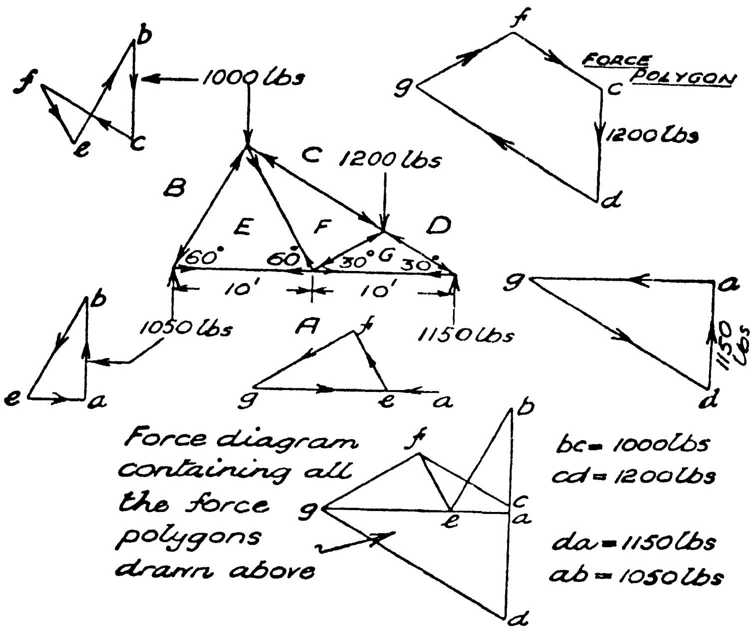


FIG. 202.—COMPOSITION OF A STRESS DIAGRAM.

The triangle of forces '*abe*' is drawn for the three forces acting at the left-end reaction point. We find that member 'BE' is a 'strut,' i.e. it exerts a thrust at the joint. The magnitude of the thrust may be found by scaling the vector line '*be*.' Member 'BE' will also exert this thrust at the apex joint of the truss. This fact enables us to construct the vector line '*eb*' (see top left-hand diagram), and knowing that '*bc*' must scale 1000 lb. (the external load at the apex), we are enabled to complete the force polygon for the four forces at the apex joint. By taking the various joints in a proper sequence—and using the results obtained at one joint to aid the solution of the next—the force polygon for each joint of the given truss has been constructed, as

indicated in Fig. 202. We have thus completely 'solved' the truss.

Now examine the lower diagram shown in the figure. The reader should have no difficulty in tracing out, in this diagram, each of the five force polygons previously drawn for the respective five joints of the truss. Such a diagram—a composition of a number of force polygons—is termed a '*stress diagram.*' We have now to consider how to construct a stress diagram directly, without having to draw a number of separate force polygons.

Rules for Constructing a Stress Diagram

Taking the typical example given in Fig. 202, the steps in the procedure are as follows:

(1) Draw a scale diagram of the frame, indicating the loads acting at the various joints.

(2) Obtain the support reactions and mark these clearly at their respective positions in the frame (or 'space') diagram.

(3) Letter-up the spaces by Bow's notation. Throughout the book, the letter 'A' will be allocated to the space between the two reactions. The letter 'B' will occupy the first space at the left end of the truss and lettering will proceed round the frame in a clockwise manner, completing first the external spaces formed by the loads.

(4) Draw the '*load line bcd*' to a suitable force scale. Insert the letter '*a*' on this line by marking off '*da*' to represent '1150 lb.' (i.e. the right-end reaction). (The completed load line represents the '*force polygon*' for the external load system.)

(5) Consider the truss joint by joint. First take the left-end reaction joint. From '*b*' (in the 'load line') draw '*be*' parallel to 'BE' and from '*a*' draw '*ae*' parallel to 'EA.' (In order to get the intersection point '*e*' we have to draw from the point '*a*' towards the left.)

(6) Having completed the portion of the stress diagram for a given joint, determine whether the members concerned are '*struts*' or '*ties.*' (The rules for this are given later.)

(7) Take the next convenient joint in the frame—the apex joint in the given example. Considering the forces round the joint in clockwise order, we have 'BC,' 'CF,' 'FE' and 'EB.'

Draw '*cf*' and '*ef*' from '*c*' and '*e*' respectively parallel to forces 'CF' and 'FE.' This fixes the point '*f*' in the stress diagram. The forces in clockwise order round the central joint in the bottom tie are 'AE,' 'EF,' 'FG' and 'GA.' 'AE' and 'EF' are already represented in the stress diagram, so we draw '*fg*' parallel to 'FG' until it meets '*ag*' (parallel to 'GA') in the point '*g*.' Thus the point '*g*' is fixed. The rafter joint at which the '1200-lb.' load acts and the right-end reaction joint are similarly dealt with. The accuracy of work in the construction of a stress diagram is checked by the fact that a '*stress diagram must close.*' In the example taken, the accuracy will be checked by ascertaining whether the vector line drawn from '*d*' parallel to member 'DG' passes through the point '*g*,' which has been already fixed in the stress diagram.

(8) The stress diagram being now complete, it is usual to draw up a table, as indicated in Fig. 203, giving the force in each member. The force in any given member will be obtained by scaling (to the force scale of the 'load line') the corresponding vector line in the stress diagram.

Determination of 'Struts' and 'Ties'

The method will be explained by taking one particular joint in the previous example, i.e. the apex joint of the truss.

Take the member letters in *clockwise order* round the joint, thus: 'CF,' 'FE' and 'EB.' Considering, firstly, the member 'CF,' examine the stress diagram and ascertain in which direction you have to proceed along the appropriate vector line '*cf*' in order to obtain the same sequence of letters. This is clearly '*c*' to '*f*' (*not* '*f*' to '*c*'), i.e. upwards towards the left. Place an arrow *at the apex joint on the member 'CF'* in this direction. The arrow is pointing towards the joint, indicating that the member is pushing (see Fig. 202). Member 'CF' is therefore a '*strut.*' For member 'FE,' the order of letters indicates that we must proceed from '*f*' to '*e*' in the stress diagram, i.e. downwards towards the right. The arrow on 'FE' at the apex joint therefore denotes a 'pull,' i.e. member 'FE' is a '*tie.*' Similarly, the nature of the force in any member at any joint may be determined

General Remarks.—The following points should be carefully noted :

(i) Concentrate on one joint only at a time. See that all the ' letters ' found are placed in the stress diagram.

(ii) Place the arrow heads on the frame diagram before you leave a given joint. The arrow heads should be placed on the members fairly near the joint considered, to avoid possible error.

(iii) If you are held up at a joint because there are too many ' letters ' unfixed in the stress diagram, it is probably because you have missed a joint which will provide you with the necessary letters.

The reader is strongly advised to draw the stress diagram for the example given in Fig. 202.

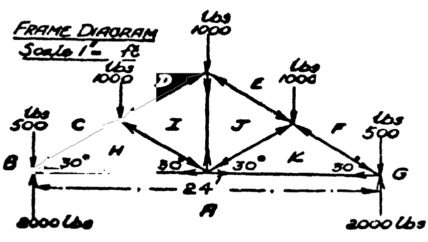
Suggested scales :

$1'' = 4$ ft. for the frame diagram, and
$1'' = 400$ lb. for the stress diagram.

Types of Stress Diagrams

1. Truss with Symmetrical Loads

In this case the two support reactions are equal and the point ' *a* ' is the mid-point of the load line. Fig. 203 shows the stress diagram for a king-post roof truss. The reader should develop this diagram himself, following the rules given on page 138.



MEMBER	FORCE	TYPE
CH	3000 Lbs	STRUT
DI	2000 "	"
EJ	2000 "	"
FK	3000 "	"
AK	2600 "	TIE
AH	2600 "	"
HI	1000 "	STRUT
[illegible]	1000 "	TIE
JK	1000 "	STRUT

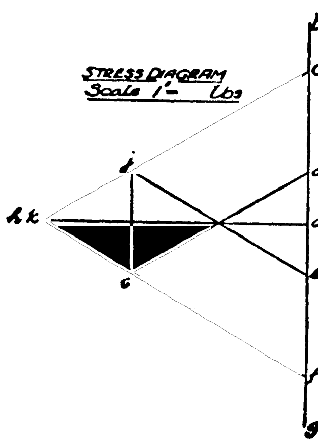


FIG. 203.—TRUSS WITH SYMMETRICAL LOADS.

A suitable table for recording the force in each member is shown in the figure. In the case of a symmetrical truss with symmetrical loads the stress diagram will be a symmetrical figure about a horizontal line through the point '*a*.' It is advisable in examination work, if time permits, to complete the whole stress diagram even though half the diagram would be sufficient to obtain all the member forces. The 'closure' check may thereby be applied.

2. Truss with Unsymmetrical Loads

Fig. 204 shows an example of this case. Another example is given in Fig. 205. In such examples the support reactions must first be obtained. By means of the reaction values, the point '*a*' is fixed (see rule 4, page 138).

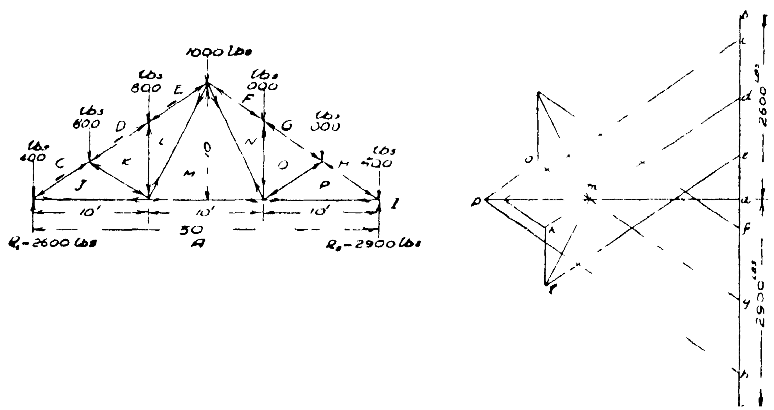


FIG 204—UNSYMMETRICAL LOADS.

Note that the letter 'A' is associated with all the bottom horizontal members of the truss. The reader should find no difficulty in tracing the development of the stress diagram. In Fig. 204 the three joints on the left-hand rafter were first considered and, as the apex joint could not then be proceeded with, the next joint taken was the lower joint 'JKLMA.'

3. Truss with Redundant Members

A '*redundant member*' is one which could be removed from a frame without, theoretically, causing its collapse, the remaining

members being capable of maintaining static equilibrium—provided the load system remain unaltered. Distortion of the frame, on a change of the load system, might make such a member a *practical* necessity. Figs. 205 and 206 illustrate typical cases of redundant members.

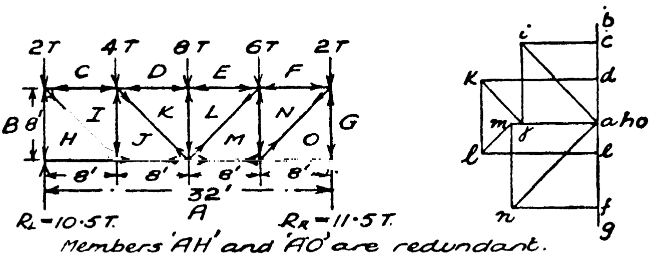


FIG. 205.—REDUNDANT MEMBERS.

When dealing with complicated load systems, e.g. combined vertical and inclined loads, it is sometimes convenient to break up into simpler component systems. A member may be redundant for one of these load systems and not for another.

In Fig. 205, members 'AH' and 'AO' are redundant. The vector lines representing these forces in the stress diagram must be of zero length. We therefore place the letters '*h*' and '*o*' on the load line at the position of the letter '*a*.'

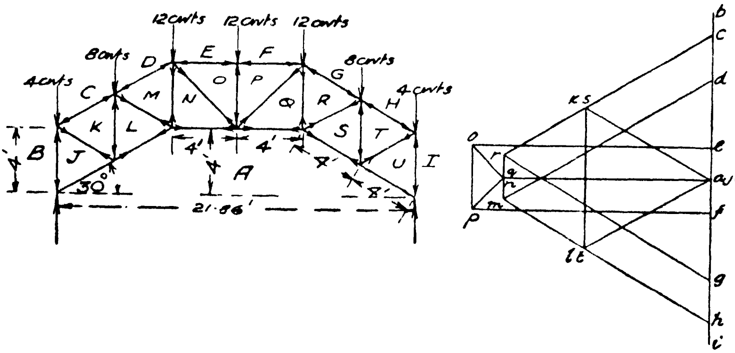


FIG. 206.—FOOT BRIDGE TRUSS.

If we consider equilibrium at the left end of member 'AH,' we find that the forces in member 'BH' and reaction 'AB' are both vertical. Therefore for horizontal equilibrium member 'AH' cannot be exerting any force. Similarly, in Fig. 206, the

force in member 'AJ' must be zero. If member 'AJ' did exert a force, it would have a horizontal component. This is impossible, as there is no other horizontal force at the left-end reaction point to balance it. Member 'AU' is also redundant.

4. Truss with Loads on the Bottom Tie

It is convenient in this case (Fig. 207) to place the letter 'A' in the space nearest to the left-end reaction. Note that in this example there are several letters beneath the bottom tie. In setting out the '*load line*' the vector lines must be drawn with due regard to the direction of the forces concerned. Thus '*gh*' is measured upwards (because the reaction 'GH' acts upwards), and '*hi*' is plotted downwards (because load 'HI' acts downwards).

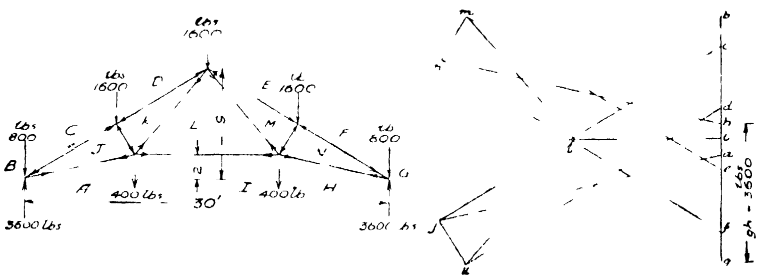


FIG. 207.—LOADS ON BOTTOM TIE.

Having fixed the point '*a*' in the load line, the vector line '*ab*' should be verified—it should scale 3600 lb. The setting out of the load line having been checked, the stress diagram is constructed in the usual way.

5. Truss with Unsymmetrical Loads Solved with the Aid of a Link Polygon

The position of the point '*a*' in the load line (Fig. 208) is fixed by drawing in the closer line in the link polygon and drawing '*Oa*' parallel to the closer (see page 124).

6. Cantilever Truss

In trusses of the type given in Fig. 209 we may proceed with the stress diagram without first obtaining the reactions. The

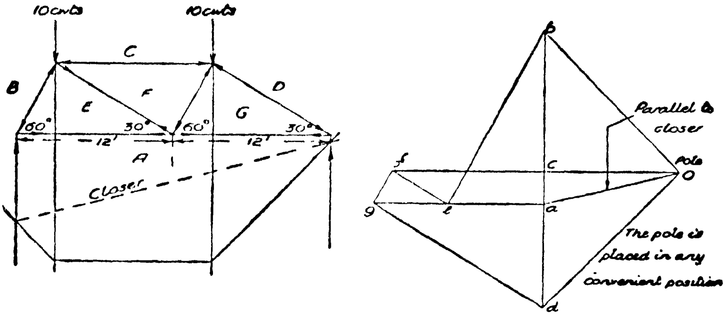


FIG. 208.—REACTIONS BY LINK POLYGON.

load line '*bcdefg*' having been drawn, the joint at the extreme right end of the truss is first dealt with. The stress diagram does not begin to grow from the top of the load line in this example, but from the position where the vector line '*de*' is situated. It will be noted that, in this case, the stress diagram develops to the right of the load line. If the truss has a vertical member connecting the reaction points, we require to know the direction of one of the reactions before this member can be solved.

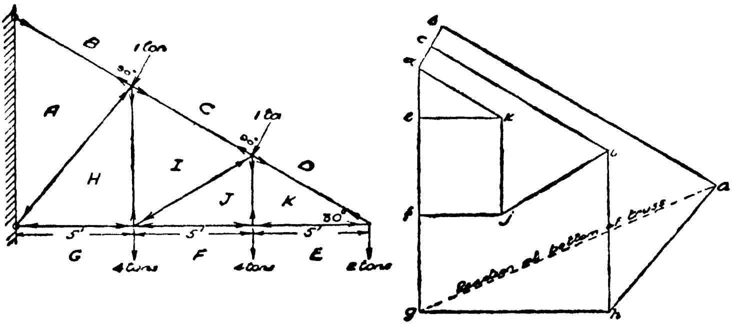


FIG. 209.—CANTILEVER TRUSS.

7. Truss Overhanging its Supports

In Fig. 210, the truss overhangs the supports at both ends. The reader should construct the stress diagram to the following scales: 1" = 4 ft. and 1" = 4 cwts.

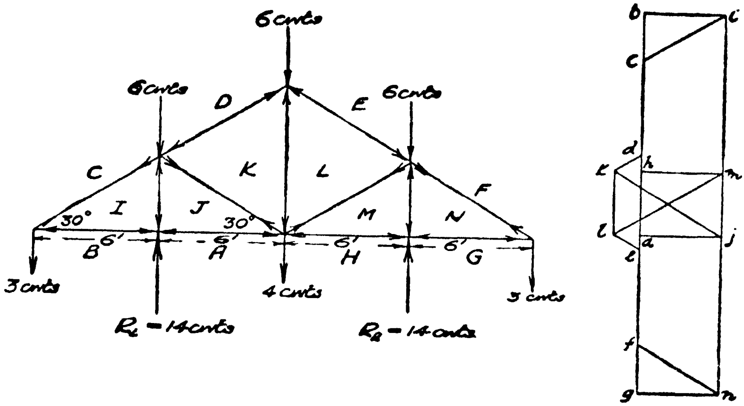


FIG. 210.—OVERHANGING TRUSS.

8. **Truss Solved by Means of a Calculated Member**

It would not be possible to draw a complete stress diagram for the truss given in Fig. 211A by the usual methods. When joint ' DEPONM ' was being considered, it would be found that more than two members were unknown. This would hold up the construction of the stress diagram. There are various methods of getting over the difficulty. A good method in such cases is to calculate the force in a suitable member of the truss. In this case the tie ' AR ' is a suitable member.

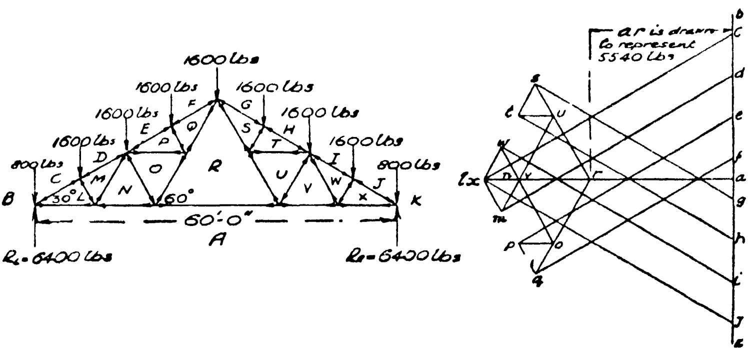


FIG. 211A.—AID OF CALCULATED MEMBER.

If the tie were removed, the truss would collapse by turning about the apex joint as a fulcrum (see Fig. 211B). The tie

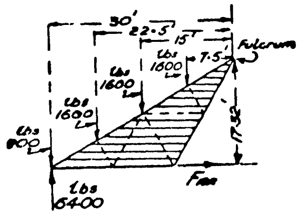


FIG. 211B.

member must therefore be pulling with just sufficient force to maintain the left-hand portion of the truss in equilibrium.

Let 'F lb.' be the pull in the tie member 'AR.'

Taking moments about the apex of truss:

$$(F \times 17{\cdot}32) + (1600 \times 7{\cdot}5) + (1600 \times 15) + (1600 \times 22{\cdot}5) + (800 \times 30) = 6400 \times 30$$

$$17{\cdot}32F + 12000 + 24000 + 36000 + 24000 = 192000$$

$$17{\cdot}32F = 192000 - 96000 = 96000$$

$$F = \frac{96000}{17{\cdot}32} = 5540 \text{ lb.}$$

The force in member 'AR' is therefore 5540 lb. The vector line '*ar*,' which represents this force in the stress diagram, must scale 5540 lb. This fixes the point '*r*' in the stress diagram. The joint 'ANORA' can now be completed and the remainder of the stress diagram finished off in the usual way.

Alternative Solution.—A method known as the '*reversal of diagonal*' method may be employed to get over the difficulty of the completion of the stress diagram in this case. Member 'PO,' which is a diagonal of a quadrilateral formed by four members, is assumed to be temporarily removed and to be replaced so as to form the other diagonal of the quadrilateral. This method is further explained in the example given later (see Fig. 217).

Wind Load Stress Diagrams

Determination of Reactions

When a frame is subjected to loads which are inclined to the vertical, it is not possible for both reactions to be vertical. One, or both, of the reactions must contribute a horizontal force to

balance the horizontal effect of the inclined loads. If one end of the truss is so supported as to allow practical freedom for horizontal movement—so that it may be regarded as being supported by a roller bearing—it is usual to assume that the reaction at this end is vertical. Such freedom is sometimes necessary, in order to allow for expansion due to temperature change. In this case the reaction at the '*fixed*' end must supply the necessary horizontal force to prevent the lateral movement of the truss.

If both ends are *similarly fixed* on rigid supports, it is a usual assumption that each reaction is parallel to the resultant force acting on the truss.

In the case of a roof truss supported by steel columns (of approximately equal stiffnesses), it is usual to assume that the horizontal components of the reactions are equal.

Regulations differ in their requirements in respect to the combination of *dead* and *wind* loads. It will be clear that the actual design load for any given member of the frame will be the algebraic sum of the loads produced independently (and simultaneously) by the dead load and by the wind loads. Some regulations require the 'negative' wind loads ('*suction loads*') to be taken without the accompaniment of the positive wind loads. When the various loads are treated independently careful tabulation will be necessary in order to arrive at the maximum possible load for any particular member.

The general principles of stress-diagram construction for wind loads are illustrated in the examples which follow.

* Examples of Roof Trusses Supporting Wind Loads

Positive Wind Loads Only

In this case (Fig. 212) we can determine the direction of the right-end reaction by the rule given on page 37. A line is drawn from '*e*,' in the load line, parallel to the ascertained direction to meet a vertical line (representing the direction of the free-end reaction) drawn from '*b*.' The lines meet in the point '***a***.' The stress diagram is then readily constructed. Member 'HI' is redundant.

* **More advanced students should consult B.S. 449**

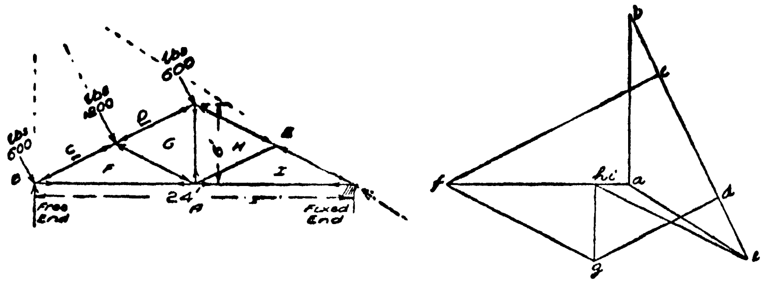


FIG. 212.—POSITIVE WIND LOADS.

Positive Wind Loads and Dead Loads Combined

The example given in Fig. 213 illustrates a method of dealing with dead loads and positive wind loads acting simultaneously. The wind is assumed to act from the left. The wind loads and dead loads, acting at the respective joints on the left slope of the truss, are combined into resultant single loads by parallelograms of forces. In this case the direction of the fixed-end reaction is determined by the construction of a *link polygon.*

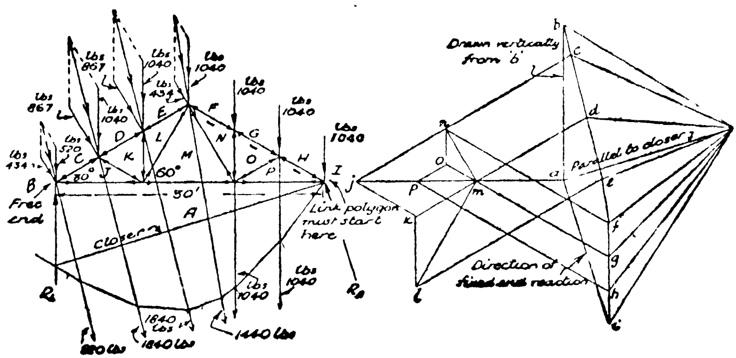


FIG. 213.—POSITIVE WIND AND DEAD LOADS.

To avoid the difficulty of having to find where the link in space 'H' cuts the fixed-end reaction line of action, it is necessary to commence drawing the link polygon from the fixed-end reaction point (which, of course, will be on the reaction). As we start drawing the links from this point, the link corresponding to space 'I' is not utilised. The reader should very carefully check that

the links are drawn in their proper spaces in these examples. The reaction problem of this example was previously dealt with on page 127.

Negative Wind Loads and Dead Loads Combined

The example given in Fig. 214 assumes wind acting from the left, so that the negative wind loads act outwards on the right slope of the truss. Instead of drawing complete parallelograms of forces, the negative wind loads and dead loads have been combined by the drawing of one vector line at the end of the other (see page 10). The resultant force on the truss is represented by '*bg*' and each reaction is assumed to be parallel to '*bg*' in this example.

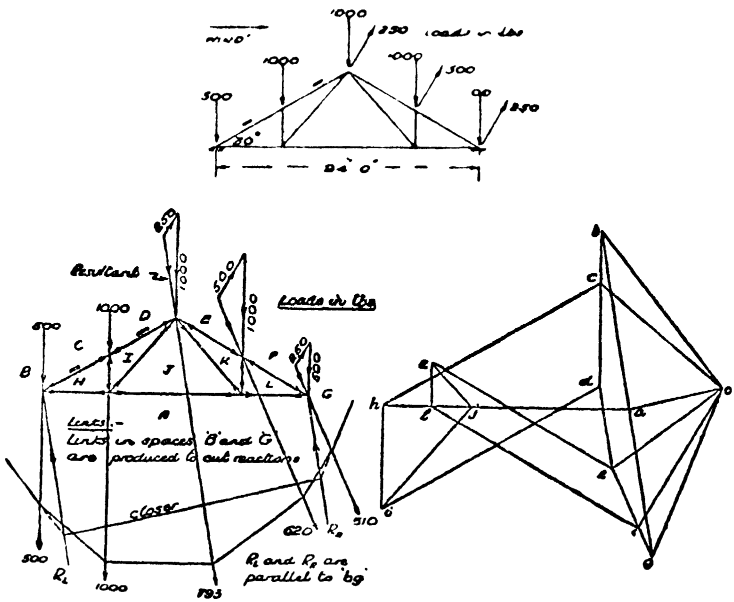


FIG 214 —NEGATIVE WIND AND DEAD LOADS.

There is no need to start the link polygon at any particular reaction point in this case, as the directions of the reactions can be drawn-in on the frame diagram. The point '*a*' is on the line '*bg*.'

It will be noted that the links in spaces 'B' and 'G' respec-

tively have to be produced till they cut the reaction lines in order that the closer line may be drawn-in.

Positive Wind Loads, Negative Wind Loads and Dead Loads Combined

In the example illustrated in Fig. 215 the wind is taken as acting from the right. Both reactions are assumed to be parallel to the resultant thrust on the truss.

In this example the *successive vector line* method has been adopted to reduce the forces acting, at any particular joint, to one resultant force. The reader should particularly notice the compounding of the three forces at the apex joint of the truss.

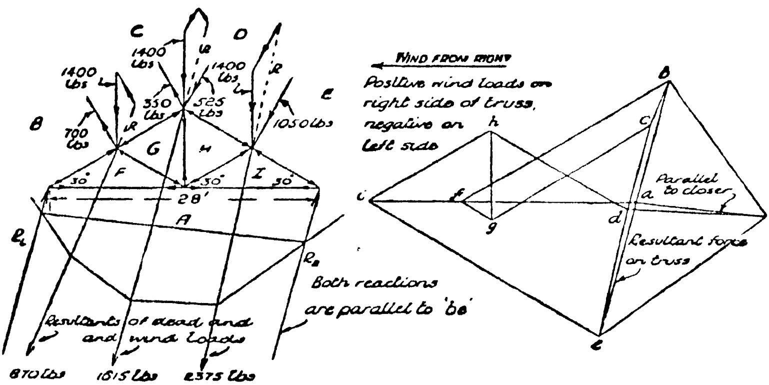


FIG. 215.—POSITIVE WIND, NEGATIVE WIND AND DEAD LOADS.

Having drawn the load line '*bcde*' to represent the respective resultant joint loads 870 lb., 1615 lb. and 2375 lb., the points '*e*' and '*b*' are joined. '*be*' represents the resultant force on the truss. We thus have the direction of each reaction fixed.

No loads were given for the eaves joints of the truss in this case. Such loads are sometimes omitted in problems on stress-diagram construction on the supposition that the common rafters between the roof trusses transmit their bottom end loads, not to purlins carried by the truss, but to wall plates.

Example Involving Equal Horizontal Reactions

The roof truss shown in Fig. 216 is supported on steel columns of equal stiffnesses. The particular point to note in such prob-

lems is the effect, on the values of the reactions, of the overturning moment caused by the combined horizontal components of the positive and negative wind loads. The lower diagram in Fig. 216 shows the load system reduced to resultant vertical and horizontal components. (The reader should check these components graphically or by the '*cosine*' rule.)

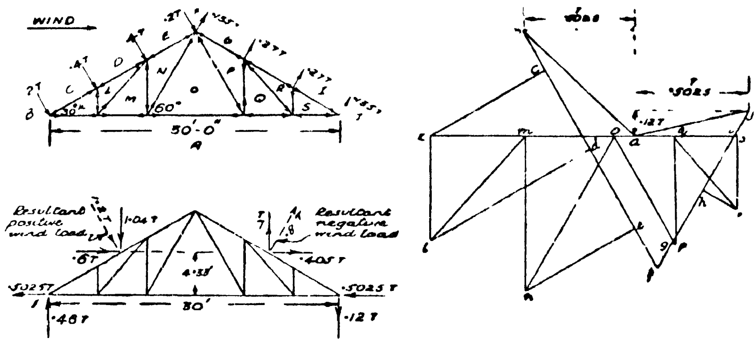


FIG. 216.—EQUAL HORIZONTAL REACTIONS.

Vertical Reactions (*due to vertical loads only*)

$$R_L \times 30 = (1{\cdot}04 \times 22{\cdot}5) - ({\cdot}7 \times 7{\cdot}5) \text{ [Note the minus sign.]}$$
$$= 23{\cdot}4 - 5{\cdot}25$$
$$= 18{\cdot}15$$
$$R_L = \frac{18{\cdot}15}{30} \text{ tons} = {\cdot}605 \text{ tons } (\textit{upwards}).$$
$$R_R \times 30 = (1{\cdot}04 \times 7{\cdot}5) - ({\cdot}7 \times 22{\cdot}5)$$
$$= 7{\cdot}8 - 15{\cdot}75$$
$$= -\,7{\cdot}95$$
$$R_R = -\frac{7{\cdot}95}{30} \text{ tons} = -\,{\cdot}265 \text{ tons } (\textit{downwards}).$$

Vertical Reactions (*due to overturning moment*)

Total horizontal force acting on truss

$$= ({\cdot}6 + {\cdot}405) = 1{\cdot}005 \text{ tons.}$$

Height of truss $= 15 \tan 30° = 8{\cdot}66$ ft.

Arm of overturning moment $= \frac{8{\cdot}66}{2} = 4{\cdot}33$ ft.

Taking moments about right-end reaction point :

$$R_L \times 30 = 1{\cdot}005 \times 4{\cdot}33 = 4{\cdot}352$$

$$\therefore R_L = \frac{4{\cdot}352}{30} \text{ tons} \qquad = {\cdot}145 \text{ tons } (\textit{downwards}).$$

Moments about left-end reaction point :

$$R_R \times 30 = 1{\cdot}005 \times 4{\cdot}33 = 4{\cdot}352$$

$$\therefore R_R = \frac{4{\cdot}352}{30} \text{ tons} \qquad = {\cdot}145 \text{ tons } (\textit{upwards}).$$

Net Vertical Reactions

$$R_L = {\cdot}605 \text{ tons (upwards)} - {\cdot}145 \text{ tons (downwards)}$$
$$= {\cdot}46 \text{ tons (upwards)}.$$
$$R_R = {\cdot}265 \text{ tons (downwards)} - {\cdot}145 \text{ tons (upwards)}$$
$$= {\cdot}12 \text{ tons (downwards)}.$$

Horizontal Reactions

The horizontal reactions being equal at both supports, the value of each reaction $= \frac{{\cdot}6 + {\cdot}405}{2}$ tons $= \frac{1{\cdot}005}{2}$ tons $= {\cdot}5025$ tons.

Construction of Stress Diagram

The load line '*bcdefghij*' is constructed in the manner indicated in Fig. 216. From '*j*' a vector line is drawn horizontally towards the left to represent ·5025 tons, the horizontal component of the right-end reaction. At the end of this line a vector line is drawn vertically downwards to represent ·12 tons, the vertical (downward) component of the right-end reaction. We thus get '*ja*' as the vector line representing the total right-end reaction. By joining '*a*' to '*b*' the complete polygon of forces, for the external load system acting on the truss, is formed. The stress diagram is constructed in the usual way off this polygonal diagram.

Calculation of Wind Joint Loads

The principles of the calculations for wind loads are the same as for dead loads. As an example we will take the truss illustrated in Fig. 213.

The dead load was taken as 18 lb. per sq. ft. of roof area. The

wind load was assumed to be 15 lb. per sq. ft. inwards on the windward side. The trusses were assumed to be at 10-ft. centres.

Length of principal rafter $= \frac{15}{\cos 30^\circ} = 17{\cdot}32$ ft.

Area of roof slope on each side $= (17{\cdot}32 \times 10)$ sq. ft.
$= 173{\cdot}2$ sq. ft.

Dead load: at 18 lb. per sq. foot, the total load on each roof slope $= (18 \times 173{\cdot}2)$ lb. $= 3120$ lb.

Full joint load $= \frac{3120}{3}$ lb. $= 1040$ lb.

$\therefore$ Half joint load $= 520$ lb.

Positive wind load: at 15 lb. per sq. foot, total load on windward roof slope $= (15 \times 173{\cdot}2)$ lb. $= 2600$ lb.

Full joint load $= \frac{2600}{3}$ lb. $= 867$ lb.

$\therefore$ Half joint load $= 434$ lb.

French Truss by 'Reversal of Diagonal' Method

In the example illustrated in Fig. 211A, the truss carried dead loads only and was solved by the calculation of the force in one of the members of the truss.

In Fig. 217 we have a similar type of truss with positive wind and dead loads. Both reactions are assumed to be parallel to the resultant force on the truss. The stress diagram can be constructed until the joint 'NMDEPO' is reached. The procedure then is as follows:

Remove member 'PO' and replace by the diagonal member shown by a broken line. The amended space notation is shown in the separate small inset diagram.

The original member 'PQ' becomes redundant and is ignored. Its redundancy can be verified by considering equilibrium conditions at its lower end. Joint DE1NM may now be completed and the point 1 fixed.

The stress diagram may be developed by taking the joint 'EF21,' the temporary member 12 being represented by the broken line 12 in the stress diagram

The letter '*q*' is substituted for the figure 2, the diagonal

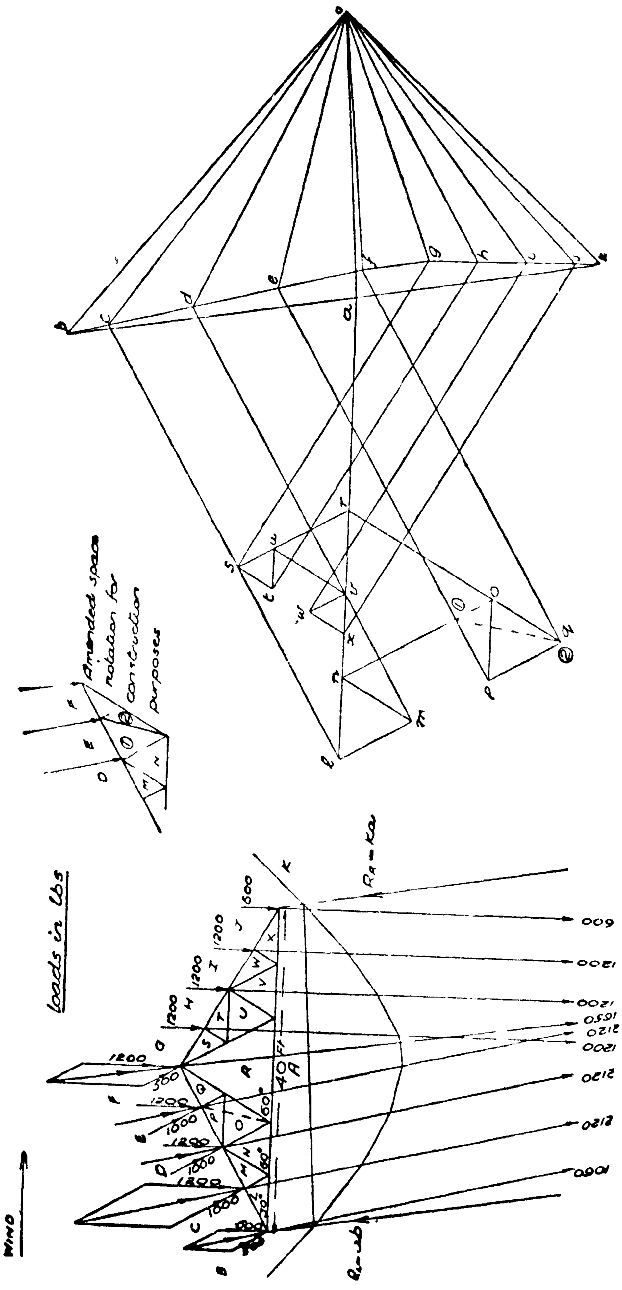


FIG 217 —REVERSAL OF DIAGONAL METHOD.

member 'PO' is replaced in its correct position and the stress diagram completed. The justification of the method lies in the fact that the reversal of the diagonal 'PO' does not affect the loads carried by the members of the truss *outside the quadrilateral* of which it is the diagonal. This means that 'FQ' is correctly represented by 'F(2),' i.e. 'Fq' in the stress diagram.

Exercises 8

(The reader is strongly advised to draw-out the stress diagrams already dealt with in this chapter.)

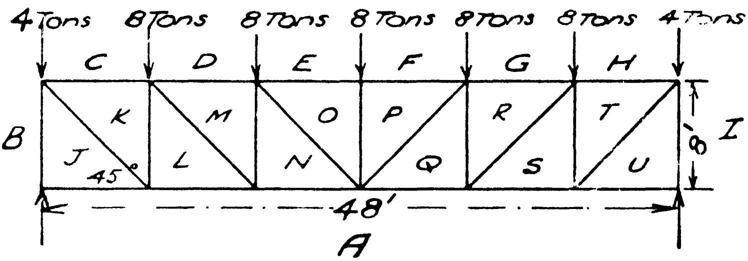


Fig. 218.

(1) Construct the stress diagram for the symmetrically loaded braced girder given in Fig. 218. Tabulate the forces in the various members, differentiating between struts and ties.

(2) Fig. 219 shows a form of roof truss which provides good

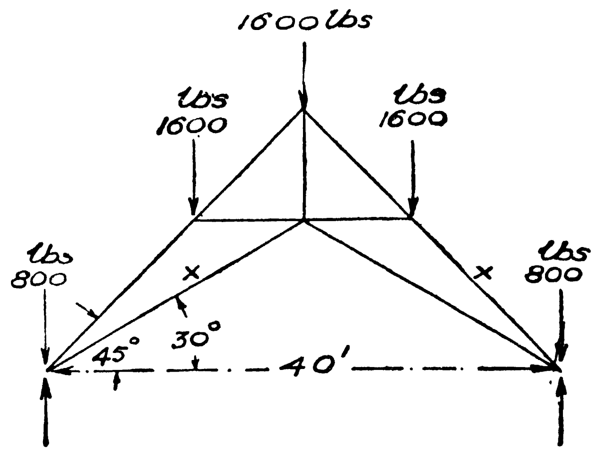


Fig. 219.

headroom at mid-span. Draw the stress diagram for the truss. Determine the force in each of the two members marked by a cross and state whether they are respectively in tension or compression.

(3) Determine the forces in members 'AF,' 'IJ' and 'JK' respectively, of the loaded frame given in Fig. 220, by the construction of the complete stress diagram.

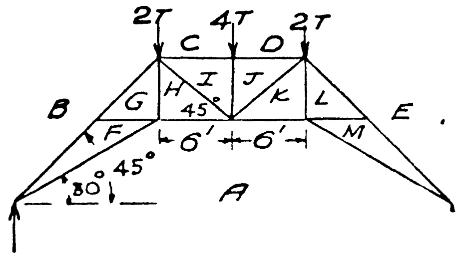


FIG. 220.

(4) Draw the stress diagrams for the given cantilever trusses (Fig. 221 (*a*) and Fig. 221 (*b*)). Tabulate the various member forces (stating whether the members are 'struts' or 'ties') in the case of Fig. 221 (*a*).

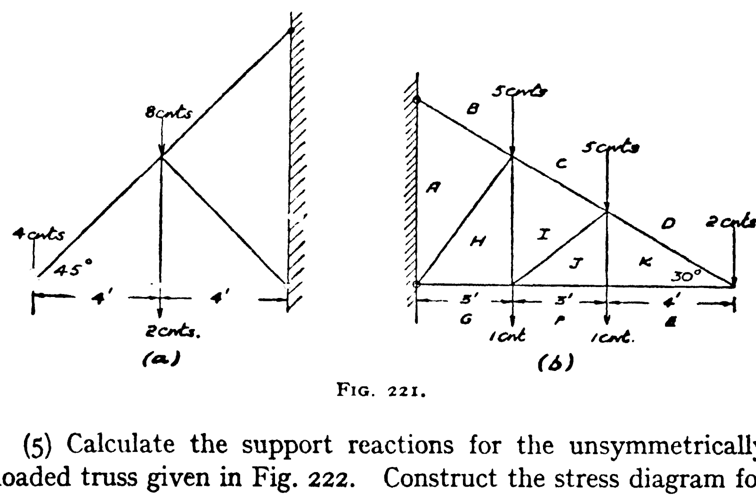


FIG. 221.

(5) Calculate the support reactions for the unsymmetrically loaded truss given in Fig. 222. Construct the stress diagram for the truss and determine the force in the inclined member indicated by a cross.

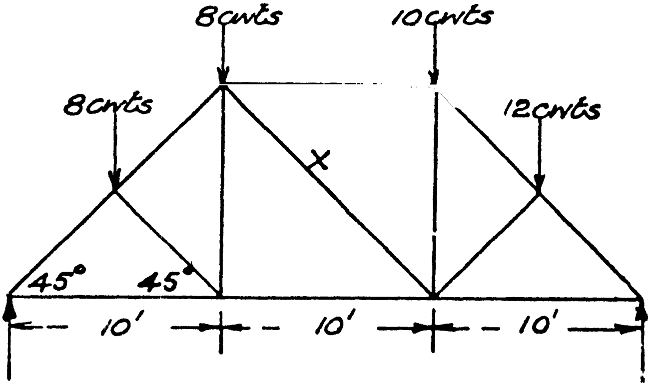


FIG. 222

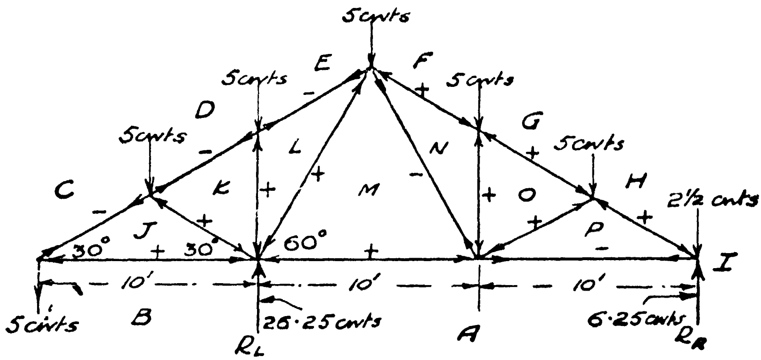


FIG. 223.

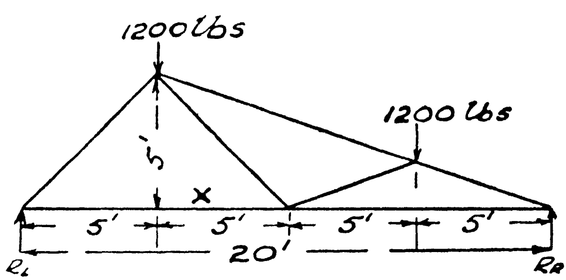


FIG. 224

(6) The reactions for the overhanging roof truss given in Fig. 223 have been evaluated and the struts and ties determined. The ties are denoted by a negative sign and the struts by a

positive sign. Check the reaction values and verify the correctness of the strut and tie indications by constructing a complete stress diagram for the truss.

(7) Construct the stress diagram for the roof truss given in Fig. 224. Find the force in the member indicated by a cross. State whether the member is a 'strut' or a 'tie.'

(8) Draw the stress diagram for the truss shown in Fig. 225. The truss carries dead loads and positive wind loads. The reactions are to be assumed to be parallel to the resultant force on the truss.

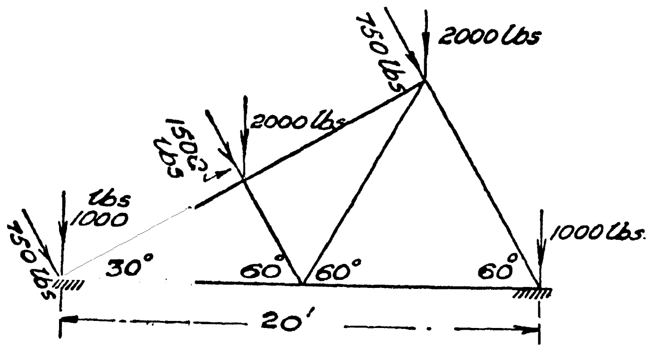


FIG. 225.

(9) The king-post truss (Fig. 226) carries dead loads and positive wind loads. The wind load is to be taken at 15 lb. per sq. foot of roof area, and the total dead load at 16 lb. per sq. foot of

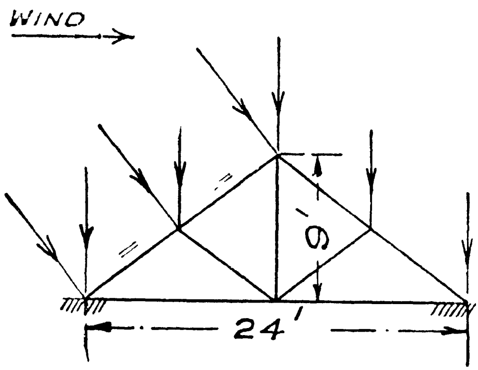


FIG. 226.

roof area. Calculate the respective joint loads for the truss, assuming the trusses to be spaced at 8-ft. centres.

(10) Draw the stress diagram for the braced girder given in Fig. 227. Indicate struts and ties on your frame diagram by writing alongside the members a plus sign for a strut and a minus sign for a tie.

Give the force in each of the inclined members of the girder.

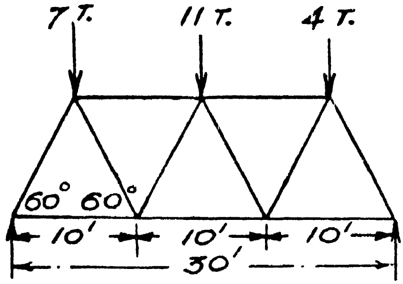


FIG. 227.

(11) Using the 'reversal of diagonal' method, construct the stress diagram for the truss shown in Fig. 211A.

CHAPTER IX

STRESS AND STRAIN. YOUNG'S MODULUS

Nature of Stress

A STRESS diagram of the type considered in the last chapter enables us to find the force or '*total stress*' in any given member of a loaded frame. We now have to investigate the implication of the term '*stress*' as normally employed in structural calculations.

Member 'NA' in the frame shown in Fig. 218 carries a tensile load of 32 tons. Is the member unsafely loaded? We cannot answer this question until the net cross-sectional area of the member is known. It is not the load in the member that ultimately matters but the *intensity* of the loading, i.e. how much the member has to carry for every square unit (usually taken as 'sq. inch') of sectional area. Assuming that the net sectional area of the member referred to equals 4 sq. ins., the load carried by one sq. inch is 8 tons. This is quite safe - assuming the material of the member to be mild steel. The intensity of loading expressed as '*so many units of load per unit of sectional area*' is termed the *stress* in the material of the given member.

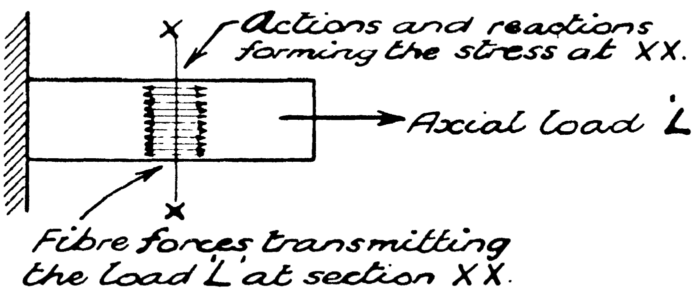


FIG. 228—NATURE OF STRESS.

Uniform Stress.—In cases in which the load carried by a member may be regarded as being uniformly distributed over a given cross-sectional area of the member, the stress at that section may be obtained by dividing the total load by the area of the section (Fig. 228).

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}$$

Units for Expressing Stress Values

Stress values for structural steelwork are usually expressed in *tons per sq. inch* (tons/in.^{2}). The units '*lb. per sq. inch*' (lb./in.^{2}) and '*cwts. per sq. inch*' (cwts./in.^{2}) are both used in timber calculations. For reinforced concrete work the common unit is '*lb. per sq. inch.*' Masonry calculations and foundation pressure problems frequently involve the units 'lb./ft.^{2}' and 'tons/ft.^{2}.'

Non-uniform Stress.—If the load distribution over a given section be not uniform, the value obtained by dividing the 'total load' by the 'area of section' would merely give an *average* stress value. The structural designer does not think in terms of 'average stress' values, but has to consider the intensity of stress in the most heavily loaded fibres of the member he is designing. In some cases the formula used for determining this maximum stress may not look much like the basic expression *load/area*, but the nature of stress remains the same, i.e. it is an '*intensity*' or '*rate*' of loading.

If a tiny beam fibre have a stress intensity of 3 tons per sq. inch, the implication is that if the sectional area of the fibre were enlarged up to one sq. inch and the load were increased in the same proportion, the sq. inch would be carrying a load of 3 tons.

In this chapter we will confine our attention to uniform stress.

Three Types of Simple Stress

The three basic types of stress are (i) *tensile*, (ii) *compressive*, and (iii) *shear*.

Tensile Stress.—This stress occurs in the fibres of a member which is subjected to a pull.

In Fig. 229, 'A' sq. inches of cross-sectional area at section 'XX' have to carry 'L' tons

$\therefore$ each sq. inch carries L/A tons
$\therefore$ *tensile stress* at 'XX' = L/A tons/in.^{2}.

Compressive Stress.—When a member transmits a thrust the material of the member is subjected to this form of stress (Fig. 230).

$$\text{Compressive stress at section 'XX'} = \frac{\text{Load}}{\text{Area}} = \frac{\text{L}}{\text{A}} \text{ tons/in.}^2$$

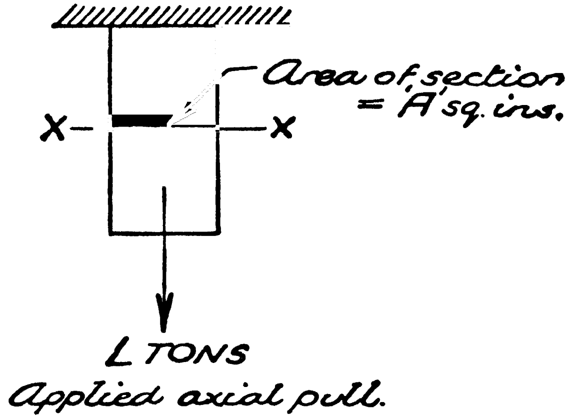


FIG. 229.—TENSILE STRESS.

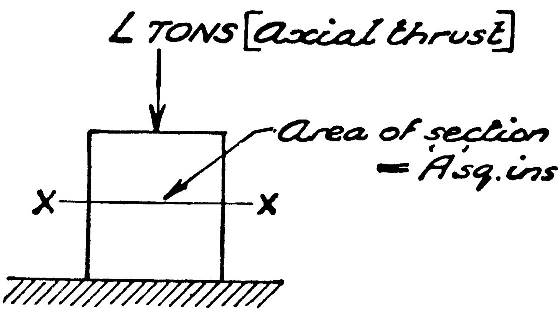


FIG 230.—COMPRESSIVE STRESS.

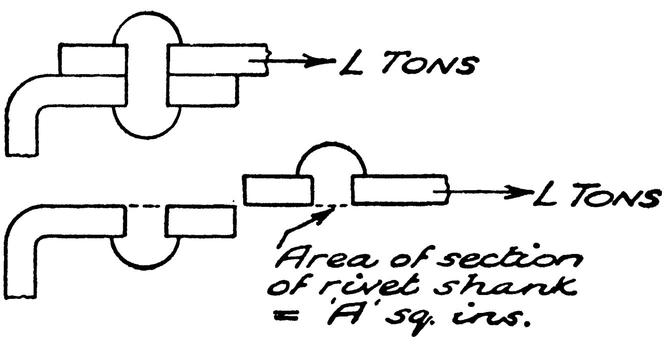


FIG 231.—SHEAR STRESS.

Shear Stress.--This is the type of stress that occurs in the steel of a rivet which is loaded as indicated in Fig. 231. When one portion of a member tends to slide over another portion at a certain plane in the material, the fibres at that plane are said to be in '*shear.*' Beam webs are subjected to 'shear stress.'

If the rivet shank have a sectional area of 'A' sq. ins. and the applied shear load be 'L' tons, the shear stress in the rivet steel will be 'L/A' tons per sq. inch.

EXAMPLES:

(i) *A steel tie-member has a solid rectangular section, 4" × $\frac{3}{4}$". It carries an axial (i.e. centrally applied) load of 21 tons. Calculate the stress in the steel.*

$$\text{Tensile stress} = \frac{\text{Load}}{\text{Area}} = \frac{21 \text{ tons}}{(4 \times \frac{3}{4}) \text{ sq. ins.}}$$

$$= \frac{21}{3} \text{ tons/in}^2 = 7 \text{ tons/in.}^2$$

(ii) *A short concrete column of square section, 10" × 10", carries an axial load which induces a compressive stress of 600 lb. per sq. in. in the concrete. Calculate the value of the load.*

$$\text{Compressive stress} = \frac{\text{Load}}{\text{Area}}$$

$$\text{Let L lb.} = \text{load}$$

$$600 = \frac{L}{100}$$

(It is usual to leave the units out in such equations, but great

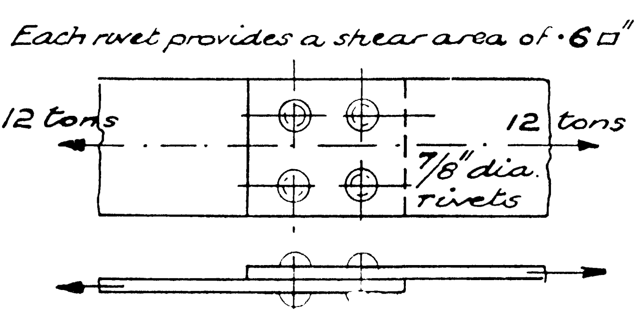


FIG 232 - LAP JOINT.

care must be taken to employ the same units on both sides of the equation.)

$$\therefore L = (600 \times 100) \text{ lb.}$$
$$= 60{,}000 \text{ lb.}$$

(iii) *Calculate the shear stress in the rivets in the lap-joint shown in Fig. 232.*

The total shear area provided is 4 times the sectional area of one rivet. Area of one rivet $= \frac{\pi d^2}{4} = \frac{\pi \times \frac{7}{8}^2}{4}$ sq. ins.

$$= \cdot 60 \text{ sq. ins.}$$

$$\therefore \text{total shear area} = (4 \times \cdot 6) \text{ sq. ins.}$$
$$= 2 \cdot 4 \text{ sq. ins.}$$

$$\text{Shear stress} = \frac{\text{Shear load}}{\text{Area under shear}}$$
$$= \frac{12 \cdot 0}{2 \cdot 4} \text{ tons/in.}^2$$
$$= 5 \text{ tons/in.}^2.$$

Working Stresses

The maximum safe value for the stress in the material of a practical structural member depends upon several factors. It will depend upon the nature of the material. It will also depend upon whether the stress is '*tensile*,' '*compressive*' or '*shear*.' The actual manner in which the member is employed in the structure will affect the maximum permissible stress, e.g. a 'long' compression member should not be so highly stressed as a 'short' one.

The nature of the loading, e.g. '*live*' or '*dead*,' will also be a determining factor.

For cases commonly occurring in structural design, regulations issued by local authorities or professional institutions specify the working stresses which should be employed.

Ultimate Stress.—Working stresses are usually fixed from the results of practical tests to destruction, carried out by means of suitable testing machines. The tests give *ultimate stress* values from which the safe working stresses are derived.

Fig. 233 shows a 25-ton Macklow-Smith Universal Testing Machine.

FIG. 233.—MACKLOW-SMITH 25-TON UNIVERSAL TESTING MACHINE

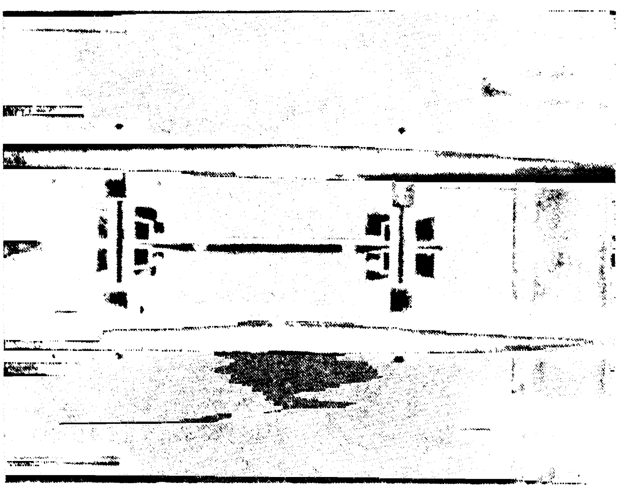


FIG 234 —STEEL TENSION TEST.

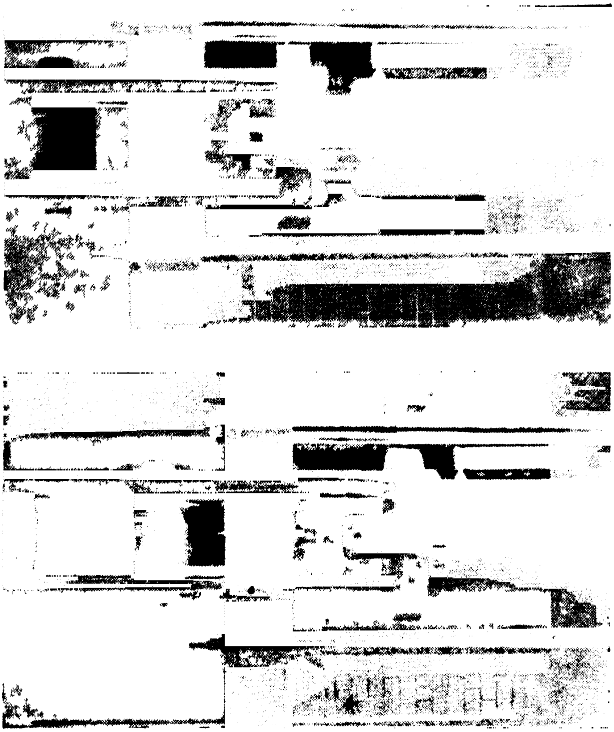


FIG 235 —SHEAR TEST ON STEEL SPECIMEN

In Fig. 234 a steel specimen is shown in position preparatory to a tensile test to destruction. The two marks on the specimen

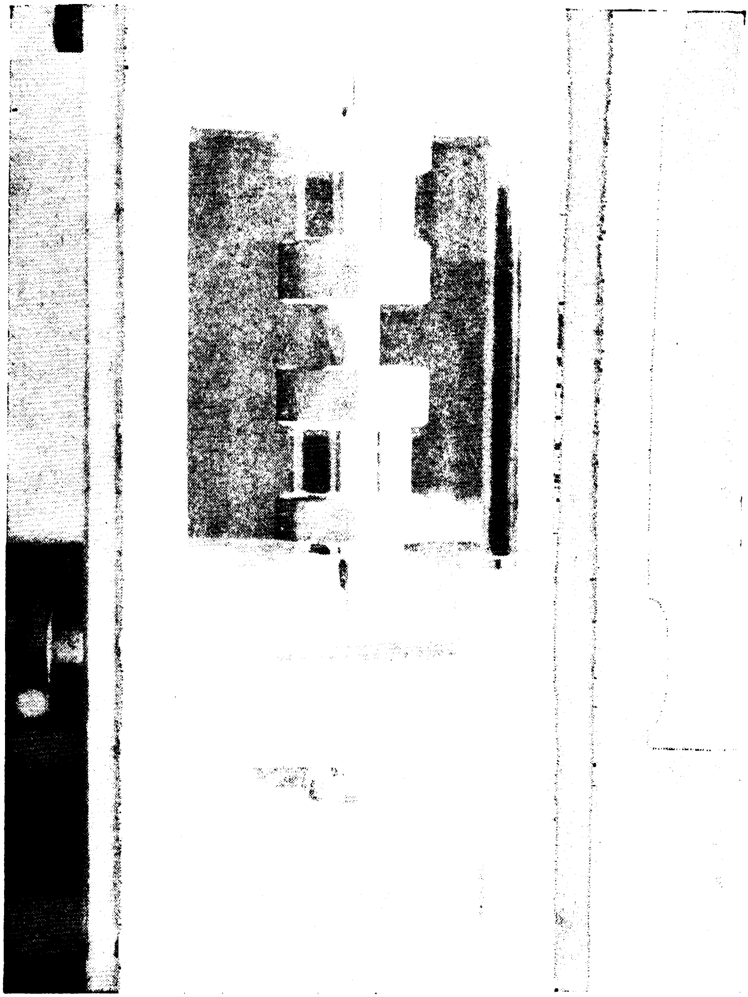


FIG. 236.—TENSION TEST ON TIMBER SPECIMEN.

are 8 ins. apart, this being the usual gauge length for measurements of elongation.

Fig. 235 shows a specimen of steel, of circular section, being tested for ultimate shear stress. The first photograph indicates

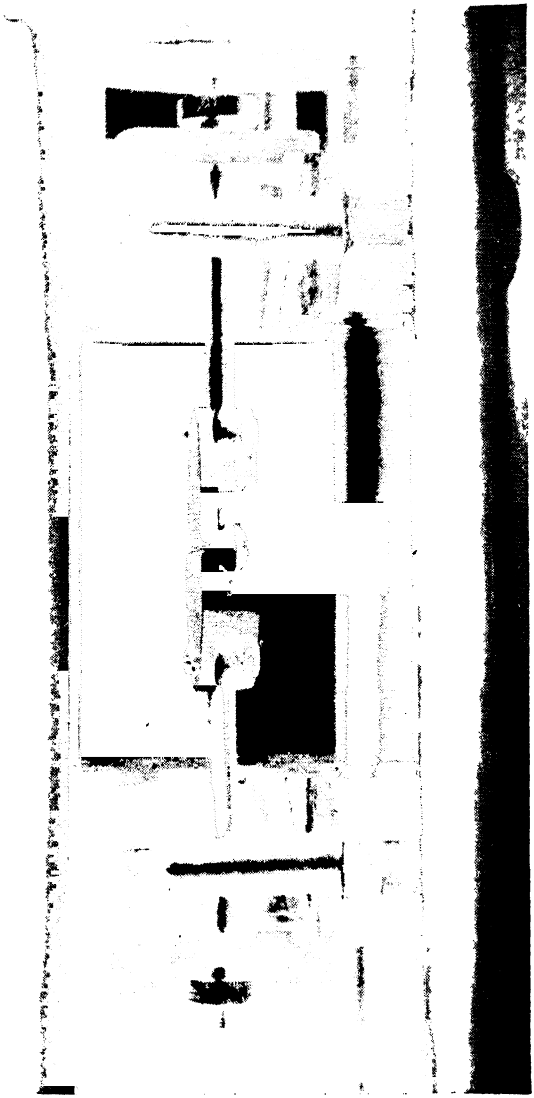


FIG 237 - CLEAVAGE TEST ON TIMBER SPECIMEN

the nature of the shear tool. In the second illustration (shear tool at right angles to the first view), the tool has made contact with the specimen.

Small clear specimens of timber are tested to a British Standard. The method of testing a timber specimen in tension is shown in Fig. 236. The specimen has enlarged circular ends which bear against split brass rings in the shackles. Timber specimens are tested along the grain and across the grain.

In Fig. 237 another form of timber test is illustrated. The object of this test is to determine the '*cleavage*' strength of timber of a given type.

Concrete is tested for compressive strength. The concrete to be tested is poured into moulds in the form of cubes, usually 6-in. cubes. After 'curing' for a definite period, say 28 days, the concrete cubes are tested to destruction. Fig. 238 shows a cube being tested in an Amsler Compression and Bending machine.* L.C.C. By-laws give full details of the manner in which the cubes shall be manufactured, cured and tested.

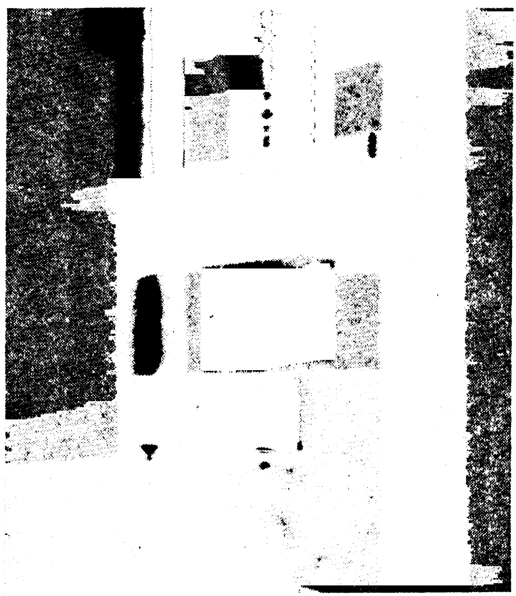


FIG. 238.—COMPRESSION TEST ON CONCRETE CUBE.

Readers who are interested in the testing of materials should consult the B.S. (British Standards) which are concerned with the particular materials in question (see Appendix I).

* Reproduced by courtesy of Messrs. T. C. Howden & Co.

'B.S. No. 15' deals with the testing of 'structural steel.' The testing of cement is the subject of 'B.S. No. 12.' The methods of testing small clear specimens of timber are dealt with in 'B.S. No. 373.' These, and other similar standards, may be obtained from the British Standards Institution, 2 Park Street, London, W.1.

Factor of Safety.—A '*factor of safety*' is a number which is divided into an 'ultimate stress' in order to obtain a suitable 'working stress.' The value of the 'factor of safety' is such as to provide an appropriate margin of safety in the employment of the derived stress. Some authorities prefer to derive the working stress from the 'yield-point' stress.

$$\text{Working stress} = \frac{\text{Ultimate stress}}{\text{Factor of safety}}.$$

EXAMPLES :

(i) *In a tensile test to destruction on a mild-steel specimen the maximum load indicated by the testing machine was* 15·75 *tons. The sectional area of the specimen was* ·5 *sq. in. Calculate the working tensile stress, for steel of this quality, assuming a factor of safety of* 3·5.

$$\text{Ultimate stress} = \frac{\text{Maximum load during test}}{\text{Sectional area of specimen}}$$

$$= \frac{15{\cdot}75 \text{ tons}}{{\cdot}5 \text{ sq. in.}} = 31{\cdot}5 \text{ tons/in.}^2.$$

$$\text{Working stress} = 31{\cdot}5/3{\cdot}5 \text{ tons/in}^2.$$

$$= 9 \text{ tons/in.}^2.$$

(ii) *A* 6-*in. cube of concrete crushed, in a compression test, at a load of* 50 *tons. Assuming a factor of safety of* 6, *determine suitable dimensions for the section of a short column, of the same quality concrete, which has to carry an axial load of* 23 *tons.*

$$\text{Ultimate compressive stress} = \frac{\text{Max. load during test}}{\text{Original sectional area}}$$

$$= \frac{50 \times 2240}{6 \times 6} \text{ lb./in.}^2 = 3111 \text{ lb./in.}^2$$

$$\text{Working stress} = \frac{\text{Ultimate stress}}{\text{Factor of safety}} = \frac{3111}{6} \text{ lb./in.}^2$$

$$= 518{\cdot}5 \text{ lb./in.}^2.$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} \quad \therefore \text{Area} = \frac{\text{Load}}{\text{Stress}}$$

$$= \frac{23 \times 2240}{518 \cdot 5} = 99 \cdot 4 \text{ sq. ins.}$$

Assuming a square section, say 10″ × 10″.

(iii) *In a shear test on rivet steel the ultimate load was 22 tons, the total area under shear being ·88 sq. in. In a joint (as in Fig. 232), 4 No. ⅝″ dia. rivets of this quality steel supported a shear load of 6·136 tons. What factor of safety does this represent?*

$$\text{Ultimate shear stress} = \frac{\text{Ultimate load}}{\text{Shear area provided}}$$

$$= \frac{22 \text{ tons}}{\cdot 88 \text{ in.}^2} = 25 \text{ tons/in.}^2$$

$$\text{Actual working stress of rivets in joint} = \frac{\text{Total load}}{\text{Total area}}$$

$$= \frac{6 \cdot 136}{4 \times \cdot 3068} \text{ tons/in.}^2 = 5 \text{ tons/in.}^2$$

(Area of one ⅝″-dia. rivet = ·3068 sq. in.)

$$\text{Factor of safety} = \frac{\text{Ultimate stress}}{\text{Working stress}}$$

$$= \frac{25 \text{ tons/in.}^2}{5 \text{ tons/in.}^2} = 5.$$

The reader is referred to regulations issued by the London County Council (L.C.C. By-laws for steelwork, reinforced concrete and timber design, etc.), and the British Standards Institution (B.S. 449, etc.), and to current codes of practice, for detailed lists of working stresses. From time to time revision of these working stresses is made necessary by improved methods of manufacture and the application of the results of research.

Strain

When a member is loaded and its fibres are put into a state of stress, some alteration is bound to take place in the dimensions or shape of the member. A member subjected to such dimensional change is said to be in a state of *strain*. The effect of load is therefore to develop in the fibres of the member both *stress* and *strain* simultaneously.

Tensile Strain.—This is the type of strain which the fibres of

a tie member experience. Tensile strain is associated with the elongation of members.

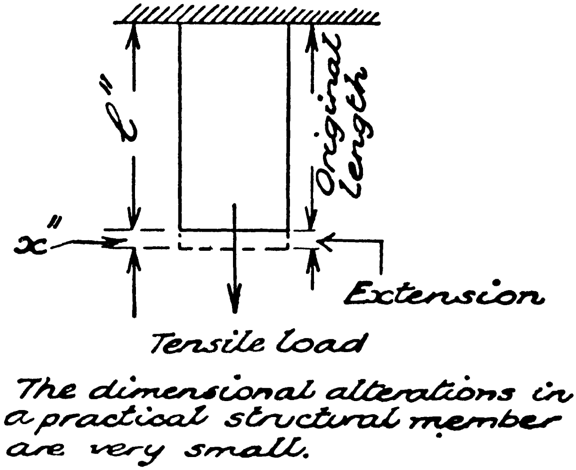


FIG. 239.—TENSILE STRAIN.

The numerical value of the '*strain*' is not the extension itself. The tensile strain in the member shown in Fig. 239 is obtained by dividing the *extension* by the *original length* of the member.

$$\text{Tensile strain} = \frac{\text{Extension}}{\text{Original length}} = \frac{x''}{l''} = x/l.$$

EXAMPLE.—*Calculate the tensile strain in a steel rod* 5 *ft. long which undergoes an extension of* ·03 *in. when the load is applied.*

$$\text{Tensile strain} = \frac{\text{Extension}}{\text{Original length}} = \frac{\cdot 03 \text{ in.}}{(5 \times 12) \text{ ins.}}$$

$$= \frac{\cdot 03}{60} = \cdot 0005 \text{ (expressed as a '\textit{number}')}.$$

Compressive Strain.—When a member shortens under the action of a thrust, the material of the member is subjected to *compressive strain*. This is the type of strain which occurs in columns (Fig. 240).

$$\text{Compressive strain} = \frac{\text{Contraction}}{\text{Original length}} = \frac{x''}{l''} = x/l.$$

EXAMPLE (i).—*A short reinforced concrete column,* 6 *ft. high, shortens by* ·0216 *in. under the applied axial load. Calculate the strain in the concrete and in the steel reinforcement.*

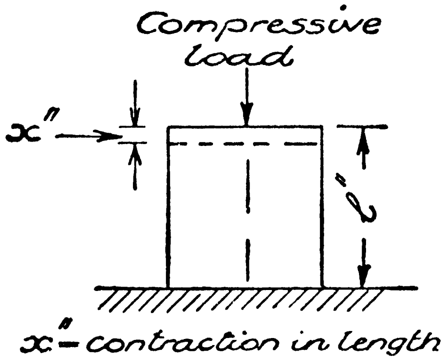


FIG 240 —COMPRESSIVE STRAIN

As the concrete and the steel contract in length equally, they are both subjected to equal '*strain.*'

Compressive strain in both materials

$$= \frac{\cdot 0216 \text{ in.}}{72 \text{ ins.}} = \cdot 0003.$$

EXAMPLE (ii).—*A 6-in. timber cube in a test is loaded so that the compressive strain is ·0004. Calculate the contraction in length of the specimen.*

$$\text{Compressive strain} = \frac{\text{Contraction in length}}{\text{Original length}}.$$

$$\text{Let } x'' = \text{Contraction}$$

$$\cdot 0004 = \frac{x''}{6''}$$

$$\therefore x = (6 \times \cdot 0004)'' = \cdot 0024''.$$

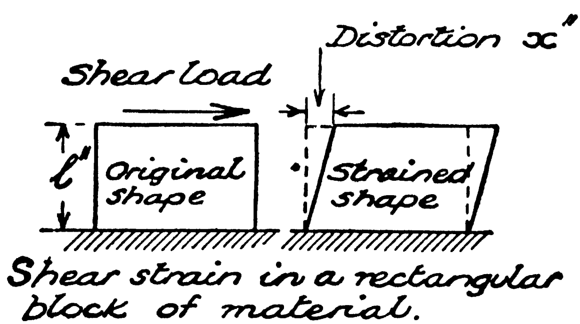


FIG. 241 —SHEAR STRAIN.

Shear Strain. The piece of material illustrated in Fig 241 is distorted by the action of the shear load shown

The value of the shear strain is obtained by the ratio x''/l'', i e x/l

When a bolt is turned by a spanner the material of the bolt is in a state of shear strain The name '*torsional strain*' is given to this particular type of shear strain

Relationship between Stress and Strain

Hooke's Law.—The two physical states '*stress*' and '*strain*'—which we have seen are always co-existent—are related by a law known as '*Hooke's Law*' It is a law concerned with bodies in an 'elastic state'

An '*elastic*' body is one which, having been deformed by an applied force, will regain its original size and shape when the deforming force is removed

As working stresses in structural work are so chosen as to ensure that the property of elasticity is maintained, it is clear that Hooke's law of elasticity is of great importance

The law states that, provided a member remain perfectly elastic, the stress induced in it will always be directly proportional to the accompanying strain This means that should the load the member is carrying alter in magnitude (and thereby cause an increase or a decrease in the stress value), the strain value would increase or decrease proportionately with the variation in stress Thus if the stress increases by 50%, the strain would increase by 50% The *stress-strain* graph for an elastic member is therefore a straight line

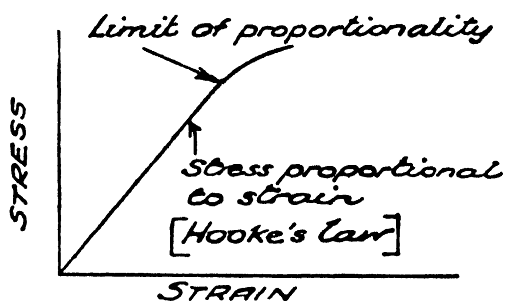


FIG 242—ELASTIC STRESS AND STRAIN.

Young's Modulus of Elasticity.—Let us suppose that we have conducted a tensile or compressive test on a suitable elastic specimen, and have tabulated corresponding 'stress' and 'strain' values as in Fig. 243. The figures shown in the table represent a perfect set of results in that they comply exactly with the terms of '*Hooke's Law*.' In the last column the values obtained by dividing a 'stress result' by the corresponding 'strain result' are shown. The value in each case is the same.

Material	Stress lb./in.²	Strain	Ratio of stress to strain = $\frac{\text{Stress}}{\text{Strain}}$
1 : 2 : 4 Concrete tested in compression	200	·0001	$\frac{200}{\cdot 0001}$ = 2,000,000 lb./in.²
	400	·0002	$\frac{400}{\cdot 0002}$ = 2,000,000 ,,
	600	·0003	$\frac{600}{\cdot 0003}$ = 2,000,000 ,,
	800	·0004	$\frac{800}{\cdot 0004}$ = 2,000,000 ,,

FIG. 243.

Thus if stress varies as strain we may say that the ratio $\frac{\text{Stress}}{\text{Strain}}$ is a constant value. If we repeated the test, using a specimen of different dimensions and form of cross-section but of the same material, we would get the same value for the 'stress-strain' ratio. Such a constant, which depends simply upon the nature of the material and not upon the dimensions of the tested specimen, is termed a '*physical constant*.' To the physical constant represented by the ratio $\frac{\text{Stress}}{\text{Strain}}$ the name 'Young's modulus' is given. The letter 'E' is used to denote 'Young's modulus':

$$E = \frac{\text{Stress}}{\text{Strain}}.$$

Just as '*density*' (another 'physical constant') tells us how much weight is associated with a certain volume of a particular substance, so Young's modulus tells us how much stress accompanies a given strain in the material of a given structural member.

Units for 'E.'—As 'strain' is merely a number, the units in

which Young's modulus values are expressed are those of 'stress,' e.g. '*lb./in.²*' or '*tons/in.²*.'

Typical values of 'E':

Structural or mild steel:	13,000 tons/in.² or 30,000,000 lb./in.².
Concrete:	2,000,000 lb./in.².
Timber (average):	1,200,000 lb./in.².
Rubber, say:	100 lb./in.².

The value for rubber has been inserted to indicate that materials which, under similar test conditions, stretch considerably more than others have much lower 'E' values.

Practical Determination of Young's Modulus

In a practical test on a material like steel or concrete, the alteration in length of a specimen will be very small if we keep the stress within the elastic range. Such alterations in length are not visible to the naked eye.

To effect accurate readings of extensions or contractions in these circumstances, special instruments known as '*extensometers*' are employed. The underlying principles, upon which the operation of extensometers depends, vary considerably. A number employ the principles of optics.

Fig. 244 (left figure) illustrates the employment of a Lamb's roller extensometer in the derivation of 'E' for a steel cylinder, tested in compression. As the specimen contracts in length the mirrors, which are attached to rollers fitted in between the moving plates, rotate and an incident ray of light from a projector, which is successively reflected by the two mirrors, is made to travel along a vertical scale. A telescope with cross hairs may be used instead of a projector. The distance of the scale from the instrument is calculated to give a suitable value to the scale graduations. The spring clips shown are in elongated holes to allow relative movement of the two plates.

EXAMPLES:

(i) *A bar of sectional area 2 sq. ins. and 5 ft. in length extended ·012 in. when an axial load of 5·2 tons was applied. Calculate Young's modulus for the material of the bar.*

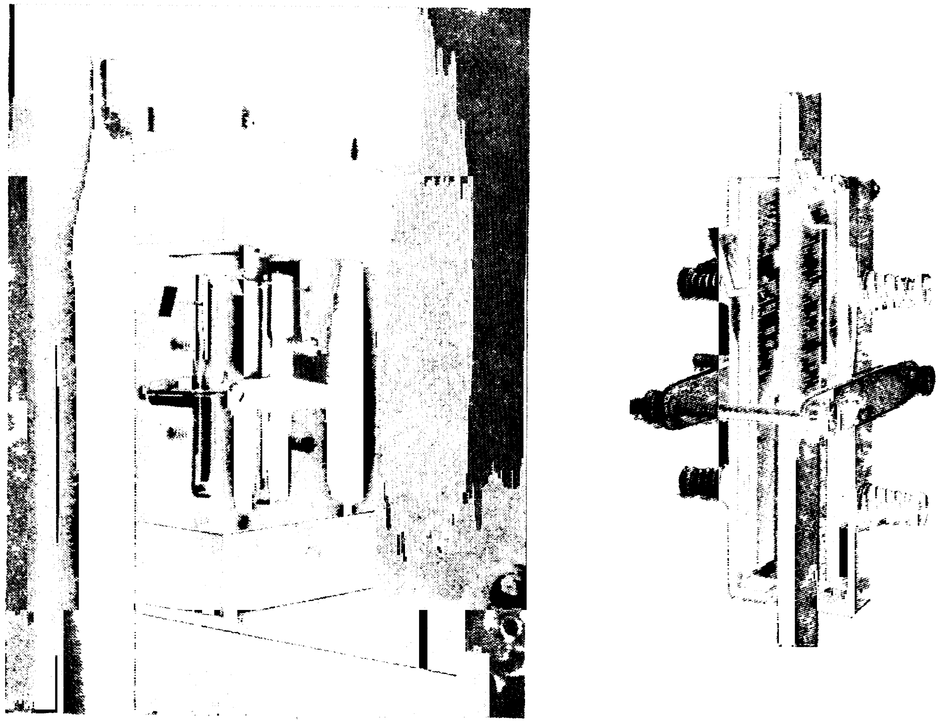


FIG. 44.—LAMB'S ROLLER EXTENSOMETER

$$E = \frac{\text{Stress}}{\text{Strain}}.$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{5{\cdot}2 \text{ tons}}{2 \text{ sq. ins.}} = 2{\cdot}6 \text{ tons/in.}^2.$$

$$\text{Strain} = \frac{\text{Extension}}{\text{Original length}} = \frac{{\cdot}012 \text{ in.}}{60 \text{ ins.}} = {\cdot}0002.$$

$$E = \frac{2{\cdot}6}{{\cdot}0002} \text{ tons/in.}^2 = 13{,}000 \text{ tons/in.}^2.$$

'Young's modulus' is one of the physical constants by which we can identify a given material. The result of the calculations in this problem indicates that the bar was composed of mild steel.

(ii) *Calculate the contraction in length of a short concrete column,* 12″ × 12″ *square section, when carrying an axial load of* 72,000 *lb. The original unloaded length of the column was* 8 *ft. Assume* '*E*' *for concrete to be* 2,000,000 *lb. per sq. inch.*

Let x'' = contraction in length.

$$\text{Compressive stress in concrete} = \frac{72000}{12 \times 12} \text{ lb./in.}^2$$

$$= \frac{72000}{144} \text{ lb./in.}^2 = 500 \text{ lb./in.}^2.$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{500}{x/96} = \frac{500 \times 96}{x}$$

$$\therefore 2{,}000{,}000 = \frac{500 \times 96}{x}$$

$$x = \frac{500 \times 96}{2{,}000{,}000} \text{ ins.} = {\cdot}024 \text{ ins.}$$

(iii) *A short timber post of rectangular section has one side of its section twice the other. When the post is loaded axially with* 2160 *lb. it contracts* ·0012 *ins. per foot of length. If* '*E*' *for this timber* = 1,200,000 *lb./in.*^{2}, *calculate the sectional dimensions of the post.*

Let x'' = smaller side, $\therefore 2x''$ = larger side.

$$\text{Area of section} = 2x^2 \text{ sq. ins.}$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{2160}{2x^2} \text{ lb./in.}^2 = \frac{1080}{x^2} \text{ lb./in.}^2.$$

$$\text{Strain} = \frac{{\cdot}0012''}{12''} = {\cdot}0001.$$

$$E = \frac{\text{Stress}}{\text{Strain}}.$$

$$1{,}200{,}000 = \frac{1080/x^2}{\cdot 0001}$$

$$\therefore \frac{1080}{x^2} = 120, \quad \therefore x^2 = \frac{1080}{120} = 9$$

$$\therefore x = 3''.$$

$\therefore$ Sectional dimensions are $3'' \times 6''$.

(iv) *Fig.* 245 *shows the section of a reinforced concrete short column. Calculate the stress in the concrete and the stress in the steel if an axial load of* 120,960 *lb. be applied to the column.*

' E ' for steel = 30,000,000 *lb./in.*^{2},
' E ' for concrete = 2,000,000 *lb./in.*^{2}.

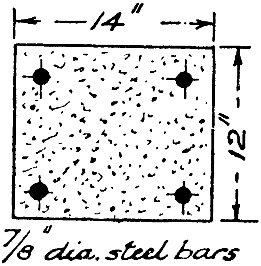


FIG. 245—REINFORCED CONCRETE COLUMN SECTION.

In such problems (as in the practical design of reinforced concrete members) it is assumed that there is no slipping of the steel in the concrete, i.e. that there is perfect *' bond '* between the two materials. The steel and the concrete will therefore be subjected to equal strain as the column shortens under the load.

Let *' c '* lb./in.^{2} = stress in concrete and *' t '* lb./in.^{2} be the stress in the steel reinforcement.

Area of steel = $(4 \times \cdot 6)$ sq. ins. = $2\cdot 4$ sq. ins.

$\therefore$ Load carried by steel bars = $(2\cdot 4 \times t)$ lb.

Area of concrete = $[(14'' \times 12'') - 2\cdot 4]$ sq. ins. = $165\cdot 6$ sq. ins.

$\therefore$ Load carried by concrete = $(165\cdot 6 \times c)$ lb.

But load carried by steel plus load carried by concrete = total load on column,

$\therefore 2\cdot 4t + 165\cdot 6c = 120{,}960$ . . . . . (i)

We get a second equation connecting *' t '* and *' c '* from the equal-strain property.

$$30{,}000{,}000 = \frac{t}{\text{Strain in steel}}$$

$$\therefore \text{Strain in steel} = \frac{t}{30{,}000{,}000}$$

$$\text{Also } 2{,}000{,}000 = \frac{c}{\text{Strain in concrete}}$$

$$\therefore \text{ Strain in concrete} = \frac{c}{2{,}000{,}000}$$

$$\therefore \text{ As strains are equal, } \frac{t}{30{,}000{,}000} = \frac{c}{2{,}000{,}000}$$

$$\therefore t = c \times \frac{30{,}000{,}000}{2{,}000{,}000} = 15c \quad . \quad . \quad . \quad \text{(ii)}$$

This is a well-known and important result much used in reinforced concrete calculations. Substituting in (i):

$$(2{\cdot}4 \times 15c) + 165{\cdot}6c = 120{,}960$$
$$201{\cdot}6c = 120{,}960$$
$$c = 600 \text{ lb./in.}^2$$
$$\text{and } t = 15 \times c = 9000 \text{ lb./in.}^2.$$

Definitions

Ductility.—A specimen of mild steel when tested in tension will not show any *visible* signs of dimensional alteration during the early stages of the test, i.e. during the 'elastic range of stress.' Shortly after the maximum elastic stress has been reached the specimen stretches visibly and extensions can be measured by means of dividers. The cross-sectional area becomes much smaller and the area at fracture may be less than half the original

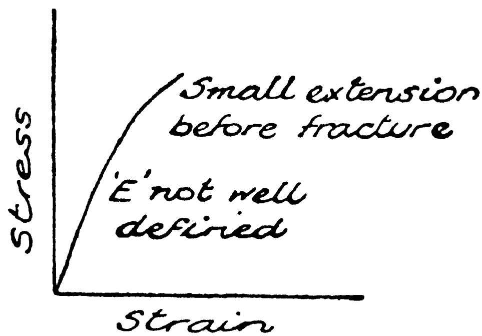


FIG. 246.—BRITTLE SPECIMEN IN TENSILE TEST.

sectional area. This property of marked dimensional change with increasing applied stress is termed '*ductility*.' Steel to be used in a steel-framed building must exhibit the property of ductility when representative specimens are tested.

Brittleness.—'*Brittleness*' denotes absence of ductility. Cast iron is brittle and specimens of cast iron tested to destruction

undergo very little dimensional change. Fig. 246 shows a typical stress-strain graph for a brittle specimen tested in tension.

Elastic Limit.—The elastic limit stress is the stress intensity up to which it is perfectly safe to go without causing the material to lose its property of perfect elasticity. If we stress a piece of material beyond its elastic limit stress, it will not regain its original size or shape when the deforming force producing the stress is removed.

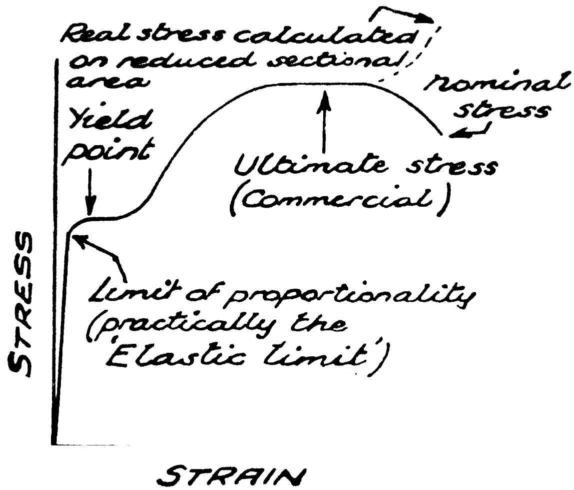


FIG. 247.—TENSILE TEST. MILD STEEL.

Permanent Set.—This is the dimensional change which persists after all load is removed. It is due to stressing the material beyond its elastic limit.

Limit of Proportionality.—As the term implies, this is the stress value at which the proportional law of Hooke breaks down. In most cases little practical distinction need be made between this stress and the 'elastic limit stress.'

Yield Point.—As will be seen in Fig. 247, at a point just above the 'limit of proportionality' a considerable increase in strain takes place in ductile materials with little increase of stress. The stress value at which this big increase takes place is termed the '*yield point*' of the material. As the '*working stress*' in a member must be kept well below the yield point, in order to be able to take advantage of the high ultimate stresses of certain

modern steels, it is necessary to show by test specimens that the 'yield point' has also been suitably raised.

The reader is referred to books on '*Strength of Materials*' for a more detailed account of the behaviour of various structural materials under test.

Examples of Bolted and Riveted Joints in Structural Steelwork

The general principles involved in the calculation of the strength of riveted (or bolted) joints will be explained by means of a few typical examples.

Value of One Rivet (or Bolt)

In the usual type of structural connection—in which steel plates are lapped or butted together—a rivet has two types of strength: (i) *a shear strength*, (ii) *a bearing strength.*

Shear Strength of One Rivet

A rivet may be (i) in *single* shear or (ii) in *double* shear, according to the type of joint.

Single Shear.—When the shearing tendency is across one sectional plane only, the rivet is said to be in '*single shear.*'

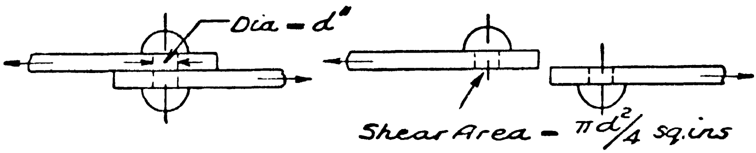


FIG. 248.—RIVET IN SINGLE SHEAR.

Double Shear.—If the rivet is subjected to shear across two sections, as in Fig. 249, the rivet is in '*double shear.*'

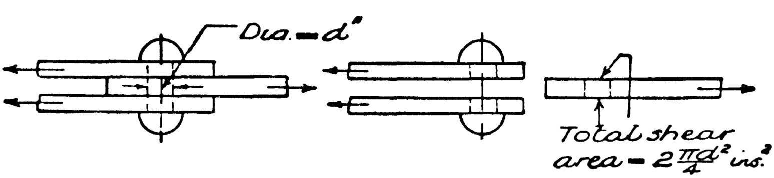


FIG. 249.—RIVET IN DOUBLE SHEAR.

If 'f_s' tons/in.² be the safe working stress in shear in the rivet material and 'd' ins. be the rivet diameter, the safe shear load, in tons per rivet, will be as follows :

(i) Single shear : Safe load $=$ Working stress $\times$ sectional area
$= (f_s \times \pi d^2/4)$ tons.

(ii) Double shear : Safe load $= (f_s \times 2\pi d^2/4)$ tons.

$\therefore$ *Single shear strength of one rivet (or bolt)* $= \dfrac{\pi d^2}{4} f_s$ tons.

Double shear strength „ „ „ $= \dfrac{2\pi d^2}{4} f_s$ tons.

Bearing Strength of One Rivet

A thin plate pressing against the shank of a rivet may do greater damage by imposing a high intensity of '*bearing*' or contact stress than by tending to cause shear in the manner already explained.

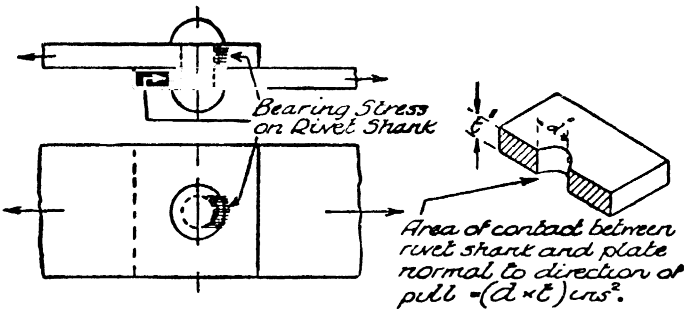


FIG. 250.—BEARING STRESS IN RIVETS.

It is necessary therefore to limit the intensity of the bearing stress on the rivet.

If 'f_b' tons/in.² be the maximum safe bearing stress, and the area resisting bearing be taken as the projected area normal to the direction of pressure (see Fig. 250), the safe bearing strength of one rivet (or bolt) will be given by the expression $d \times t \times f_b$ tons, where t'' = plate thickness.

Summary

S.S. strength of one rivet (or bolt) $= \dfrac{\pi d^2}{4} f_s$ tons.

D.S. „ „ „ „ $= \dfrac{2\pi d^2}{4} f_s$ tons.

Bearing „ „ „ „ $= dtf_b$ tons.

SHEARING and BEARING VALUES FOR RIVETS AND BOLTS

BASED ON B.S. 449 REVISED 1948

Effective Diameter of Rivet or Bolt in inches	Area in square inches	Shearing Value @ 6 tons/inch² Single Shear	Shearing Value @ 6 tons/inch² Double Shear	Simple Bearing Value @ 12 tons/inch² on Thickness in inches of plate passed through: 1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1
3/8	0.1104	0.66	1.33	1.13	1.41	1.69								
7/16	0.1503	0.90	1.80	1.31	1.64	1.97	2.30							
1/2	0.1964	1.18	2.36	1.50	1.88	2.25	2.63	3.00						
9/16	0.2485	1.49	2.98	1.69	2.11	2.53	2.95	3.38	3.80					
5/8	0.3068	1.84	3.68	1.88	2.34	2.81	3.28	3.75	4.22					
11/16	0.3712	2.23	4.45	2.06	2.58	3.09	3.61	4.13	4.64	5.16				
3/4	0.4418	2.65	5.30	2.25	2.81	3.38	3.94	4.50	5.06	5.63	6.19			
13/16	0.5185	3.11	6.22	2.44	3.05	3.66	4.27	4.88	5.48	6.09	6.70	7.31		
7/8	0.6013	3.61	7.22	2.63	3.28	3.94	4.59	5.25	5.91	6.56	7.22	7.88		
15/16	0.6903	4.14	8.28	2.81	3.52	4.22	4.92	5.63	6.33	7.03	7.73	8.44	9.84	
1	0.7854	4.71	9.42	3.00	3.75	4.50	5.25	6.00	6.75	7.50	8.25	9.00	10.50	12.00
1 1/16	0.8866	5.32	10.64	3.19	3.98	4.78	5.58	6.38	7.17	7.97	8.77	9.56	11.16	12.75

Effective Diameter of Rivet or Bolt in inches	Area in square inches	Shearing Value @ 5 tons/inch² Single Shear	Shearing Value @ 5 tons/inch² Double Shear	Simple Bearing Value @ 10 tons/inch² on Thickness in inches of plate passed through: 1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4	7/8	1
3/8	0.1104	0.55	1.10	0.94	1.17	1.41								
7/16	0.1503	0.75	1.50	1.09	1.37	1.64	1.91							
1/2	0.1964	0.98	1.96	1.25	1.56	1.88	2.19	2.50						
9/16	0.2485	1.24	2.49	1.41	1.76	2.11	2.46	2.81	3.16					
5/8	0.3068	1.53	3.07	1.56	1.95	2.34	2.73	3.13	3.52					
11/16	0.3712	1.86	3.71	1.72	2.15	2.58	3.01	3.44	3.87	4.30				
3/4	0.4418	2.21	4.42	1.88	2.34	2.81	3.28	3.75	4.22	4.69	5.16			
13/16	0.5185	2.59	5.18	2.03	2.54	3.05	3.55	4.06	4.57	5.08	5.59	6.09		
7/8	0.6013	3.01	6.01	2.19	2.73	3.28	3.83	4.38	4.92	5.47	6.02	6.56		
15/16	0.6903	3.45	6.90	2.34	2.93	3.52	4.10	4.69	5.27	5.86	6.45	7.03	8.20	
1	0.7854	3.93	7.85	2.50	3.13	3.75	4.38	5.00	5.63	6.25	6.88	7.50	8.75	10.00
1 1/16	0.8866	4.43	8.87	2.66	3.32	3.98	4.65	5.31	5.98	6.64	7.30	7.97	9.30	10.63

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BASED ON B.S. 449 REVISED 1948

SHEARING and BEARING VALUES FOR RIVETS AND BOLTS

Effective Diameter of Rivet or Bolt in inches	Area in square inches	Shearing Value @ 6 tons/inch^2		Enclosed Bearing Value @ 12 tons/inch^2 + 25% on Thickness in inches of enclosed plate								
		Single Shear	Double Shear	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
3/8	0·1104	0·66	1·33	1·41	1·76							
7/16	0·1503	0·90	1·80	**1·64**	2·05	2·46						
1/2	0·1964	1·18	2·36	**1·88**	**2·34**	2·81	3·28					
9/16	0·2485	1·49	2·98	**2·11**	**2·64**	3·16	3·69					
5/8	0·3068	1·84	3·68	**2·34**	**2·93**	**3·52**	4·10	4·69				
11/16	0·3712	2·23	4·45	**2·58**	**3·22**	**3·87**	4·51	5·16				
3/4	0·4418	2·65	5·30	**2·81**	**3·52**	**4·22**	**4·92**	5·63	6·33			
13/16	0·5185	3·11	6·22	3·05	**3·81**	**4·57**	**5·33**	**6·09**	6·86	7·62		
7/8	0·6013	3·61	7·22	3·28	**4·10**	**4·92**	**5·74**	**6·56**	7·38	8·20		
15/16	0·6903	4·14	8·28	3·52	4·39	**5·27**	**6·15**	**7·03**	**7·91**	8·79	9·67	
1	0·7854	4·71	9·42	3·75	4·69	**5·63**	**6·56**	**7·50**	**8·44**	**9·38**	10·31	11·25
1 1/16	0·8866	5·32	10·64	3·98	4·98	**5·98**	**6·97**	**7·97**	**8·96**	**9·96**	10·96	11·95

Effective Diameter of Rivet or Bolt in inches	Area in square inches	Shearing Value @ 5 tons/inch^2		Enclosed Bearing Value @ 10 tons/inch^2 + 25% on Thickness in inches of enclosed plate								
		Single Shear	Double Shear	1/4	5/16	3/8	7/16	1/2	9/16	5/8	11/16	3/4
3/8	0·1104	0·55	1·10	1·17	1·46							
7/16	0·1503	0·75	1·50	**1·37**	1·71	2·05						
1/2	0·1964	0·98	1·96	**1·56**	**1·95**	2·34	2·73					
9/16	0·2485	1·24	2·49	**1·76**	**2·20**	2·64	3·08					
5/8	0·3068	1·53	3·07	**1·95**	**2·44**	**2·93**	3·42	3·91				
11/16	0·3712	1·86	3·71	**2·15**	**2·69**	**3·22**	3·76	4·30				
3/4	0·4418	2·21	4·42	**2·34**	**2·93**	**3·52**	**4·10**	4·69	5·27			
13/16	0·5185	2·59	5·18	2·54	**3·17**	**3·81**	**4·44**	**5·08**	5·71	6·35		
7/8	0·6013	3·01	6·01	2·73	**3·42**	**4·10**	**4·79**	**5·47**	6·15	6·84		
15/16	0·6903	3·45	6·90	2·93	**3·66**	**4·39**	**5·13**	**5·86**	**6·59**	7·32	8·06	
1	0·7854	3·93	7·85	3·13	3·91	**4·69**	**5·47**	**6·25**	**7·03**	**7·81**	8·59	9·38
1 1/16	0·8866	4·43	8·87	3·32	4·15	**4·98**	**5·81**	**6·64**	**7·47**	**8·30**	9·13	9·96

Constructional Steelwork Association and the British Steelmakers.

* See note at foot of page 188.

The actual safe load for a rivet (or bolt) in any given case will be the *lesser* of its 'shear' and 'bearing' values.

Working Stresses for Rivets and Bolts

The tables given on pages 184 and 185 are based on the working stresses specified at the head of the respective tables. The reader will have no difficulty in amending the tabular values for the case of a working stress not included in either of the three tables given. The safe value for a bolt or a rivet is directly proportional to the appropriate working stress selected.

EXAMPLE (i).—*Calculate the safe load in (i) double shear, (ii) bearing for a* $\frac{3}{4}''$ *dia. rivet, assuming a plate thickness of* $\frac{9}{16}''$ *and the following working stresses:* $f_s = 6{\cdot}5$ *tons/in.*^{2}, $f_b = 13$ *tons/in.*^{2}.

(a) *By use of formulæ:*

$$\text{D.S. value} = 2\frac{\pi d^2}{4} f_s \text{ tons} = 2 \times \frac{\pi \times \frac{3}{4}^2}{4} \times 6{\cdot}5 \text{ tons}$$
$$= 5{\cdot}74 \text{ tons.}$$
$$\text{Bearing value} = dtf_b \text{ tons} = \tfrac{3}{4} \times \tfrac{9}{16} \times 13 \text{ tons.}$$
$$= 5{\cdot}48 \text{ tons.}$$

(b) *By use of tables:*

Using the first table, evaluated for $f_s = 6$ tons per sq. inch and $f_b = 12$ tons per sq. inch, we have, for the given stresses:

$$\text{D.S. value} = \left(5{\cdot}3 \times \frac{6{\cdot}5}{6}\right) \text{ tons} = 5{\cdot}74 \text{ tons.}$$

$$\text{Bearing value} = (5{\cdot}06 \times \tfrac{13}{12}) \text{ tons} = 5{\cdot}48 \text{ tons.}$$

The safe load for a $\frac{3}{4}''$ dia. rivet in the given circumstances will be 5·48 tons.

EXAMPLE (ii).—*Calculate the safe load for the joint shown in Fig.* 251, *from the point of view of the rivets in the connection.*

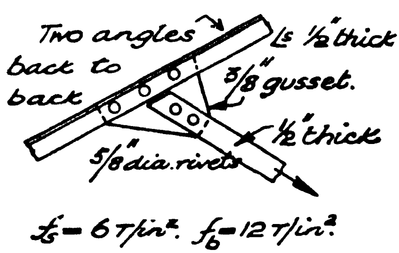


FIG. 251.—RIVETED JOINT.

In this example the tie-member laps on to the gusset plate so that the two connecting rivets are in single shear. These rivets are also under bearing stress in plate thicknesses, respectively, of $\frac{3}{8}''$ and $\frac{1}{2}''$. The $\frac{3}{8}''$ thickness would clearly produce the higher stress. In the case of a lap joint with plates of different thicknesses, take the *smaller thickness* for bearing-strength calculations.

Single shear strength of one $\frac{5}{8}''$ dia. rivet at 6 tons/in.^{2}

$$= \frac{\pi d^2}{4} f_s = \frac{\pi \times \frac{5}{8}^2}{4} \times 6 = 1{\cdot}84 \text{ tons.}$$

Bearing strength of one $\frac{5}{8}''$ dia. rivet in $\frac{3}{8}''$ plate thickness $= dtf_b = \frac{5}{8} \times \frac{3}{8} \times 12 = 2{\cdot}81$ tons.

The *lesser* of these two strengths $= 1{\cdot}84$ tons, therefore one rivet is worth $1{\cdot}84$ tons in the joint.

$\therefore$ Safe total load $= 2 \times 1{\cdot}84 = 3{\cdot}68$ tons.

The connection between the gusset plate and the angles is clearly stronger because we have here 3 rivets in *double shear* or bearing in $\frac{3}{8}''$ thickness.

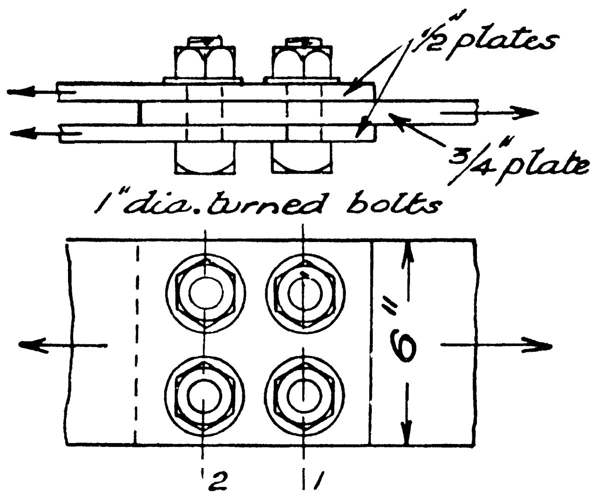


FIG. 252.—BOLTED JOINT.

EXAMPLE (iii).—*Fig.* 252 *shows a plate,* 6" *wide* $\times$ $\frac{3}{4}''$ *thick, connected to two* $\frac{1}{2}''$ *plates by means of* 4 *No.* 1" *dia. turned and fitted bolts. Calculate the safe load the connection can transmit.* $f_s = 6$ *tons/in.*^{2}, $f_b = 12$ *tons/in.*^{2}, $f_t = 8$ *tons/in.*^{2}

In this case three distinct strengths will have to be considered :

(i) Bolt strength, (ii) $\frac{3}{4}''$ plate strength, (iii) the combined strength of the two $\frac{1}{2}''$ plates.

Bolt strength:

The bolts are in double shear and bearing in $\frac{3}{4}''$ plate thickness. The $\frac{1}{2}''$ plate thickness is not taken for bearing-strength calculation, although less than $\frac{3}{4}''$, because the two $\frac{1}{2}''$ plates act in conjunction and are equivalent to a thickness of $\frac{1}{2}'' + \frac{1}{2}'' = 1''$.

Double shear strength of one 1″ dia. bolt $= 2 \times \frac{\pi d^2}{4} f_s$ tons

$$= 2 \times \frac{\pi \times 1^2}{4} \times 6 \text{ tons} = 9{\cdot}42 \text{ tons.}$$

* *Bearing strength* of one 1″ dia. bolt in $\frac{3}{4}''$ plate thickness $= dtf_b$ $= (1 \times \frac{3}{4} \times 12)$ tons $= 9$ tons

∴ Actual value of one bolt $= 9$ tons

∴ Bolt strength of connection $= 4 \times 9$ tons $= 36$ tons.

$\frac{3}{4}''$ plate strength:

This plate is in tension and will tend to fail at a section weakened by bolt holes. When there are several sections weakened by reduction of sectional area, each section should be separately considered.

Section 1.—The '*effective*' or solid plate width at this section $= 6'' - (2 \times 1)'' = 4''$

∴ Effective or net area $= 4'' \times \frac{3}{4}'' = 3$ sq. ins.

Safe tensile load = net area × working tensile stress $= (3 \times 8)$ tons $= 24$ tons.

Section 2.—If the plate failed at this section it could not come out from between the two $\frac{1}{2}''$ plates owing to the two bolts at section 1 acting as stop pegs. Therefore the strength of these two bolts should be added on to the $\frac{3}{4}''$ plate strength at section 2, in order to compute the total strength there (see also Example (iv)). The plate is therefore stronger at section 2.

Combined $\frac{1}{2}''$ plates:—(no 'stop peg' allowance this time).

Tensile strength = Effective area × 8 tons/in.²

$$= [(6 - 2) \times (2 \times \tfrac{1}{2})] \times 8 \text{ tons} = 32 \text{ tons.}$$

Summary: Bolt strength $= 36$ tons.
$\frac{3}{4}''$ pl. strength $= 24$ tons.
$2/\frac{1}{2}''$ pl. strength $= 32$ tons.

* When rivets are in double shear the permissible bearing stress on the central thickness of metal may be increased by 25 per cent. This allowance will not be claimed in this book.

The safe load for the connection is the smallest of all these values, i.e. 24 tons.

EXAMPLE (iv).—*Calculate the safe load for the double-covered butt joint given in Fig. 253. Use the following working stresses:* $f_s = 6$ *tons/in.*^{2}, $f_b = 12$ *tons/in.*^{2}, $f_t = 7$ *tons/in.*^{2}.

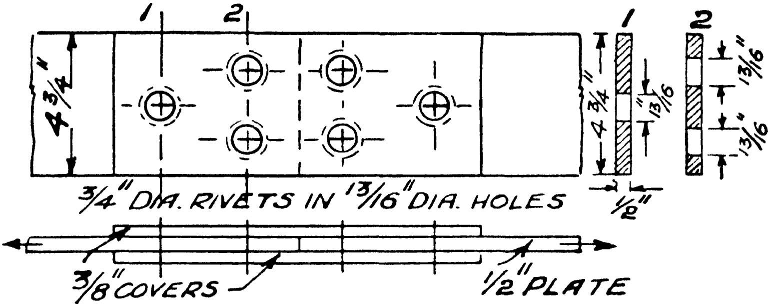


FIG. 253.—DOUBLE-COVERED BUTT JOINT.

The rivet arrangement, giving a single or '*leading*' rivet at section 1, results in an efficient or economical joint, as only one rivet hole is deducted from the solid plate section without some compensating rivet strength (see below). The reader will note the usual allowance of $\frac{1}{16}''$ clearance on the *nominal* rivet diameter to allow easy entry of the rivet (when hot) into the rivet hole. Some regulations permit the '*finished*' rivet size, i.e. $\frac{13}{16}''$ in this case, to be used for strength calculations.

Rivet strength:

D.S. value of one rivet $= 2 \times \dfrac{\pi d^2}{4} \times f_s = 2 \times \dfrac{\pi \times \frac{3}{4}^2}{4} \times 6$

$= 5{\cdot}3$ tons.

* Bearing value of one rivet $= dtf_b = \frac{3}{4}'' \times \frac{1}{2}'' \times 12 = 4{\cdot}5$ tons

$\therefore$ Strength of one rivet $= 4{\cdot}5$ tons

$\therefore$ Total rivet strength $= 3$ (*not* 6) $\times 4{\cdot}5 = 13{\cdot}5$ tons.

$\frac{1}{2}''$ plate strength:

Section 1.—Tensile strength $= (4{\cdot}75 - \frac{13}{16}) \times \frac{1}{2} \times 7 = 13{\cdot}78$ tons.

Section 2.—Total strength $= (4{\cdot}75 - 2 \times \frac{13}{16}) \times \frac{1}{2} \times 7$ *plus* the strength of one rivet $= 10{\cdot}93 + 4{\cdot}5 = 15{\cdot}43$ tons

$\therefore$ Actual $\frac{1}{2}''$ plate strength $= 13{\cdot}78$ tons.

* 12 tons/in.^{2} taken for this problem for f_b, as noted at foot of page 188.

Cover-plate strength:

The weakest section for the covers is 'section 2.' There is no rivet strength to be added on. The 'leading rivet' at section 1 would simply come away with the fractured joint.

Tensile strength $= (4 \cdot 75 - 2 \times \frac{13}{16}) \times 2 \times \frac{3}{8} \times 7$ tons $= 3 \cdot 125 \times 5 \cdot 25 = 16 \cdot 4$ tons.

Summary: Rivet strength $= 13 \cdot 5$ tons.
$\frac{1}{2}''$ plate strength $= 13 \cdot 78$ tons.
Cover-plate strength $= 16 \cdot 4$ tons.

Actual safe load for joint $= 13 \cdot 5$ tons.

EXAMPLE (v).—*Calculate the safe uniformly distributed load* '*W*' (Fig. 254) *from the point of view of the end connections of the steel beam.* $f_s = 6$ *tons/in.*^{2}, $f_b = 12$ *tons/in.*^{2}.

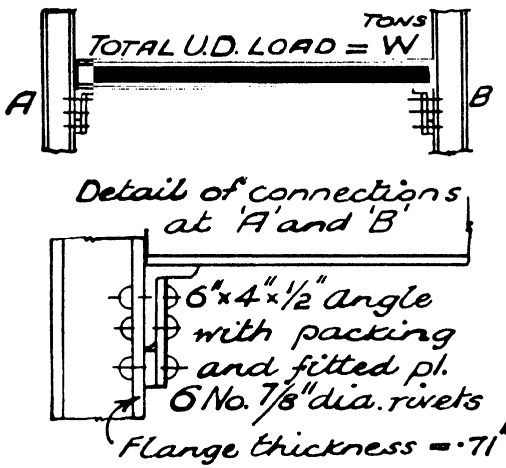


FIG. 254.—BRACKET CONNECTION.

In practical connections of the type illustrated, the single shear strength of the rivets is the criterion. Thus in this case:

S.S. strength of one rivet $= \dfrac{\pi \times \frac{7}{8}^2}{4} \times 6 \qquad = 3 \cdot 61$ tons

Bearing strength of one rivet $= dtf_b = \frac{7}{8} \times \frac{1}{2} \times 12 = 5 \cdot 25$ tons

$\therefore$ Actual strength $= 3 \cdot 61$ tons

[Sometimes a slight reduction is made in the rivet strength to allow for a bending tendency on the bracket.]

$\therefore$ Total rivet strength $= 6 \times 3 \cdot 61 = 21 \cdot 66$ tons.

This value is the maximum safe reaction for the beam, therefore the maximum safe value of 'W' $= 2 \times 21{\cdot}66 = 43{\cdot}32$ tons.

Exercises 9

(1) A tie-bar has a rectangular section, $4'' \times \frac{5}{8}''$. It is subjected to an axial pull of 20 tons. Find the stress in the material of the tie-bar. Replace the $\frac{5}{8}''$ thickness given by another thickness which would correspond to a stress of 5 tons per sq. inch.

(2) A mild steel tie-rod has to carry an axial load of 5·4 tons. The maximum permissible stress = 9 tons per sq. inch. Calculate a suitable diameter for the rod.

(3) A reinforced concrete column transmits a total axial load of 106·5 tons to a footing slab of 6 tons estimated weight. Assuming the ground to have a safe bearing pressure of 2 tons per sq. foot, calculate suitable dimensions, in plan, for the slab.

(4) A short square pier is built of blue brickwork in cement mortar. Calculate the necessary side of the square section if the pier is to carry a concentric load of 12·5 tons. Assume 10 tons per sq. foot to be the safe load for the brickwork. (Make no allowance for self-weight of pier.)

(5) How many $\frac{3}{4}''$ dia. rivets in single shear would be required to transmit a shear load of $26\frac{1}{2}$ tons, if the maximum permissible stress were 6 tons per sq. inch?

(6) Calculate Young's modulus in compression for a specimen of timber, $4'' \times 4''$ in section, which shortened by ·0016" on a gauge length of 8" when the test load (axially applied) was 3840 lb.

(7) A mild steel tie-rod 10 ft. long, having a sectional area of 1·5 sq. ins., is subjected to an axial pull of 9·75 tons. Assuming 'E' to be 13,000 tons per sq. inch, calculate the extension in the rod.

(8) A short compression specimen is composed of concrete and steel. The total sectional area = 24 sq. ins., comprised of 4 sq. ins. of steel surrounded by 20 sq. ins. of concrete. The specimen is subjected to an axial test load which produces a stress of 600 lb./in.^{2} in the concrete. Calculate (i) the stress in the steel, (ii) the test load referred to.

$E_{steel} = 30{,}000{,}000$ lb./in.^{2}, $E_{concrete} = 2{,}000{,}000$ lb./in.^{2}.

(9) The following results were obtained in a laboratory test on a steel specimen :

Sectional area of specimen = ·66 sq. ins.
Gauge length = 8 ins.
Applied load = 2·145 tons.
Corresponding extension on gauge length = ·002 ins.
Maximum load in test = 21·12 tons.

Calculate (i) Young's modulus, (ii) ultimate stress, (iii) necessary thickness for a tie-bar of this quality of steel, 4″ wide, to safely carry an axial load of 16 tons, using a factor of safety of 4.

(10) A specimen of mild steel, 4″ × $\frac{3}{4}$″, broke in a tensile test at a maximum load of 90 tons. Calculate the necessary diameter of a circular bar of the same quality steel which has to carry an axial pull of 1·8 tons. Use a factor of safety of 5.

(11) Briefly explain the meaning of the following : 'ultimate stress,' 'factor of safety' and 'Young's modulus.' Draw a fully referenced diagram indicating the stress-strain relationship for a mild steel specimen tested to destruction.

A short compression specimen of concrete, 4″ × 4″ in section, was tested between the same compression plates as a timber specimen, 6″ × 6″ in section.

Calculate the load carried by the concrete and the load carried by the timber if the testing machine registered a total load of 3760 lb. $E_{concrete} = 2{,}000{,}000$ lb./in.^{2}, $E_{timber} = 1{,}200{,}000$ lb./in.^{2}

(12) Fig. 255 shows the end connection of a strut member. Calculate the safe load for the strut from the point of view of the

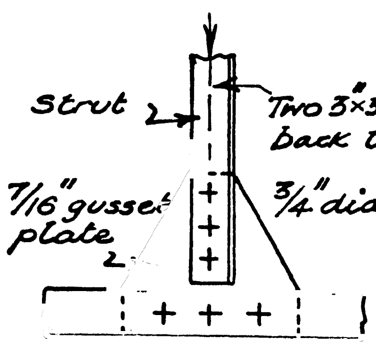


FIG. 255.

rivets at its end, assuming the working stresses in shear and bearing to be 6 tons/in.^{2} and 12 tons/in.^{2} respectively.

(13) Calculate the safe load for the tie-bar joint given in Fig. 256. The tensile stress has to be limited to 8 tons/in.^{2}.

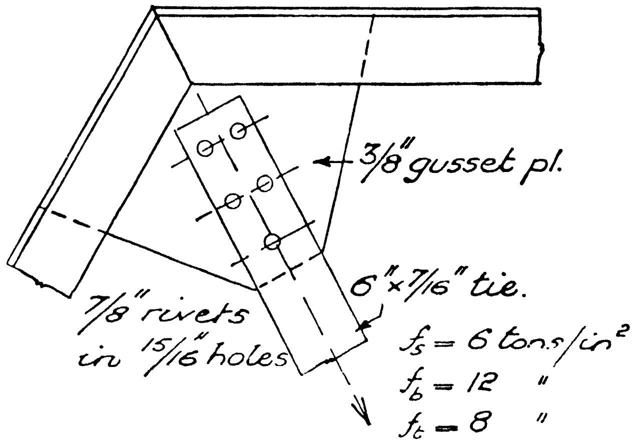


FIG. 256.—GUSSET CONNECTION.

(14) Obtain the safe end reaction for the beam shown in Fig. 257 from the point of view of the strength of the bolts in the web cleats.

$$f_s = 6 \text{ tons/in.}^2, f_b = 12 \text{ tons/in.}^2$$

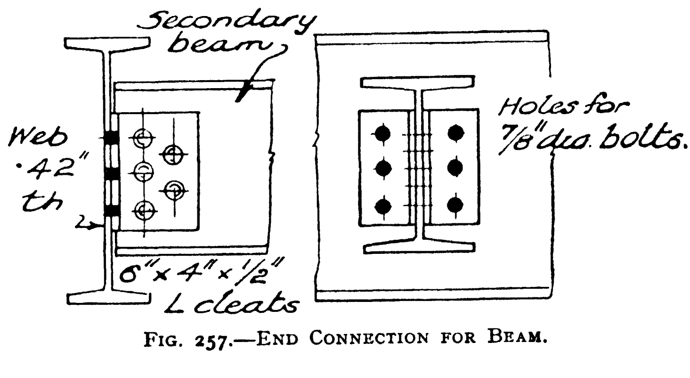


FIG. 257.—END CONNECTION FOR BEAM.

CHAPTER X

SIMPLE BEAMS. BENDING MOMENT AND SHEAR FORCE

THE theory underlying the calculation of the load-carrying capacity of a simple beam will be developed by means of an experimental model beam. The more mathematical treatment of the theory of bending will be found in other text-books on this subject.

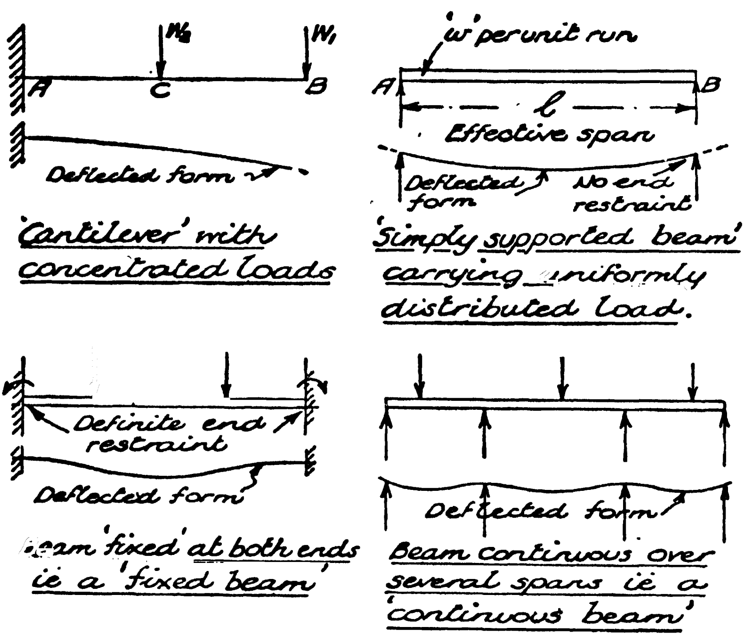


FIG. 258.—TYPES OF BEAMS.

Fig. 258 illustrates various types of beams. The type most commonly occurring in beam calculations is the simply supported beam of one span. The end connections of such a beam are not assumed to be able to develop any appreciable degree of '*fixity*' in the beam.

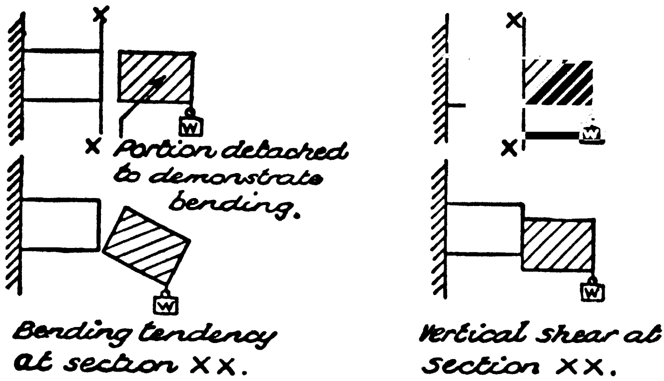


FIG. 259.—BENDING MOMENT AND SHEAR FORCE.

Failure Tendency at a Beam Section

Consider section ' XX ' of the cantilever shown in Fig. 259. If we saw-cut through the cantilever at this section, collapse of the portion to the right of the section plane would take place.

The failure may be attributed to two distinct reasons : (i) the load ' W ' produces a moment at section ' XX ' which the cantilever, when cut, is unable to balance ; (ii) the load ' W ' is able to cause vertical sliding at the section when the ' *shear* ' resistance there is destroyed by the saw-cut.

It is important to realise that to prevent failure of a beam at a section such as ' XX ' we must *independently* counterbalance the two failure tendencies indicated above. This is borne out by trial with the beam model shown in Fig. 260.

(i) If the pull in the vertical string be reduced the cantilever fails. The tension in the chains and the thrust in the horizontal bars do not assist vertical equilibrium.

(ii) If either the chains or bars be removed collapse takes place. The pull in the vertical string is not able to maintain equilibrium.

A further experimental result should be carefully noted. If an additional weight, say 2 lb., be placed on the detached portion of the cantilever, 2 lb. extra weight will have to be placed in the scale pan to preserve equilibrium. But the exact *position* in which the 2-lb. weight is placed on the cantilever is quite immaterial provided it is placed to the right of the vertical string (see the definition of ' *shear force* ' later).

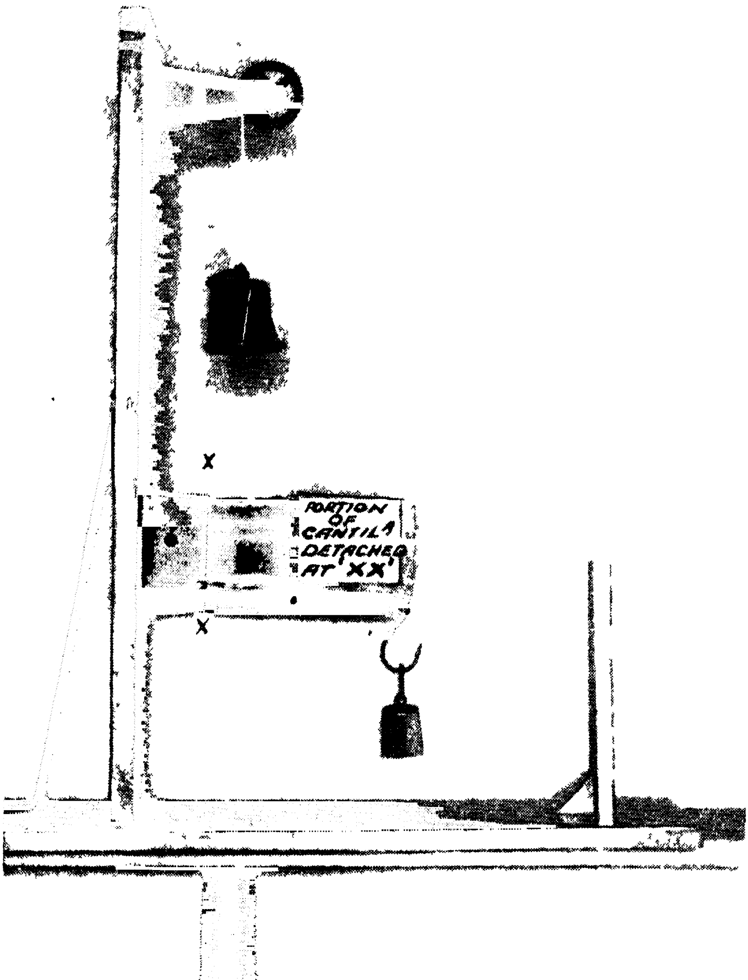


FIG 260.—FORCES ACTING AT A BEAM SECTION.

To demonstrate the failure tendencies with the beam model, it is necessary to have a gap in the cantilever as shown in Fig. 260. In the following development of the theory this gap is assumed to be extremely small, so that section 'XX' may be taken to be at the left end of the detached portion of the cantilever.

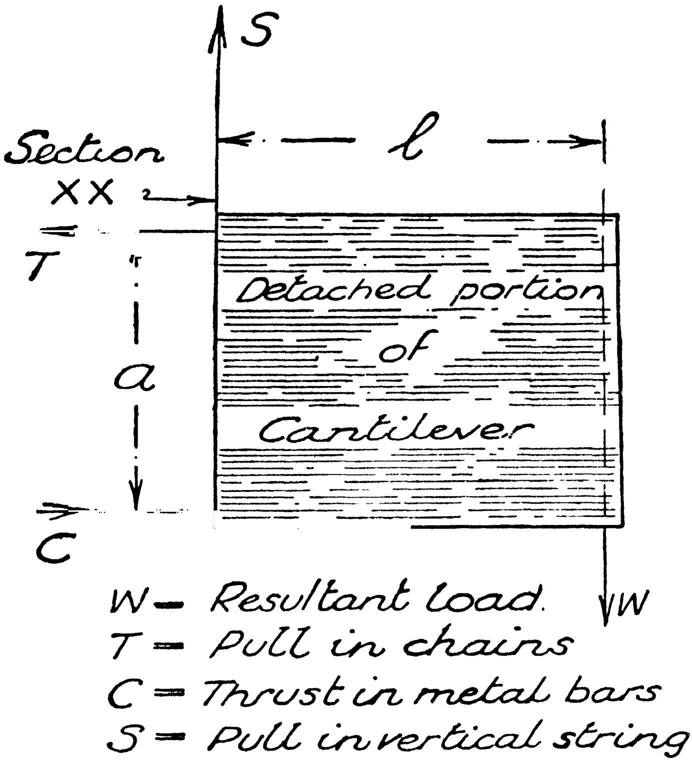


FIG. 261.—B.M., S.F. AND M.R.

Bending Moment at Section 'XX' (Fig. 261)

The load 'W' and the string tension 'S,' being equal and unlike parallel forces acting out of line, form a couple. It is the moment of this couple which causes the '*bending-off*' tendency of failure at section 'XX.' This moment, the magnitude of which is 'W × *l*,' is termed the '*bending moment*' at section 'XX.'

Shear Force at Section 'XX'

The load 'W,' the resultant force tending to cause vertical shearing at 'XX,' is termed the '*shear force*' at the section.

The first portion of the work on beams will be devoted to the investigation of the magnitudes of the two quantities '*bending moment*' and '*shear force*' respectively, in certain beam examples.

Later we will have to consider how the internal fibre forces at a beam section, such as ' XX,' build up a ' *couple,*' able to perform the task of the chains and the metal bars in the model beam apparatus.

Definitions

Bending Moment (B.M.).—*The bending moment at any section of a beam is the resultant moment about that section of all the forces acting to one side of the section.*

Shear Force (S.F.).—*The shear force at any section of a beam is the resultant vertical force of all the forces acting to one side of the section.*

Note.—It is necessary, first of all, to decide which side of the given beam section is going to be taken. Both in ' B.M.' and ' S.F.' calculations, whichever side be selected, the respective answers will be the same. In cantilevers it is easier to consider the forces which act to the ' free end ' side of the section considered. Shear force calculations, as indicated in the definition of S.F. above, merely involve the algebraic addition of vertical loads. The actual positions of the loads, therefore, do not affect the S.F. value, provided they lie to one side of the section (cf. experimental result on page 195).

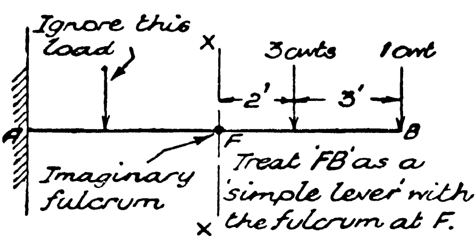


FIG. 262.—BENDING MOMENT AND SHEAR FORCE.

EXAMPLE.—*Calculate the B.M. and S.F. at section ' XX ' of the given cantilever (Fig. 262).*

We consider the forces to the *right* of section ' XX,' completely ignoring those to the left. We then imagine a hinge at the point ' F ' and treat the portion ' FB ' as a simple lever with the fulcrum at ' F.'

Resultant moment about 'F' = (1 cwt. × 5 ft.)
+ (3 cwts. × 2 ft.)
= (5 + 6) cwts. ft.
= 11 cwts. ft.

∴ B.M. at section 'XX' = 11 cwts. ft.

Bending moment values are expressed in the usual units for '*moments.*'

The resultant vertical force, considering all the forces which lie to the right of section 'XX,' = 3 cwts. + 1 cwt. = 4 cwts.

∴ S.F. at section 'XX' = 4 cwts.

Shear force values are, of course, expressed in *force* units.

Convention of Signs for B.M. and S.F.

Fig. 263 illustrates the two possible types of curvature which a given bending moment may impose on a beam. These are distinguished by employing '*plus*' and '*minus*' signs. The convention adopted throughout the book is shown in the figure.

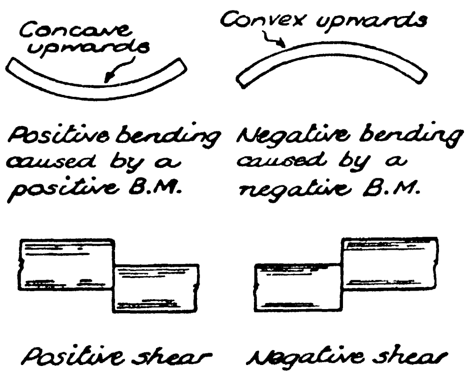


FIG. 263 —CONVENTION OF SIGNS.

Shear force may tend to cause shear in the two ways indicated. The convention adopted in this case is also shown in Fig. 263.

B.M. and S.F. Diagrams

It is sometimes necessary to draw a graph showing the variation of bending moment along the span of a beam. Such a graph is known as a '*bending moment diagram.*' A 'B.M. diagram' has

two independent scales: (i) a linear scale for the span (e.g. 1″ = 4 ft.), and (ii) a B.M. scale for the vertical ordinates (e.g. 1″ = 2 tons ft.).

In the same way 'S.F. diagrams' are constructed to '*linear*' and '*force*' scales. The general form of B.M. and S.F. diagrams in certain standard cases will now be considered.

Cantilever with Single Point Load at the Free End

Fig. 264 shows a cantilever 'AB' with a point load of 2 tons at 'B.'

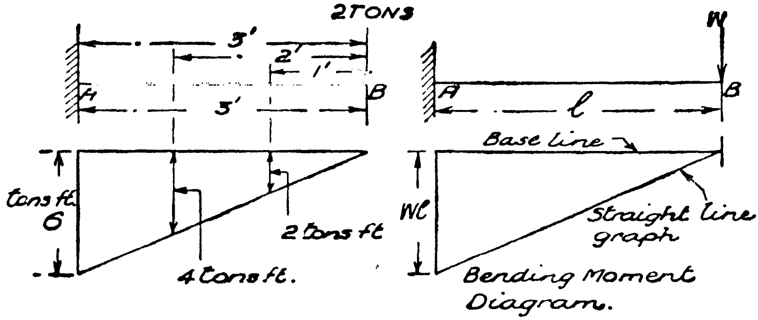


FIG. 264.—BENDING MOMENT DIAGRAM.

B.M. values:

At 'B,' the free end:	B.M. = 2 tons × 0 ft. = 0 tons ft.
„ 1 ft. from 'B':	B.M. = 2 „ × 1 „ = 2 „ „
„ 2 „ „ „	B.M. = 2 „ × 2 „ = 4 „ „
„ 3 „ „ „	B.M. = 2 „ × 3 „ = 6 „ „

The form of the graph is now clear, as each ordinate representing a B.M. value is proportional to the distance of the particular section from 'B.'

The maximum bending moment will obviously be at the support in this case.

$$\text{B.M.}_{\text{max.}} = 2 \text{ tons} \times 3 \text{ ft.} = 6 \text{ tons ft. (negative)}.$$

If 'W' = the point load and 'l' = the length of the cantilever, the max. B.M. will be given by the expression 'Wl.' The B.M. diagram will be bounded by a straight line as indicated in Fig. 264.

S.F. Values

In cases in which the length of bearing of a load on a beam is small compared with the beam length, the load may be regarded as acting at a '*point*' for purposes of B.M. and S.F. diagram construction. This accounts for the sudden vertical jumps which are common in S.F. diagrams. In actual fact, of course, there must be a transition from one 'shear' value to another, however rapid, and no point in a beam can simultaneously have two S.F. values.

Just inside point 'B' (Fig. 265) the S.F. = 2 tons, i.e. the load to the right of the section.

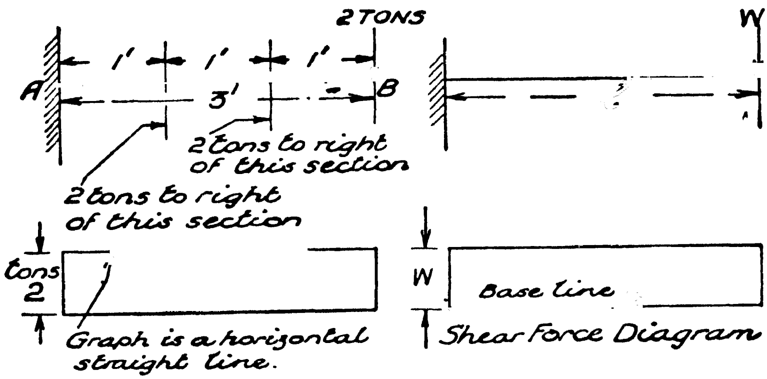


FIG. 265.—SHEAR FORCE DIAGRAM.

At 1 ft. from 'B,' S.F. = load to right of section = 2 tons
,, 2 ,, ,, ,, ,, = ,, ,, ,, = 2 ,,

The shear force is 2 tons for every section in the cantilever. Expressed in symbols, S.F. = constant = 'W.' The S.F. graph will be a straight line parallel to base (Fig. 265). The B.M. diagram is drawn below the base line and the S.F. diagram above the base line in agreement with the convention of signs adopted.

Addition of B.M. (or S.F.) Values and Diagrams

When a beam carries several loads, or more than one type of load system, it is sometimes convenient to deal separately with suitable component portions of the total load. The B.M. (or S.F.) values for a given beam section, contributed by the various component loads, are then algebraically added. Thus if a total

load system be divided up into three component systems which taken *quite independently* produce, respectively, B.M. values of 5 cwts. ft., 3 cwts. ft. and $-$ 2 cwts. ft. at a given section 'C' in a beam, the actual B.M. at 'C' $= (5 + 3 - 2)$ cwts. ft. $=$ 6 cwts. ft.

In like manner B.M. (or S.F.) diagrams may be algebraically added by geometrical means.

Cantilever with Several Point Loads

Fig. 266 shows the combination of two B.M. diagrams in order to arrive at the actual final B.M. diagram. Also the figure illustrates the building up of the net S.F. diagram. The component-diagram method of treatment is not commonly used in this type of example. It has been used here to demonstrate the nature of the complete diagrams.

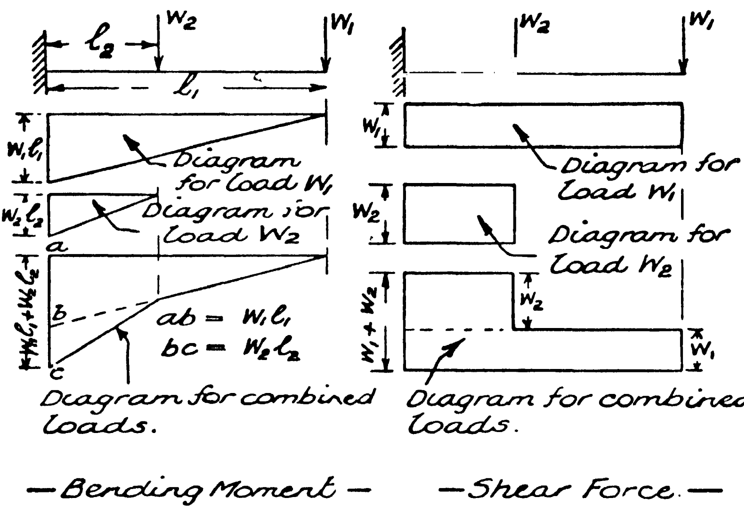


FIG. 266.—ADDITION OF COMPONENT DIAGRAMS.

As the diagrams are both bounded by straight lines, they are more quickly constructed by direct calculation of the values at the points where the loads occur (see following example).

EXAMPLE.—*Construct the B.M. and S.F. diagrams for the cantilever given in Fig. 267. Calculate the B.M. and S.F. for a*

section 2 ft. from the support. Write down the values of $B.M._{max.}$ *and* $S.F._{max.}$ *respectively.*

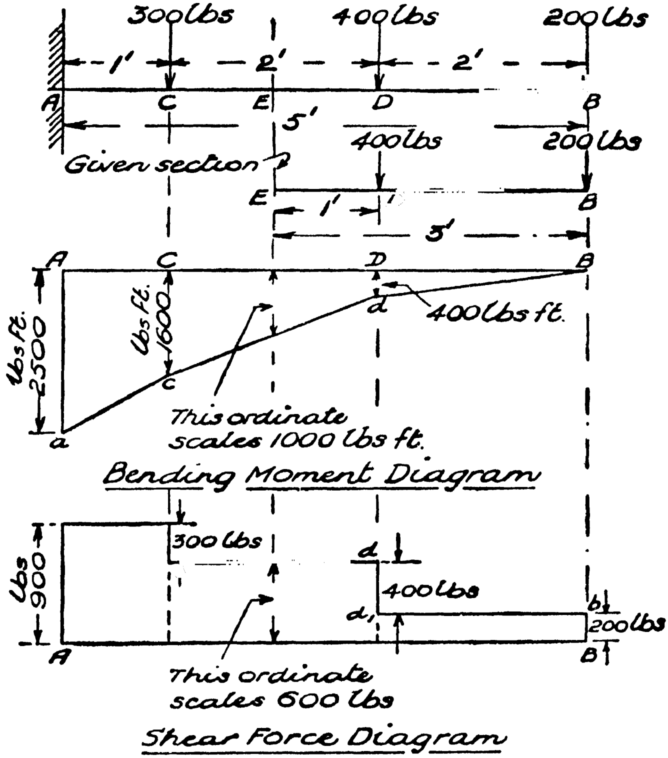


FIG 267.—CANTILEVER WITH POINT LOADS

B.M. at ' B ' = 0. The bending moment is zero at the free end of a cantilever.

B.M. at ' D.' Regard ' D ' as a fulcrum and ' DB ' as a simple lever.

$$B.M._D = 200 \text{ lb.} \times 2 \text{ ft.} = 400 \text{ lb. ft.}$$

B.M. at ' C.' Regard ' C ' as a fulcrum and ' CB ' as a simple lever.

$$B.M._C = [(200 \times 4) + (400 \times 2)] \text{ lb. ft.}$$
$$= (800 + 800) \text{ lb. ft.} = 1600 \text{ lb. ft.}$$

B.M. at the support ' A.' Regard ' A ' as a fulcrum.

$$\begin{aligned} \text{B.M.}_{A} &= [(200 \times 5) + (400 \times 3) + (300 \times 1)] \text{ lb. ft.} \\ &= (1000 + 1200 + 300) \text{ lb. ft.} \\ &= 2500 \text{ lb. ft.} \end{aligned}$$

(All these bending moments are negative, according to the convention of signs we have adopted.)

Special section 2 ft. from support (point 'E'):

Regard 'E' as the fulcrum of a simple lever 'EB.'

$$\begin{aligned} \text{B.M.}_{E} &= [(200 \times 3) + (400 \times 1)] \text{ lb. ft.} \\ &= (600 + 400) \text{ lb. ft.} = 1000 \text{ lb. ft.} \end{aligned}$$

Shear Force.—Between points 'B' and 'D' the shear force is 200 lb. At any point between 'D' and 'C' there is a total load of (200 + 400) lb. to the right, hence the S.F. between 'D' and 'C' = 600 lb. S.F. at 'E' = 600 lb.

S.F. between 'C' and 'A' = (200 + 400 + 300) lb. = 900 lb.

$$\text{B.M.}_{max.} = 2500 \text{ lb. ft.}$$
$$\text{S.F.}_{max.} = 900 \text{ lb.}$$

Construction of Diagrams

Bending Moment.—As B.M.s are all negative, the ordinates are drawn downwards from the base line. At 'D' draw 'D*d*' to represent 400 lb. ft. At 'C' draw 'C*c*' to represent 1600 lb. ft., and at the support draw 'A*a*' to represent 2500 lb. ft. Join up B*dca*.

Shear Force.—At 'B' draw 'B*b*' to represent 200 lb. and draw 'bd_1' parallel to base. 'd_1d' must then be drawn to scale to represent 400 lb. At each load point the S.F. diagram jumps vertically upwards by an amount which represents the corresponding load, to the force scale of the diagram.

The diagrams should be constructed to the following scales:

B.M. diagram: 1″ = 1 ft. and 1″ = 800 lb. ft.

S.F. diagram: 1″ = 1 ft. and 1″ = 400 lb.

The values calculated for the special section may be checked by means of the two diagrams.

Cantilever with Uniformly Distributed Load

The cantilever given in Fig. 268 carries a load of '2 cwts. per foot run.' To find the B.M. at any particular section of the

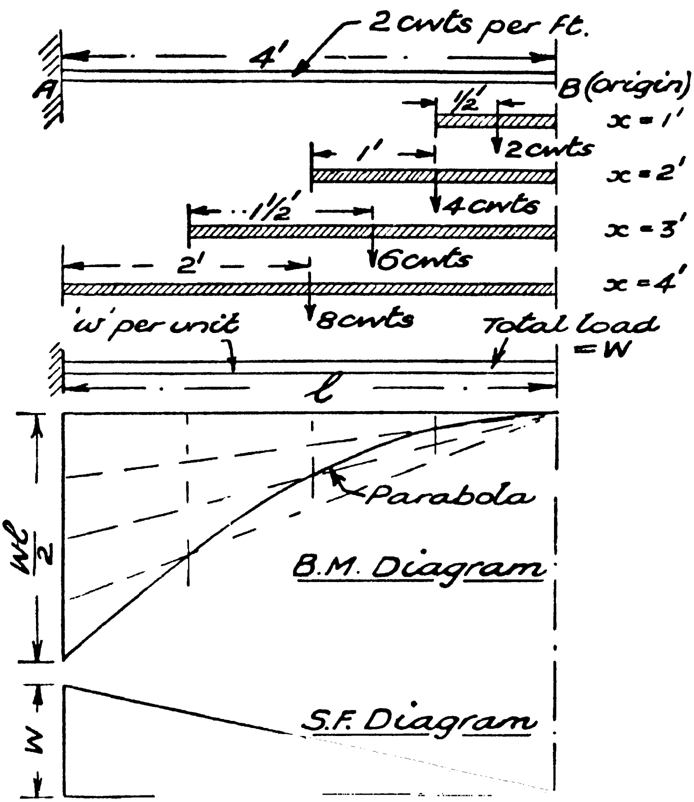


FIG. 268.—CANTILEVER WITH UNIFORM LOAD.

cantilever we imagine, as in the last case, a fulcrum at the section and the portion to the right to be a simple lever.

For purposes of taking *simple moments* a uniformly distributed load may be regarded as being concentrated at the centre of its length (i.e. its centre of gravity).

Let ' x ' = distance from ' B ' of a typical section of the beam. When $x = 1$ ft. (see Fig. 268), (2×1) cwts. lie to the right of the section. The centre of this load is $\frac{1}{2}$ ft. from the section, hence the B.M. at the section = 2 cwts. $\times \frac{1}{2}$ ft. = 1 cwt. ft.

When $x = 2$ ft., B.M. = $[(2 \times 2)$ cwts. $\times 1$ ft.$]$ = 4 cwts. ft.
„ $x = 3$ ft., B.M. = $[(2 \times 3)$ cwts. $\times \frac{3}{2}$ ft.$]$ = 9 „ „
„ $x = 4$ ft., B.M. = $[(2 \times 4)$ cwts. $\times 2$ ft.$]$ = 16 „ „

The 'x' values are in the ratio 1 : 2 : 3 : 4.
The B.M. values „ „ „ 1 : 4 : 9 : 16, i.e. $1^2 : 2^2 : 3^2 : 4^2$.

This means that the B.M. value varies as the square of the distance from 'B.' The B.M. graph is therefore a *parabola*—a curve satisfying the given conditions.

If 'w' = load per unit run and 'l' = length of the cantilever, the total load carried by the cantilever = wl = W.

B.M.$_{max.}$ will occur at the support and will equal $W \times l/2 = \frac{Wl}{2}$.

Geometrical Construction of a Parabola

The base line representing the length of the cantilever is divided into a convenient number of equal parts and the ordinate which represents the maximum bending moment is divided into the *same number* of equal parts. Verticals and radials are drawn in, as shown in Fig. 268. The intersections of corresponding

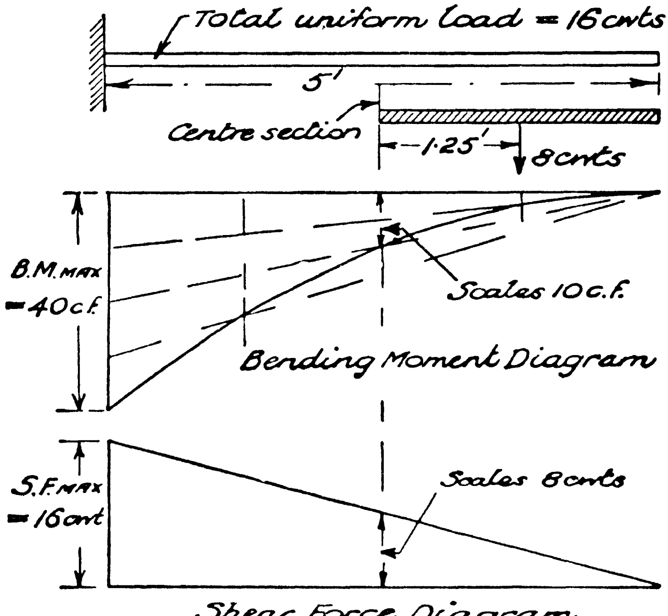


FIG. 269.—CANTILEVER EXAMPLE.

verticals and radials, in the manner indicated, give points on the parabola.

Shear Force.—Referring to Fig. 268, when $x = 1$ ft. the load to right of the section = 2 cwts. When $x = 2$ ft. the load = 4 cwts. In this case these loads, which represent S.F. values, increase in direct proportion with the distance from the free end of the cantilever. The shear force graph is therefore an inclined straight line. The max. S.F. will occur at the support and will equal ' W,' the total load on the cantilever.

Note.—The max. S.F. for a cantilever with *any* load system equals the total load carried by the cantilever.

EXAMPLE.—*Draw the B.M. and S.F. diagrams for a cantilever 5 ft. long, which carries a uniformly distributed load of total value 16 cwts. Calculate the B.M. and S.F. values for the centre point of the cantilever.*

$$\text{B.M.}_{\text{max.}} = \frac{Wl}{2} = \frac{16 \times 5}{2} \text{ c.f.} = 40 \text{ c.f (negative).}$$

$\text{S.F.}_{\text{max.}}$ = total load = W = 16 cwts. (positive).

The diagrams are shown in Fig. 269.

B.M. at centre of cantilever = $(8 \times 1{\cdot}25)$ c.f. = 10 c.f.

S.F. ,, ,, ,, = 8 cwts.

Suggested scales for diagrams :

1" = 1 ft., 1" = 10 c.f. (B.M.), 1" = 4 cwts. (S.F.).

Beams Simply Supported at Each End

The B.M. and S.F. definitions, given on page 198, may be expressed in simplified form for the case of simply supported beams.

Bending Moment = ' *Reaction moment — load moments.*

The ' reaction ' and ' loads ' must be taken to one side of the section and ' moments ' taken about the section.

Shear Force = ' *Left-end reaction — sum of loads up to section.'*

If the left side of section be taken always for S.F. computation, the correct *sign* will be obtained for the S.F. value.

Simply Supported Beam with Single Non-central Point Load

The usual first step in all beam problems is to calculate the support reactions.

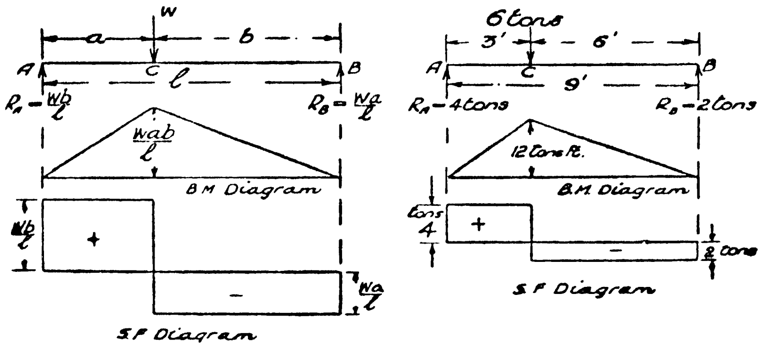


FIG. 270.—SIMPLE BEAM WITH NON-CENTRAL POINT LOAD.

Left-end Reaction.—Moments about 'B':

$$R_A \times l = W \times b. \quad \therefore R_A = \frac{Wb}{l}.$$

Right-end Reaction.—Moments about 'A':

$$R_B \times l = W \times a. \quad \therefore R_B = \frac{Wa}{l}.$$

Consider portion 'AC' of the beam.

At 1 ft. from end A, B.M. $= \frac{Wb}{l} \times 1 = \frac{Wb}{l}$.

" 2 " " " " B.M. $= \frac{Wb}{l} \times 2 = \frac{2Wb}{l}$.

" 3 " " " " B.M. $= \frac{Wb}{l} \times 3 = \frac{3Wb}{l}$.

The B.M. value increases in *direct proportion* with the distance from end 'A' and at the point 'C' the value is $\frac{Wb}{l} \times a = \frac{Wab}{l}$.

Similarly, regarded from end 'B,' the B.M. at 'C' $= \frac{Wa}{l} \times b = \frac{Wab}{l}$. The B.M. diagram will be triangular in form.

Between points 'A' and 'C' the S.F. $= R_A - 0 = R_A = \frac{Wb}{l}$.

Between 'C' and 'B' the S.F. $= R_A - W$

$$= \frac{Wb}{l} - W = \frac{Wb}{a+b} - W = \frac{Wb - Wa - Wb}{a+b} = -\frac{Wa}{l} = -R_B.$$

The S.F. diagram will therefore be as indicated in Fig. 270.

The B.M. and S.F. diagrams are drawn to agree with the convention of signs laid down.

EXAMPLE.—*A simply supported beam of 9-ft. effective span carries a concentrated load of 6 tons at 3 ft. from the left support. Construct the B.M. and S.F. diagrams for the beam.* (*See Fig.* 270.)

Note.—The '*effective span*' of a beam is the span '*centre to centre*' of the bearings. This span is used in diagram construction and for calculation of beam loading, etc.

$$R_A \times 9 = 6 \times 6 = 36$$
$$R_A = 36/9 = 4 \text{ tons.}$$
$$R_B \times 9 = 6 \times 3 = 18$$
$$R_B = 18/9 = 2 \text{ tons.}$$
$$\text{B.M.}_C = R_A \times 3 \text{ or } R_B \times 6$$
$$= (4 \times 3) \text{ or } (2 \times 6) \text{ tons ft.}$$
$$= 12 \text{ tons ft.}$$

By formula: $\text{B.M.}_C = \dfrac{Wab}{l} = \dfrac{6 \times 3 \times 6}{9} = 12$ tons ft.

The B.M. diagram is a triangle with a maximum height of 12 tons ft. (at point 'C'). To construct the S.F. diagram, erect at 'A' an ordinate to represent 'R_A,' i.e. '4 tons.' The completion of the diagram is shown in Fig. 270. General rules for the construction of shear force diagrams are given on page 221.

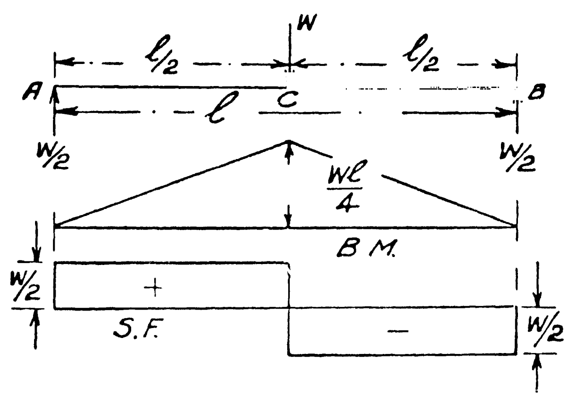


FIG. 271.—SIMPLE BEAM WITH CENTRAL POINT LOAD.

Simply Supported Beam with Single Central-point Load

The max. B.M. for this case may be derived from the previous case by putting $a = b = \frac{l}{2}$. (Fig. 271.)

$$\text{B.M.}_C = \frac{Wab}{l} = \frac{W \times \frac{l}{2} \times \frac{l}{2}}{l} = \frac{Wl}{4}.$$

Alternatively, by direct method, B.M._C = Reaction moment $= \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$.

For the shear force diagram we have : $R_A = R_B = \frac{W}{2}$.

Simply Supported Beam with Several Point (or Concentrated) Loads

The nature of the complete B.M. and S.F. diagrams in this

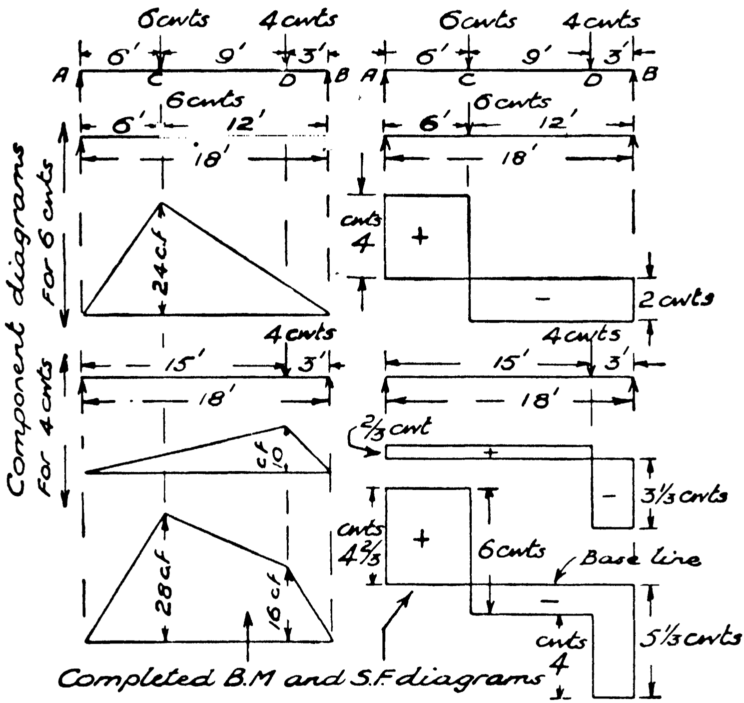


FIG. 272.—COMPOSITION OF B.M. AND S.F. DIAGRAMS.

case may be demonstrated by using the principle of the ***algebraic addition of component diagrams.***

Although this method of 'diagram addition' is not normally employed, it is a very useful method in dealing with beams with complicated load systems or systems of an unusual character. In this particular case it will be noted that the final B.M. diagram is bounded by straight lines which change their slopes at the load points. If we calculate, and plot to scale, the net B.M. values at the respective load points, the B.M. diagram may be completed by merely joining up the tops of the ordinates.

The S.F. diagram is a stepped diagram, the vertical drops taking place at the load points and representing the corresponding loads to scale. The reader should check the values given on the component diagrams.

EXAMPLE.—*Fig.* 273 *shows a simply supported beam carrying a concentrated load system. Calculate the B.M. and S.F. values for a section* 6 *ft. from the left end. Check the values obtained by constructing the B.M. and S.F. diagrams for the beam.*

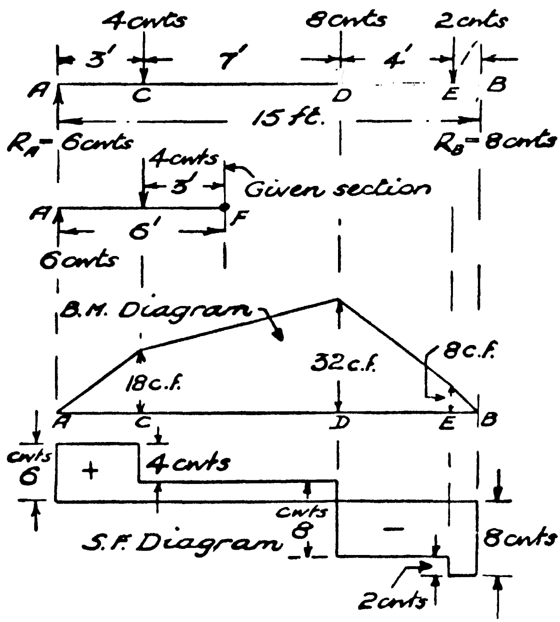


FIG. 273.—SIMPLE BEAM WITH SEVERAL POINT LOADS.

The B.M. and S.F. diagrams for this example will be developed in the straightforward way usually adopted.

$$R_A \times 15 = (4 \times 12) + (8 \times 5) + (2 \times 1)$$
$$= 48 + 40 + 2 = 90$$
$$R_A = \frac{90}{15} \text{ cwts.} = 6 \text{ cwts.}$$
$$R_B \times 15 = (2 \times 14) + (8 \times 10) + (4 \times 3)$$
$$= 28 + 80 + 12 = 120$$
$$R_B = \frac{120}{15} \text{ cwts.} = 8 \text{ cwts.}$$
$$\text{B.M.}_C = \text{Reaction moment} - \text{Load moment}$$
$$= [(6 \times 3) - 0] = 18 \text{ c.f.}$$
$$\text{B.M.}_D = \text{Reaction moment} - \text{Load moment}$$
$$= [(6 \times 10) - (4 \times 7)] \text{ c.f.}$$
$$= (60 - 28) \text{ c.f.} = 32 \text{ c.f.}$$

Alternatively, taking forces to right of section 'D,'

$$\text{B.M.}_D = \text{Reaction moment} - \text{Load moment}$$
$$= [(8 \times 5) - (2 \times 4)] \text{ c.f.}$$
$$= (40 - 8) \text{ c.f.} = 32 \text{ c.f.}$$
$$\text{B.M.}_E = \text{Right-end reaction moment}$$
$$= (8 \times 1) \text{ c.f.} = 8 \text{ c.f.}$$

Construction of B.M. Diagram.—At 'C' an ordinate is erected to scale 18 c.f., at 'D' to scale 32 c.f. and at 'E' to scale 8 c.f. The completed diagram is given in Fig. 273.

To obtain the S.F. diagram an ordinate must be erected at 'A' to represent 6 cwts. At 'C' the diagram must drop vertically a distance representing '4 cwts.' and so on. The diagram is checked by the fact that it should give '$-R_B$' as the S.F. value at the right end.

Values at the special section at 'F'!

The forces are taken to the left or the right of section as most convenient.

Taking forces to the left :

$$\text{B.M.}_F = \text{Reaction moment} - \text{Load moment}$$
$$= [(6 \times 6) - (4 \times 3)] \text{ c.f.}$$
$$= (36 - 12) \text{ c.f.} = 24 \text{ c.f.}$$

[The reader should check this value by taking the forces to the right of 'F.']

$$\text{S.F.}_F = \text{Left-end reaction} - \text{Loads up to section}$$
$$= (6 - 4) \text{ cwts.} = 2 \text{ cwts. (positive).}$$

Suggested scales for diagrams :

$\frac{3}{8}'' = 1$ ft., $1'' = 8$ c.f. (B.M.), $1'' = 4$ cwts. (S.F.).

Simply Supported Beam with Uniformly Distributed Load

A convenient method of dealing with this case is to investigate the value of the B.M. (or S.F.) at a typical section of the beam. The value is expressed in terms of 'x,' the distance of the section from 'A.' The form of the expression obtained gives the clue to the nature of the B.M. (or S.F.) diagram.

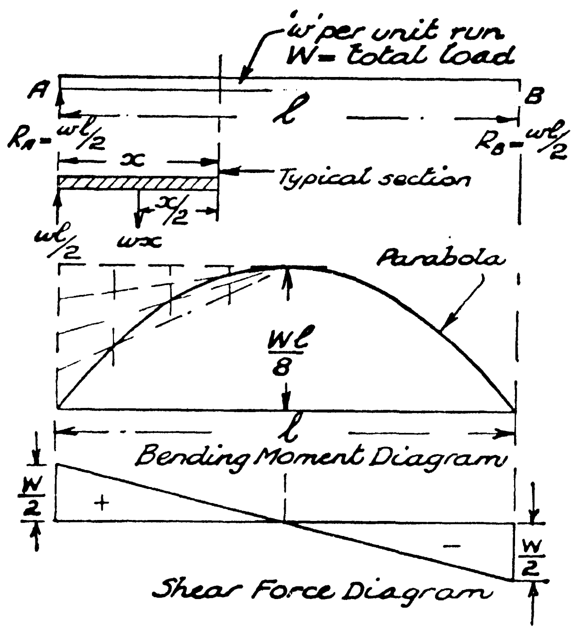


FIG. 274.—SIMPLE BEAM WITH UNIFORM LOAD.

$$\text{B.M.}_x = \textit{Reaction moment} - \textit{Load moment}$$
$$= \left(\frac{wl}{2} \times x\right) - \left(wx \times \frac{x}{2}\right) = \frac{wlx}{2} - \tfrac{1}{2}wx^2.$$

If we plotted this expression for values of 'x' from '0' (i.e. at 'A') to 'l' (i.e. at 'B'), we would obtain a graph in the form of a *parabola*. The maximum ordinate of the parabola will occur for $x = l/2$, i.e. at mid-span.

$$\therefore \text{B.M.}_{\text{max.}} = \left(\frac{wl}{2} \times \frac{l}{2}\right) - \tfrac{1}{2}w \times \left(\frac{l}{2}\right)^2 = \frac{wl^2}{4} - \frac{wl^2}{8} = \frac{wl^2}{8}.$$

Replacing 'wl' by 'W,' the total load on the beam,

$$\text{B.M.}_{\text{max.}} = \frac{\text{Wl}}{8}.$$

The shear force at the typical section $=$ *Left-end reaction* $-$ *Load up to section* $= \dfrac{wl}{2} - wx.$

The expression indicates that the shear force graph will be of a linear character.

$$\text{When } x = 0 \text{ (i.e. at A), S.F.} = \frac{wl}{2} - 0 = \frac{wl}{2} = \frac{\text{W}}{2}.$$

$$\text{When } x = l \text{ (i.e. at B), S.F.} = \frac{wl}{2} - wl = -\frac{wl}{2} = -\frac{\text{W}}{2}.$$

In all simple beams of the type now being considered, the shear force at the left end of the beam is the 'left-end reaction,' and at the right end of the beam it is the 'right-end reaction.'

EXAMPLE.—*A simply supported beam of* 20-*ft. effective span carries a uniformly distributed load of total value* 10 *tons. Construct the B.M. and S.F. diagrams for the beam. Calculate the values of B.M. and S.F. respectively for a section* 8 *ft. from the left support.*

$$\text{B.M.}_{\text{max.}} = \frac{\text{W}l}{8} = \frac{10 \times 20}{8} \text{ tons ft.} = 25 \text{ tons ft.}$$

$$\text{S.F.}_{\text{max.}} = \pm \frac{\text{W}}{2} = 5 \text{ tons at 'A' and } -5 \text{ tons at 'B.'}$$

B.M. at 8 *ft. from left support:*

$$\begin{aligned} \text{B.M.} &= \text{Reaction moment} - \text{Load moment} \\ &= (5 \times 8) - (4 \times 4) \\ &= (40 - 16) \text{ tons ft.} = 24 \text{ tons ft.} \end{aligned}$$

S.F. at 8 *ft. from left support:*

$$\begin{aligned} \text{S.F.} &= \text{Left-end reaction} - \text{Load up to section} \\ &= 5 \text{ tons} - 4 \text{ tons} = 1 \text{ ton (positive).} \end{aligned}$$

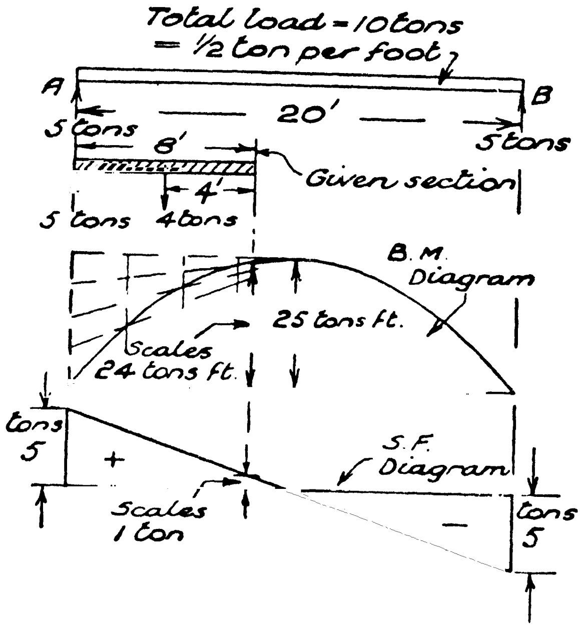


FIG. 275.—UNIFORMLY DISTRIBUTED LOAD.

Suggested scales for diagrams :

$1'' = 4$ ft., $1'' = 8$ tons ft. (B.M.), $1'' = 2$ tons (S.F.).

Example on Timber and Steel Floor

The diagram (Fig. 276) shows a floor in which timber beams are supported by steel joists. Assuming the total floor load to be 120 lb. per sq. foot (inclusive of the self-weight of the floor), draw the B.M. and S.F. diagrams for one of the timber beams. Calculate the maximum bending moment in each of the steel beams 'AB' and 'CD' respectively.

Timber Beams.—Area of floor supported by one beam

$$= 10 \text{ ft.} \times \frac{15}{12} \text{ ft.} = \frac{150}{12} \text{ sq. ft.} = 12{\cdot}5 \text{ sq. ft.}$$

U.D. load carried by one beam $= (12{\cdot}5 \times 120)$ lb. $= 1500$ lb.

$$\text{Max. B.M.} = \frac{Wl}{8} = \frac{1500 \times 10}{8} \text{ lb. ft.} = 1875 \text{ lb. ft.}$$

$$\text{Max. S.F.} = \pm \frac{W}{2} = \pm 750 \text{ lb.}$$

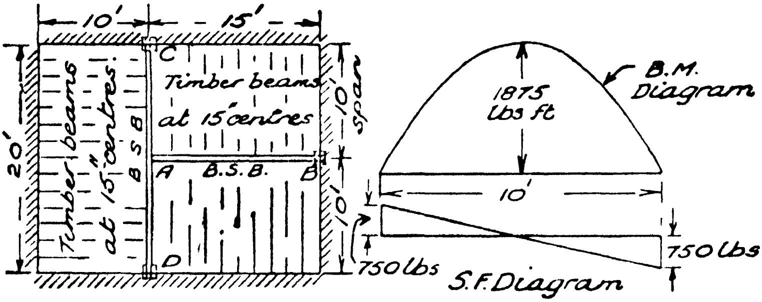


FIG 276.—TIMBER AND STEEL FLOOR.

Steel Beam 'A B.'—This beam carries the reaction loads from the timber beams in the spans on either side. It is usual to assume that such a load system may be taken as uniformly distributed for the supporting beam. Beam 'AB' will have to carry a 'uniform load' equal to that carried on a floor area $(5 + 5)$ ft. $\times$ 15 ft. = 150 sq. ft.

Total load = (150×120) lb. = 18000 lb.

Max. B.M. in beam 'AB' $= \frac{Wl}{8} = \frac{18000 \times 15}{8}$ lb. ft.

$= 33750$ lb. ft.

Steel Beam 'CD.'—Beam 'AB' is supported at end 'A' by beam 'CD.' Beam 'CD' therefore carries a reaction load, at its centre, equal to half the load carried by beam 'AB.'

$\therefore$ Load carried by beam 'CD' at its centre

$$= \frac{18000}{2} \text{ lb.} = 9000 \text{ lb.}$$

B.M. at centre of beam 'CD' due to this load alone

$$= \frac{Wl}{4} = \frac{9000 \times 20}{4} \text{ lb. ft.} = 45000 \text{ lb. ft.}$$

Beam 'CD' carries in addition the reaction loads from the timber beams in the left-end bay. As in the case of beam 'AB,' it is usual to consider such load as being uniformly distributed along 'CD.'

$\therefore$ Total 'U.D.' load = $(5 \times 20 \times 120)$ lb. = 12000 lb., i.e. half the total load on the left-end bay.

$$\text{B.M.}_{\text{max.}} \text{ in 'CD' due to this load} = \frac{Wl}{8} = \frac{12000 \times 20}{8} \text{ lb. ft.}$$
$$= 30{,}000 \text{ lb. ft.}$$

As both the maximum bending moments calculated *occur at the same beam section,* i.e. at the centre of span, we may add them together.

Total max. B.M. in beam CD = (45000 + 30000) lb. ft. = 75000 lb. ft.

EXERCISES 10

(Self-weight of beams may be neglected.)

(1) A cantilever projects 3 ft. horizontally from its support. A rope tackle fixed at the free end is capable of exerting a total vertical pull of 2 cwts. Calculate the values of B.M.$_{\text{max.}}$ and S.F.$_{\text{max.}}$ respectively. Construct the B.M. and S.F. diagrams for the cantilever.

(2) Fig. 277 shows a cantilever supporting a number of point loads. Obtain the following values: (i) maximum bending

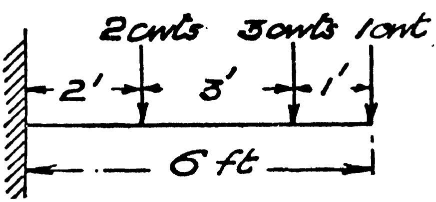


FIG. 277.

moment, (ii) maximum shear force, (iii) bending moment at mid-point of cantilever, (iv) shear force at 1 ft. from the support. Construct the bending moment and shear force diagrams for the cantilever.

(3) A solid concrete balcony floor is supported by embedded steel cantilevers at 2-ft. centres, the cantilevers and floor projecting 4′ 6″ from a wall (Fig. 278). Calculate the total safe load per sq. foot of balcony floor (inclusive of self-weight of floor) if the maximum bending moment in one cantilever is not to exceed 4050 lb. ft. Draw the B.M. and S.F. diagrams for one of the cantilevers assuming the given maximum bending moment.

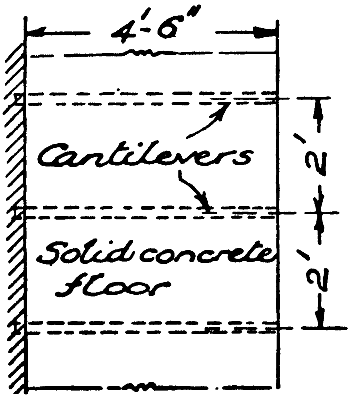


FIG. 278

(4) A simply supported beam of 8-ft. effective span carries a central-point load of 200 lb. Calculate the value of the maximum bending moment (i) by formula, (ii) by first principles (i.e. by first finding the support reactions).

Sketch the B.M. and S.F. diagrams for the beam. [All important values should be indicated on a 'sketch' diagram.]

(5) A given steel beam is able to resist safely a maximum bending moment of 45 tons ft. The beam is used to carry the loads shown in Fig. 279.

(i) Check the safety of the beam from the bending moment point of view.

(ii) Draw the B.M. and S.F. diagrams for the beam.

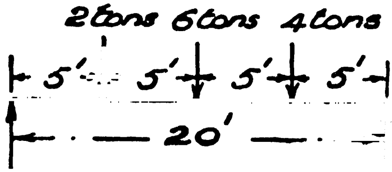


FIG. 279.

(6) Construct the B.M. and S.F. diagrams for the simply supported beam shown in Fig. 280. Calculate the B.M. and S.F. values respectively for a section 9 ft. from the right end. Verify your values by means of the diagrams you have constructed.

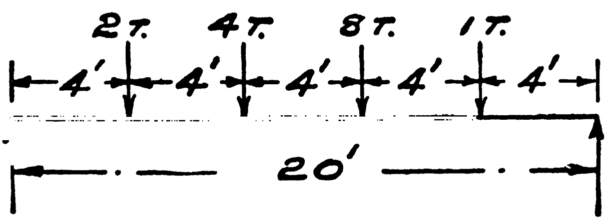


FIG. 280.

(7) A simple beam of 12-ft. effective span carries a uniform load of total value 24 cwts. Find the maximum bending moment and the maximum shear force. Draw the B.M. and S.F. diagrams

for the beam. Calculate the B.M. and S.F. values for a section 4 ft. from the left end.

(8) A simple beam of 10-ft. effective span carries a uniform load which produces a bending moment of 24 cwts. ft. at a section 4 ft. from the left support. Calculate the value of the load in cwts. per foot run. Also obtain the maximum bending moment in the beam.

(9) Fig. 281 shows a floor with timber beams of 8′ effective span supported by a steel beam. Find the maximum bending moment and maximum shear force respectively for one of the timber beams and also for the steel beam. Construct (i) the B.M. diagram for a timber beam and (ii) the S.F. diagram for the steel beam. The total inclusive floor load may be taken as 100 lb. per sq. foot.

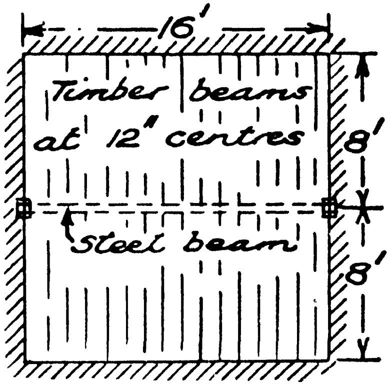


FIG. 281.

(10) The resultant upward pressure on the footing in the example given in Fig. 282 is 2 tons per sq. foot. Calculate the maximum bending moment the footing will have to resist per foot run of wall, assuming the maximum bending moment to occur at the face of the wall.

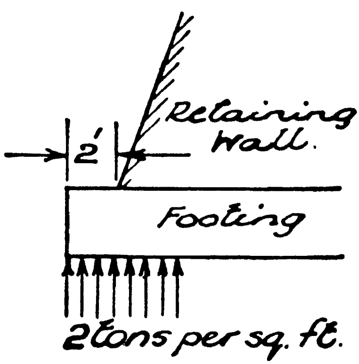


FIG. 282.

(11) A simply supported beam of 6-ft. effective span carries a point load of 4 tons at the centre. Calculate the load which would produce the same maximum bending moment if (i) uniformly distributed, (ii) concentrated at 2 ft. from the left-end support.

(12) A lintel beam of 10-ft. effective span carries a brick wall, 9 ins. thick and of 5 ft. uniform height. Assuming the average

density of the brickwork to be 120 lb. per cu. foot, obtain the B.M. and S.F. values for a section 4 ft. from the left end of beam. Construct the B.M. and S.F. diagrams for the beam.

(13) A steel chimney stack of circular section, 3-ft. external diameter, is 60 ft. high. It is subjected to a horizontal wind pressure of uniform intensity, 30 lb. per sq. foot. Assuming the area acted upon by the wind to be ·6 of the area 'height × diameter' (a common practical allowance for circular chimneys), calculate the B.M. and S.F. values for a section at mid-height of the chimney and also for the base.

(14) A crane girder carries two wheel loads of 2 tons each at a fixed distance of 4 ft. apart. The effective span of the girder is 16 ft. Calculate the two distances the left-hand wheel load may be from the left support so that the bending moment in the girder at the position of the other wheel is 12 tons ft.

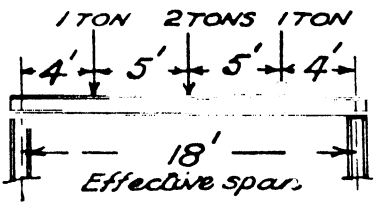


FIG. 283.

(15) Calculate the total uniformly distributed load the steel beam shown in Fig. 283 could carry, in addition to the point loads shown, if the beam were capable of resisting safely a bending moment of 22 tons ft.

Find the 'equivalent uniform load' which would produce (i) the same maximum bending moment, (ii) the same maximum shear force as the *point load* system shown in Fig. 283.

CHAPTER XI

BENDING MOMENT AND SHEAR FORCE. FURTHER EXAMPLES

THE examples of the last chapter have illustrated certain facts which will now be summarised.

(i) A concentrated load system will lead to a B.M. diagram bounded by straight lines, which change their slope at the load points.

(ii) A distributed load will have some form of curve for the B.M. diagram. If the load be uniformly distributed a parabola will be involved in the construction of the diagram.

(iii) For a concentrated load system the S.F. diagram will be stepped. A step will take place at every load point, the vertical drop representing the load to scale.

(iv) The S.F. diagram for a uniformly distributed load will be of a uniformly sloping character. For our convention of signs the slope will be downwards towards the right.

The foregoing rules will be exemplified in the following cases.

Overhanging Beam with Concentrated Load System

EXAMPLE.—*Construct the B.M. and S.F. diagrams for the overhanging beam given in Fig.* 284.

Moments about 'B':

$$(R_A \times 10) + (2 \times 5) = (8 \times 2) + (6 \times 6) + (2 \times 14)$$
$$= 16 + 36 + 28 = 80$$
$$10R_A + 10 = 80$$
$$10R_A = 80 - 10 = 70$$
$$\therefore R_A = 7 \text{ cwts.}$$

Moments about 'A':

$$(R_B \times 10) + (2 \times 4) = (6 \times 4) + (8 \times 8) + (2 \times 15)$$
$$10R_B + 8 = 24 + 64 + 30 = 118$$
$$10R_B = 118 - 8 = 110$$
$$R_B = 11 \text{ cwts.}$$

$$\textbf{B.M.}_O = 0 \text{ (free end of beam).}$$

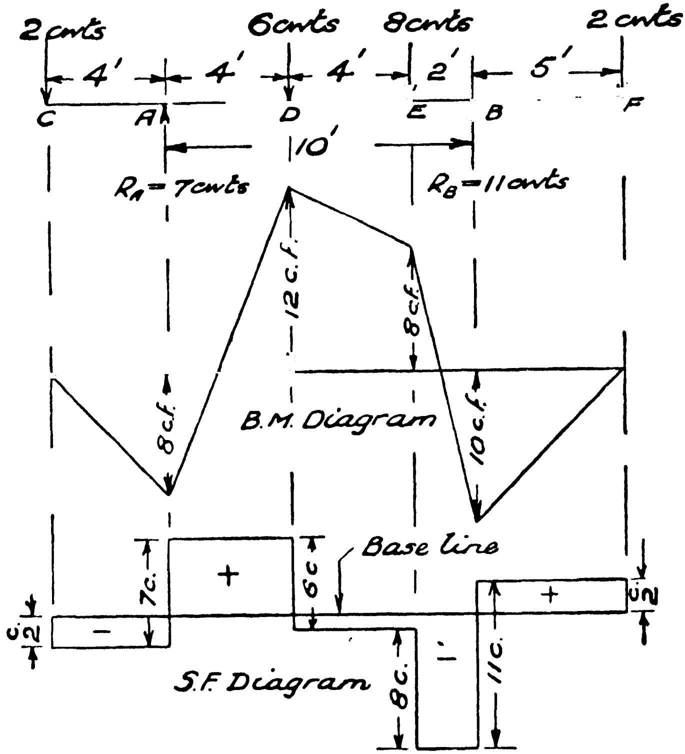


FIG. 284.—OVERHANGING BEAM WITH CONCENTRATED LOADS.

B.M._{A} = Moment of load to left of section
= (2×4) cwts. ft. = 8 cwts. ft. (*negative*).

B.M._{D} = Reaction moment − Load moment
= $(7 \times 4) - (2 \times 8)$ cwts. ft.
= $(28 - 16)$ cwts. ft. = 12 cwts. ft.

B.M._{E} = Reaction moment − Load moment
= $(11 \times 2) - (2 \times 7)$ cwts. ft. (*taking forces to right of section*)
= $(22 - 14)$ cwts. ft. = 8 cwts. ft.

B.M._{B} = (2×5) cwts. ft. = 10 cwts. ft. (*negative*).

The shear force diagram is constructed without any further detailed calculation.

Suggested scales :

$\frac{1}{4}'' = 1$ ft., $1'' = 4$ cwts. ft. (B.M.), $1'' = 4$ cwts. (S.F.).

Overhanging Beam with Uniformly Distributed Load

EXAMPLE.—*The beam given in Fig. 285 carries a uniform load of 3 cwts. per foot run. Construct the B.M. and S.F. diagrams for the beam.*

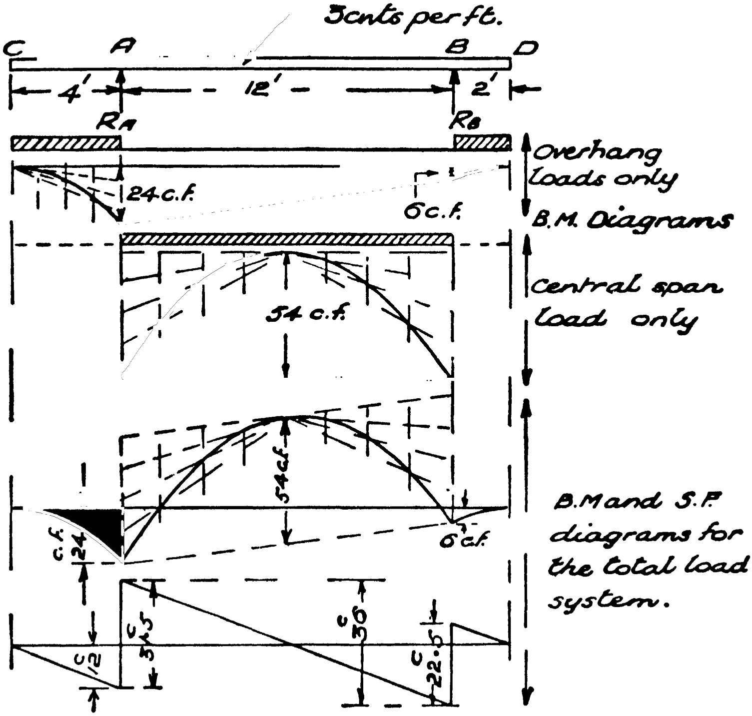


FIG. 285.—OVERHANGING BEAM WITH UNIFORM LOAD.

In order to illustrate the method of addition of component B.M. diagrams, the beam load will be divided up into two systems: (i) the loads on 'AC' and 'BD,' (ii) the load on 'AB.'

Taking component system (i) *by itself*:

$\text{B.M.}_A = [(3 \times 4) \times 2]$ c.f. $= 24$ c.f. (*negative*).

$\text{B.M.}_B = [(3 \times 2) \times 1]$ c.f. $= 6$ c.f. (*negative*).

The B.M. diagram for this component system is shown in Fig. 285 (upper diagram).

Considering component system (ii), i.e. the load on 'AB,'

$$\text{B.M.}_{\text{max.}} = \frac{Wl}{8} = \frac{3 \times 12 \times 12}{8} = 54 \text{ c.f. } (\textit{positive}).$$

We now have to combine these two B.M. diagrams together. The net B.M. diagram for the complete load system is shown in Fig. 285, which also shows the practical method of obtaining the final B.M. diagram in a more direct manner. In using this construction, care should be taken to plot the value '*54 c.f.*' (i.e. '$Wl/8$' for the central span) *vertically* from the mid-point of the temporary sloping base line shown in the diagram.

Shear Force Diagram.—We require, first of all, to calculate the support reactions.

$$(R_A \times 12) + (6 \times 1) = 3 \times 16 \times \tfrac{16}{2}$$
$$12R_A + 6 = 384$$
$$12R_A = 384 - 6 = 378$$
$$R_A = 31{\cdot}5 \text{ cwts.}$$
$$(R_B \times 12) + (12 \times 2) = 3 \times 14 \times \tfrac{14}{2}$$
$$12R_B + 24 = 294$$
$$12R_B = 294 - 24 = 270$$
$$R_B = 22{\cdot}5 \text{ cwts.}$$

From 'C' to 'A' the load on the beam = 12 cwts., hence the S.F. diagram must drop by this amount over the length 'CA.' At 'A' there is a vertical jump upwards of '31·5 cwts.' (i.e. 'R_A'). Between 'A' and 'B' the diagram must drop $3 \times 12 = 36$ cwts. At 'B' there is a vertical jump upwards of 22·5 cwts., then a final drop of '6 cwts.' over the length 'BD.'

Note.—If various portions of a beam, as in the last example, carry uniform loads of similar intensity, the *slope* of the S.F. diagram will be the same in each of these different portions.

Suggested scales :

$\frac{3}{8}'' = 1$ ft., $1'' = 12$ c.f. (B.M.), $1'' = 8$ cwts. (S.F.).

Beams with Uniformly Distributed Load Partially Covering the Span

EXAMPLE (1).—'*AB*' (*Fig.* 286) *is a simply supported beam carrying a uniformly distributed load of* 6 *cwts. per foot, which partially covers the span. Construct the B.M. and S.F. diagrams for the beam.*

The principle of the construction of the B.M. diagram in this case is to regard, *temporarily*, the U.D. load 'CD' as being a concentrated point load of value '24 cwts.,' acting at 'E,' the

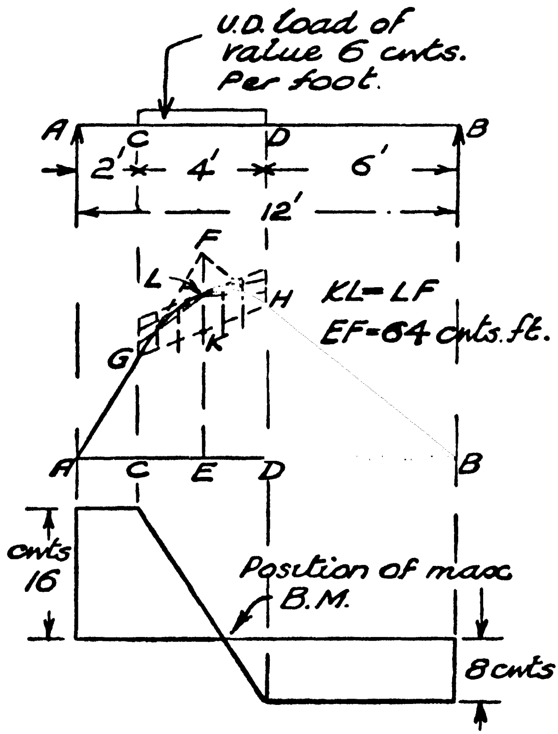


FIG. 286.—UNIFORM LOAD PARTIALLY COVERING SPAN.

mid-point of 'CD.' This is for construction purposes only, and allowance is subsequently made for the fact that the load is actually distributed.

$$\text{B.M.}_E = \frac{Wab}{l} = \frac{24 \times 4 \times 8}{12} = 64 \text{ c.f.}$$

'EF' is drawn to represent '64 c.f.' and the triangular B.M. diagram 'AFB' is completed.

The ordinates 'CG' and 'DH' are drawn and 'GH' is drawn in. 'GH' intersects 'EF' in point 'K.' 'KF' is bisected in point 'L' and the parabolic diagram is constructed as indicated in Fig. 286. The final B.M. diagram is 'AGLHB.'

The bisecting of 'KF' makes the necessary allowance referred to above.

The S.F. diagram presents no special difficulty.

$$R_A \times 12 = (24 \times 8) = 192$$
$$R_A = 16 \text{ cwts.}$$
$$R_B \times 12 = (24 \times 4) = 96$$
$$R_B = 8 \text{ cwts.}$$

Suggested scales :

1" = 2 ft., 1" = 16 c.f. (B.M.), 1" = 8 cwts. (S.F.).

EXAMPLE (ii).—*The given simply supported beam (Fig. 287) supports a partial uniformly distributed load together with a con-*

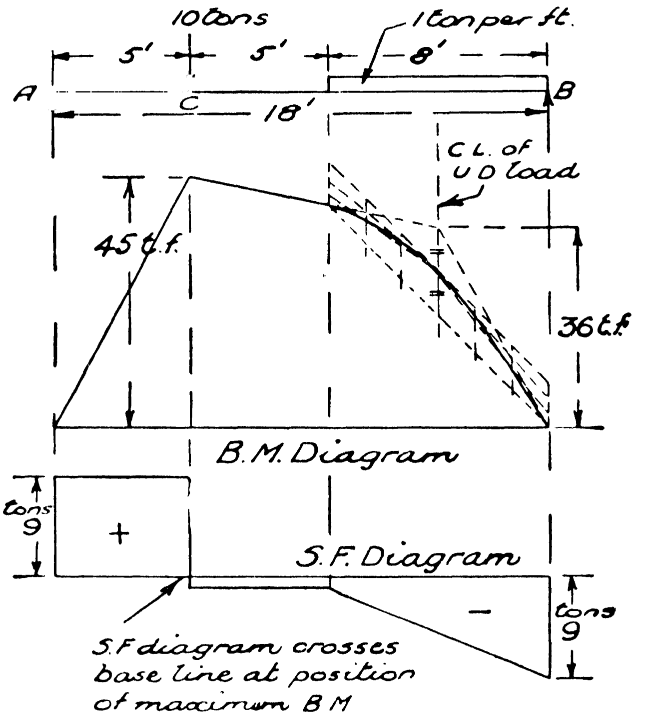


FIG. 287.—COMBINED LOADS.

centrated load. Construct the B.M. and S.F. diagrams for the beam.

As in the last example, the U.D. load is replaced temporarily, *for construction purposes,* by a point load of equal value at its

centre. The necessary final adjustment is then made as indicated in Fig. 287.

$$R_A \times 18 = (10 \times 13) + (8 \times 4) = 162$$
$$R_A = 9 \text{ tons.}$$
$$R_B \times 18 = (10 \times 5) + (8 \times 14) = 162$$
$$R_B = 9 \text{ tons.}$$
$$\text{B.M.}_C = R_A \times 5 = 9 \times 5 = 45 \text{ tons ft.}$$

B.M. at centre of U.D. load, *regarding load as concentrated*, $= R_B \times 4 = 9 \times 4 = 36$ tons ft. [Care must be exercised not to compute the *correct* B.M. at the centre of the uniform load in this method.]

The 'straight-line' B.M. diagram is completed and the portion corresponding to the uniform load modified as indicated. The S.F. diagram is easily constructed by the rules previously given.

EXAMPLE (iii).—*The cantilever shown in Fig. 288 carries a partial uniformly distributed load and, in addition, one concentrated load. Draw the B.M. and S.F. diagrams for the cantilever.*

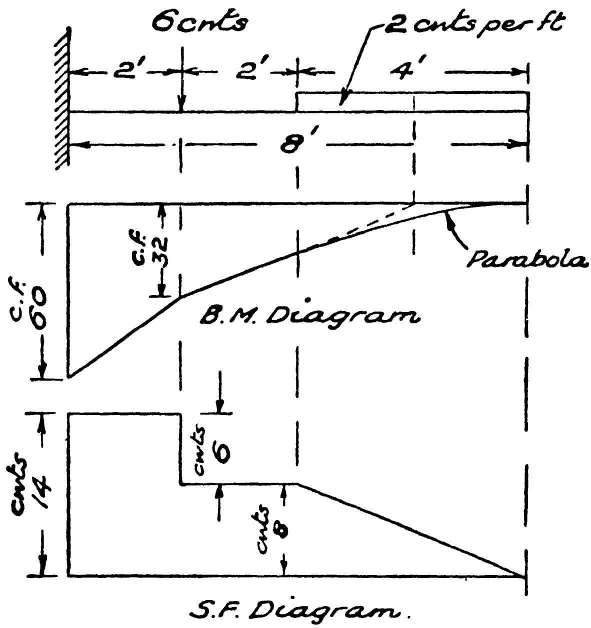


FIG. 288.—CANTILEVER WITH PARTIAL U.D. LOAD.

The treatment is as for the previous examples and the detailed calculations are left as an exercise.

Beams Carrying Complicated Load Systems

Bending Moment Diagrams

If the complete load system may be divided up into two simple systems, i.e. two systems which can be readily dealt with, the method 1 shown in Fig. 289 is sometimes adopted. The diagram

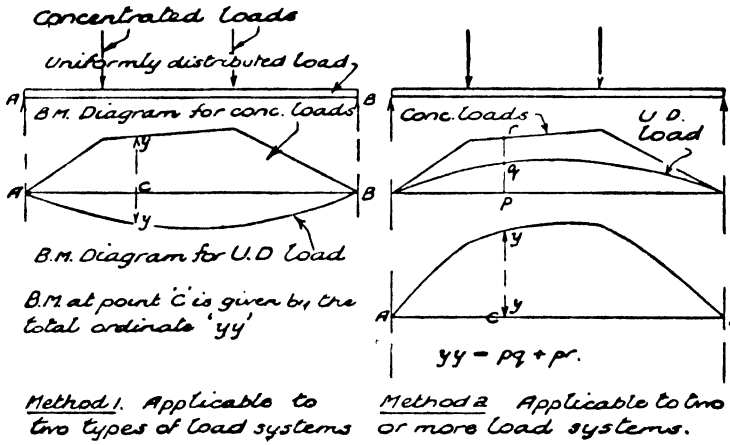


FIG. 289.—SEVERAL LOAD SYSTEMS.

for one system is drawn above the base line and for the other below (using the same B.M. scale). The total vertical depth of the diagram will then give the necessary bending moment at any particular beam section.

In cases in which there are several systems to be dealt with, the B.M. diagram for each system may be constructed on a common base (or if preferred on separate base lines) and the diagrams added together by the geometrical method of *adding corresponding ordinates* (see Fig. 289). It is essential, of course, that the B.M. scale used for each of the component diagrams shall be the same.

A further practical method for dealing with complicated load systems is to calculate the bending moment values at a suitable number of points in the beam and to draw ordinates to represent these values to a convenient B.M. scale. The reactions will have to be found and the expression '*reaction moment — load moments*'

may be used to compute the B.M. values for the various beam sections. It may be necessary to distinguish carefully between positive and negative bending moments, so that the ordinates may be drawn above or below the base line. The correct *sign* for the bending moment will be obtained if the expression given at the bottom of page 228 be employed.

Shear Force Diagrams

S.F. diagrams do not usually present much difficulty, and the rules already given cover the types of loading considered in this book.

Position of Maximum Bending Moment

If the point in the beam at which the maximum bending moment occurs, in any of the previously worked examples, be examined in conjunction with the corresponding shear force diagram, it will be found that it coincides with the point in which the shear force diagram crosses its base line. As the latter point may be readily obtained *without actually drawing the S.F. diagram*, it is clear that this relationship will give a simple method for determining the position in the beam at which B.M.$_{max.}$ occurs. The complete theory underlying this important connection between bending moment and shear force will be found in more advanced books on the theory of structures.

Rule : *The position of maximum bending moment in a beam may be found by determining the point at which the shear force is zero.*

(The case of two, or more, such points is considered later.)

The following steps illustrate the method of calculating a maximum bending moment :

(i) Evaluate the left-end support reaction.

(ii) Proceed across the span from the left end until the load taken up on the beam equals, *in total value*, the left-end reaction. This is clearly the point of zero shear and hence the position of maximum bending moment.

(iii) Calculate the B.M. at this point, by usual methods, to obtain the value of B.M.$_{max.}$

EXAMPLE (i).—*Calculate the maximum bending moment for the given simply supported beam (Fig.* 290).

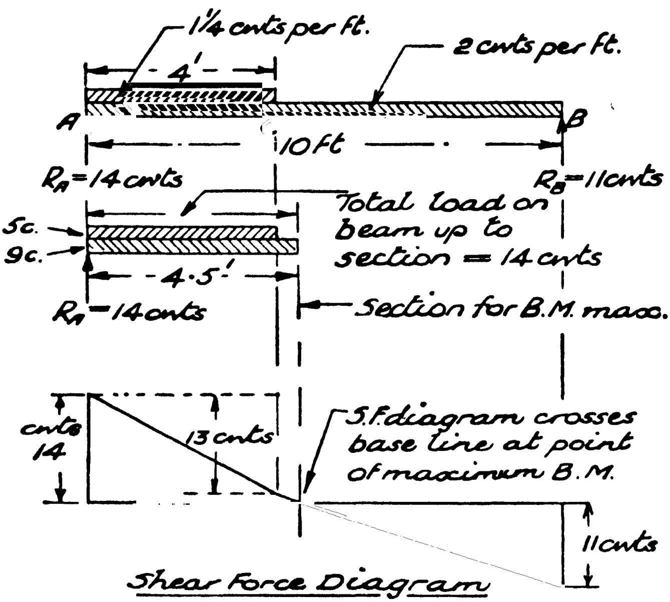


FIG. 290.—POSITION OF MAXIMUM BENDING MOMENT.

Step (i).

$$R_A \times 10 = [(4 \times 1\tfrac{1}{4}) \times 8] + [(2 \times 10) \times 5]$$
$$= 40 + 100 = 140$$
$$R_A = 14 \text{ cwts.}$$

[R_B should be calculated as a check.]

Step (ii). Up to point 'C,' from left end of beam, the load on beam $= [(1\tfrac{1}{4} \times 4) + (2 \times 4)]$ cwts.
$= (5 + 8)$ cwts. $= 13$ cwts.

We require another 1 cwt. to make up 14 cwts., which means we have to proceed ·5 ft. farther along the beam (as the load is *2 cwts. per foot*).

S.F. = 0 and B.M. = maximum at 4·5 ft. from left-end support.

$$\text{B.M.}_{4\cdot 5'} = \text{B.M.}_{\text{max.}} = \text{Reaction moment} - \text{Load moments}$$
$$= (14 \times 4\cdot 5) - (5 \times 2\cdot 5) - \left(9 \times \frac{4\cdot 5}{2}\right) \text{ cwts. ft.}$$
$$= (63 - 12\cdot 5 - 20\cdot 25) \text{ cwts. ft.}$$
$$= 30\cdot 25 \text{ cwts. ft.}$$

EXAMPLE (ii).—*Draw the shear force diagram for the given beam (Fig. 291). Find the position of zero shear* (i) *by means of the S.F. diagram,* (ii) *by direct calculation. Calculate the value of the maximum bending moment.*

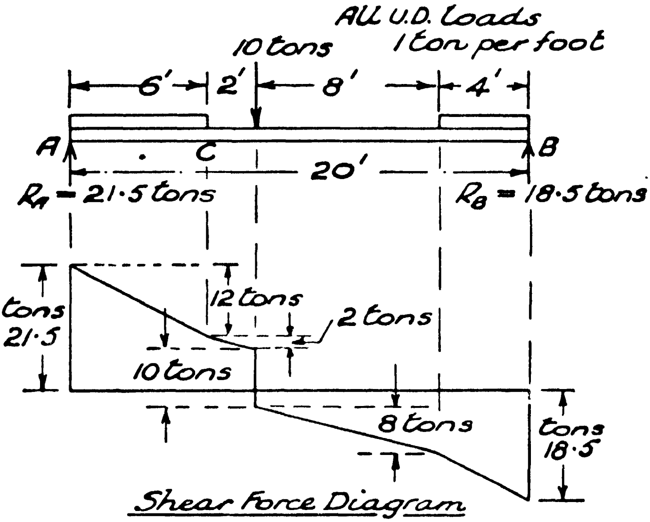


FIG. 291.—MAXIMUM B.M. EXAMPLE.

$$R_A \times 20 = [(1 \times 6) \times 17] + (10 \times 12) + [(1 \times 4) \times 2] + [(1 \times 20) \times 10]$$
$$= 102 + 120 + 8 + 200$$
$$= 430$$
$$R_A = 21{\cdot}5 \text{ tons.}$$
$$R_B \times 20 = [(1 \times 6) \times 3] + (10 \times 8) + [(1 \times 4) \times 18] + [(1 \times 20) \times 10]$$
$$= 18 + 80 + 72 + 200$$
$$= 370$$
$$R_B = 18{\cdot}5 \text{ tons.}$$

From left end up to point 'C' the load on beam = 12 tons. Up to the '10 tons' load (but not including it) the load = 12 tons + 2 tons = 14 tons. This is not sufficient. Including the 10 tons point load the total load = 24 tons. This is too great. When the inclusion of a concentrated point load makes the necessary total load exceed the required amount, *the position of B.M.$_{max.}$ is at the load point.*

$B.M._{max.}$ is therefore at the '10 tons' load point.

$$
\begin{aligned}
B.M._{max.} &= \text{Reaction moment} - \text{Load moments} \\
&= (21 \cdot 5 \times 8) - (6 \times 5) - (8 \times 4) \text{ tons ft.} \\
&= (172 - 30 - 32) \text{ tons ft.} \\
&= 110 \text{ tons ft.}
\end{aligned}
$$

The complete S.F. diagram is given in Fig. 291.

Suggested scales: 1″ = 4 ft., 1″ = 6 tons.

EXAMPLE (iii).—*Calculate the maximum bending moment which the given overhanging beam (Fig. 292) will have to resist. Construct the shear force diagram for the beam.*

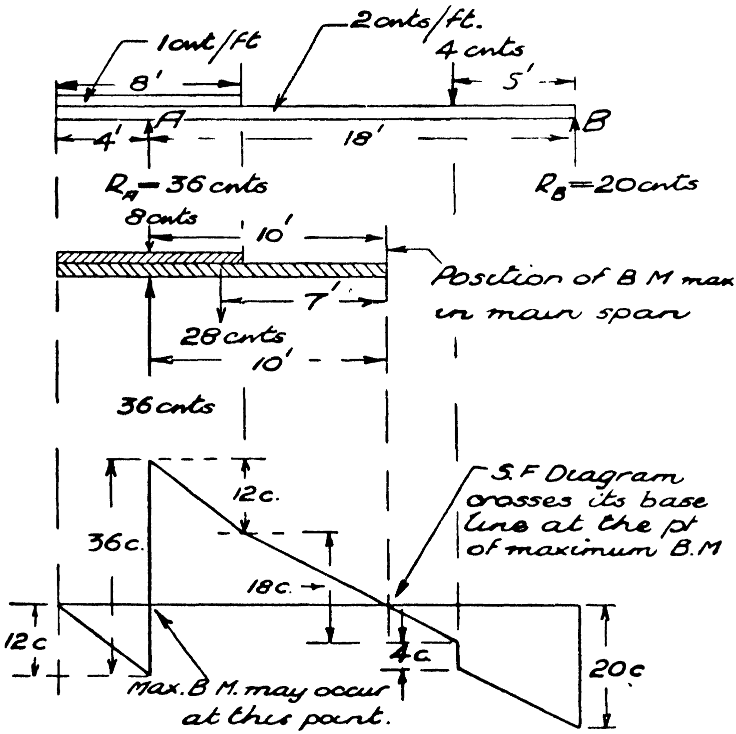


FIG. 292.—OVERHANGING BEAM.

$$
\begin{aligned}
R_A \times 18 &= [(2 \times 22) \times 11] + [(1 \times 8) \times 18] + (4 \times 5) \\
&= 484 + 144 + 20 = 648
\end{aligned}
$$

$$
R_A = \frac{648}{18} \text{ cwts.} = 36 \text{ cwts.}
$$

$$R_B \times 18 = [(2 \times 22) \times 7] + [(1 \times 8) \times 0] + (4 \times 13)$$
$$= 308 + 0 + 52 = 360$$
$$R_B = \frac{360}{18} \text{ cwts.} = 20 \text{ cwts.}$$

In the case of overhanging beams, the maximum bending moment may occur at either of the supports or in the main span of the beam. Wherever the S.F. diagram crosses the base line a '*local*' B.M.$_{max.}$ occurs.

$$\text{B.M.}_A = \text{Moment of load to left of section}$$
$$= (3 \text{ cwts. per foot} \times 4 \text{ ft.}) \times 2 \text{ ft.}$$
$$= 24 \text{ cwts. ft. } (\textit{negative}).$$

Position of B.M.$_{max.}$ in span 'AB':

The load on the beam from the *extreme left end* up to the required point must equal 36 cwts.

Up to the end of the partial U.D. load, the load on beam = (8 × 3) cwts. = 24 cwts. To obtain the additional (36 − 24) cwts. = 12 cwts., we have to proceed 6 ft. farther. B.M.$_{max.}$ therefore occurs at 10 ft. from the left support.

$$\text{B.M.}_{max.} = \text{Reaction moment} - \text{Load moments}$$
$$= (36 \times 10) - (8 \times 10) - [(2 \times 14) \times 7]$$
$$= 360 - 80 - 196$$
$$= 84 \text{ cwts. ft.}$$

This is the absolute max. bending moment in the beam.

Fig. 292 explains the various load jumps by means of which the shear force diagram is constructed.

Suggested scales : 1″ = 4 ft. and 1″ = 8 cwts.

Bending Moment Diagram by Link Polygon Method

If the link polygon be drawn for a beam with a concentrated load system and the closer line be inserted, as explained on page 124, the diagram so formed is, to some definite scale, the bending-moment diagram for the beam. This method of construction of B.M. diagrams is not so quick or direct as the methods previously described. It has, however, important applications in more advanced theory of structures, as, for example, in '*moving-load*' problems and in the graphical treatment of '*deflection of*

beams.' Fig. 293 shows how to set out the work when both **B.M.** and S.F. diagrams are required.

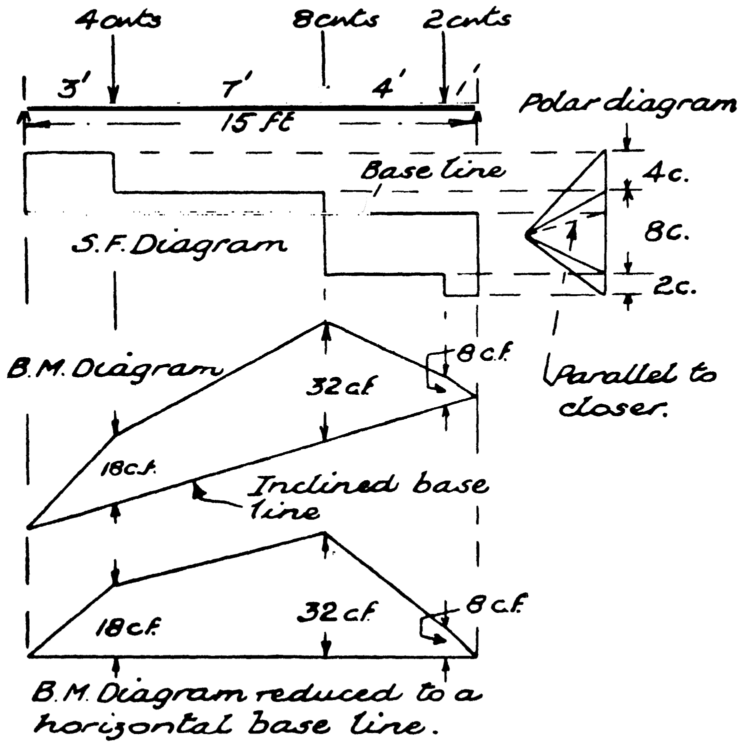


FIG. 293.—B.M. AND S.F. DIAGRAMS BY LINK POLYGON.

The pole is positioned, in the given example, to the left of the load line in order to maintain the convention of sign for bending moment which has been adopted. The position of the pole theoretically is immaterial, but it will be found that it will be advantageous to fix it at some definite *horizontal* distance from the 'load line,' say 3", in order to obtain a suitable final bending moment scale.

Having completed the polar diagram, the '*links*' and the *closer line*' are drawn in. If it is desired to obtain a horizontal base line for the B.M. diagram, it will be necessary simply to plot the *vertical* ordinates at the load points from a horizontal base as indicated in Fig. 293.

Bending Moment Diagram Scale

Usually, of course, we choose an appropriate B.M. scale according to the magnitude of the calculated B.M. values and the size of diagram that is required. In the present case we have to *calculate* the B.M. scale from the other scales used in the derivation of the diagram.

If $1'' =$ 'x' ft. for '*span*,'
$1'' =$ 'y' cwts. for '*load line*,' and
'p' ins. = actual polar distance, i.e. horizontal distance of pole from load line,
then $1''$ will represent $(x \times y \times p)$ cwts. ft. in B.M. diagram.

Thus, if $1'' = 4$ ft. for span,
$1'' = 2$ cwts. for load,
and polar distance $= 3$ ins., the B.M. scale would be

$$1'' = (4 \times 2 \times 3) \text{ cwts. ft.} = 24 \text{ cwts. ft.}$$

Note that the units for B.M. do not contain 'inches' in this case, but are compounded of the '*span*' and '*load*' units.

Shear Force Diagram

It will be clear that the vertical steps, as they represent the corresponding loads, will be given to scale in the polar diagram load line and may be projected across. It only remains to fix the position of the base line. The line drawn in the polar diagram parallel to the 'closer' divides the load line into two parts which respectively represent the support reactions, the upper part representing the left-end reaction. As the first vertical jump upwards of the S.F. diagram represents the left-end reaction, it is obvious that the base line will be obtained by projecting horizontally across in the manner indicated in Fig. 293. The shear force scale will be the same as that used for the *load line* in the polar diagram.

This example was previously solved by the direct calculation method (see Fig. 273).

EXERCISES 11

[Self-weight of beams may be neglected.]

(1) Fig. 294 shows a beam which overhangs its supports at both ends. Calculate, for the loads given, the values of the B.M.

and S.F. respectively at a section 6 ft. to the right of the left support. Draw the B.M. and S.F. diagrams for the beam.

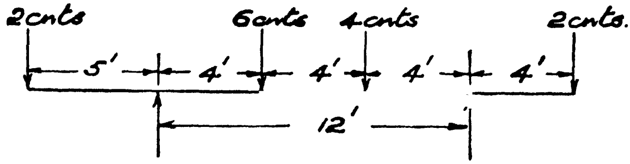


FIG. 294.

(2) Draw the B.M. and S.F. diagrams for the overhanging beam given in Fig. 295. Obtain the value of the maximum bending moment by scaling the B.M. diagram.

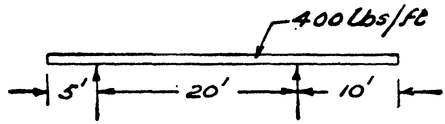


FIG. 295.

(3) Construct the B.M. and S.F. diagrams for the example given in Fig. 296.

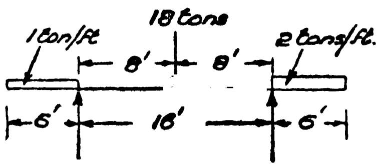


FIG. 296.

(4) A simply supported beam of 12-ft. effective span carries a uniform load which partially covers the span. The load is 4 ft. long, commences at 2 ft. from the left support and has the value of 2 cwts. per foot run. Construct the B.M. and S.F. diagrams for the beam due to this load.

(5) Find the position and magnitude of the maximum bending moment for the simply supported beam given in Fig. 297. Draw the S.F. diagram for the beam.

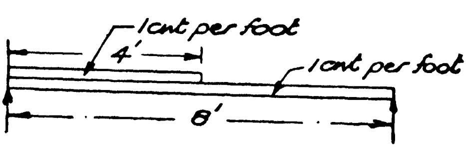


FIG. 297.

(6) Without calculating B.M. values or constructing the B.M. or S.F. diagrams for the beam given in Fig. 298, show that the maximum bending moment occurs at the position of the 8 cwts. load.

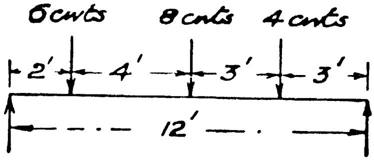


FIG. 298.

(7) Find the position of maximum bending moment for the simply supported beam shown in Fig. 299 (i) by beam load method, (ii) by constructing the S.F. diagram for the beam. Obtain the magnitude of the maximum bending moment.

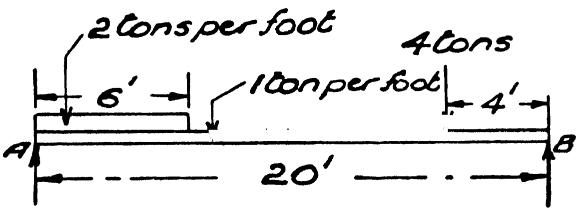


FIG. 299.

(8) Draw the S.F. diagram for the overhanging beam given in Fig. 300. Hence determine the position in the central bay at which *a* maximum bending moment occurs.

Find the value of the absolute maximum bending moment.

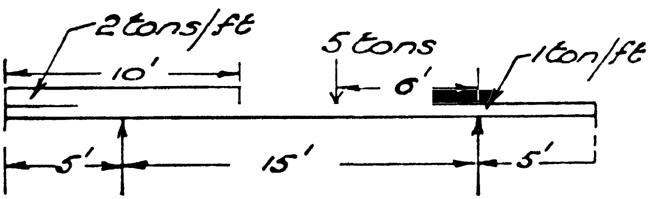


FIG 300.

(9) Find the position and magnitude of the maximum bending moment in the case of the overhanging beam given in Fig. 301. Construct the S.F. diagram for the beam.

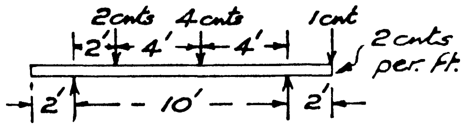


FIG. 301.

(10) Construct the bending moment and shear force diagrams for the overhanging beam shown in Fig. 302.

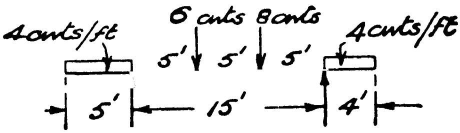


FIG. 302.

(11) A simply supported beam of 10-ft. effective span carries a uniform load of 1 cwt. per foot which covers the span. In addition there is a point load of 5 cwts. at 2 ft. from the left end. Construct the B.M. and S.F. diagrams for the beam.

(12) Calculate the maximum bending moment which must be

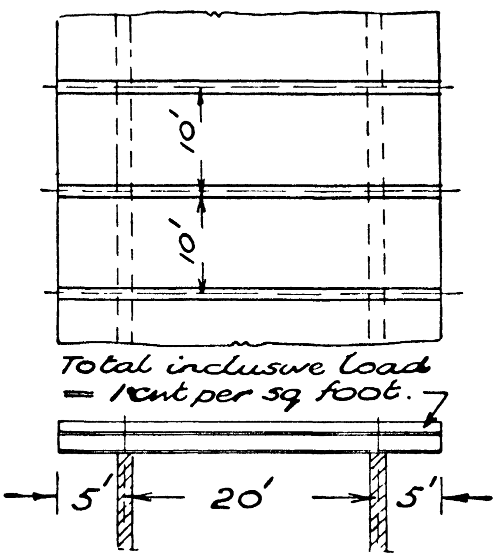


FIG. 303.

resisted by one of the beams shown in the floor diagram (Fig. 303). Draw the B.M. and S.F. diagrams for a beam.

(13) A wall 6 ft. high has to be able to resist a horizontal wind pressure of 30 lb. per sq. foot for the upper two-thirds of its height. What would be the corresponding bending moment about the base of the wall per foot run of wall, i.e. for every foot length of wall in plan? Construct the B.M. and S.F. diagrams for this foot strip of wall.

(14) Calculate the position and value of the maximum bending moment for the simply supported beam given in Fig. 304.

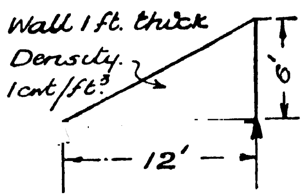


FIG 304.

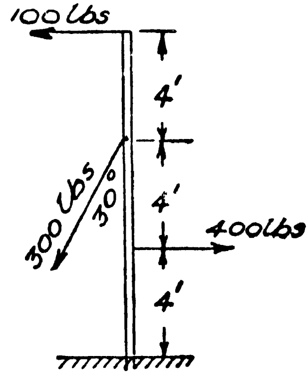


FIG. 305.

[Assume B.M.$_{max.}$ to be 'x' ft. from the left end. The wall height here will be $x/2$ ft., by proportion. Equate the weight of the wall, to the left of the section, to the left-end reaction and solve the quadratic for 'x.' Find the B.M. at the calculated section by the usual formula 'Reaction moment − load moment,' treating the load to the left of the section as acting at the C.G. of the triangle.]

(15) Draw the bending moment and shear force diagrams for the mast shown in Fig. 305. Write down the maximum bending moment the mast must be capable of resisting.

[Resolve the '300-lb.' force horizontally and ignore the vertical component. Treat as a vertical cantilever. Find B.M. at each load point, being careful with the sign of each moment. S.F. diagram is constructed by usual methods, the 'jumps' being horizontal in this case.]

CHAPTER XII

MOMENT OF RESISTANCE. DESIGN OF SIMPLE BEAMS

In the model apparatus shown on page 196, the bending moment on the detached portion of the cantilever was balanced by the couple created by the two equal horizontal forces—the *pull* in the chains and the *thrust* in the bars. The moment of this balancing couple is termed the '*moment of resistance.*' At every beam section subjected to bending moment, the moment of resistance will equal the bending moment, when the beam has deflected to its position of equilibrium. The general problem of beam design is to determine a suitable size and shape for the beam section so that the beam fibres are not excessively stressed and that the material of the beam is used economically.

Definition : *The moment of resistance (M.R.) of a beam section is the moment of the couple which is set up at the section by the longitudinal forces created in the beam by its deflection.*

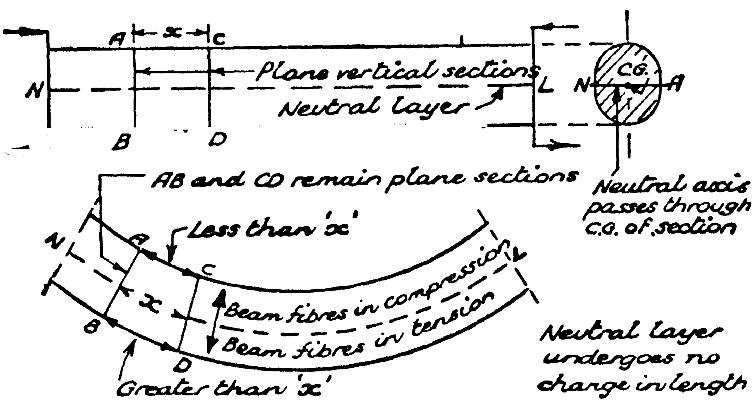


Fig. 306—Theory of Simple Bending.

General Principles of Simple Bending

The upper diagram in Fig. 306 shows the elevation of a simple beam. The lower diagram shows the beam bent under the action of bending moment. In every bent beam there is a layer of material (NL in Fig. 306) which gets neither longer nor shorter

when the beam deflects. It is termed the '*neutral layer*' of the beam.

Neutral Axis.—The '*neutral axis*' of a beam section is the straight line in which the neutral layer of the beam cuts that particular section. It represents the level in the beam section at which there is neither stress nor strain.

Variation of Strain and Stress in a Beam Section

An inspection of Fig. 306 indicates that beam layers above the neutral layer 'NL' will be shortened and those below 'NL' will be lengthened. In this case, therefore, all fibres in a beam cross section above the neutral axis will be in compressive strain and hence compressive stress, and those below will be in tensile strain and tensile stress. It will be clear that in negative bending, as in a cantilever, the upper fibres will be in tension and the lower in compression.

The full discussion of the assumptions made in the theory of bending is rather beyond the scope of this book, but one vital assumption must be appreciated. It is assumed that a vertical plane cross section of the beam before bending remains plane (i.e. flat) after bending.

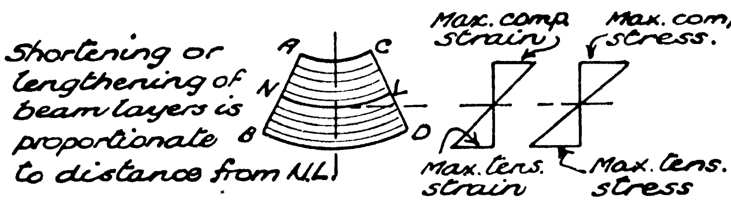


FIG. 307.—STRAIN AND STRESS DISTRIBUTION.

In Figs. 306 and 307, 'AB' and 'CD' represent two cross sections of the beam. They are assumed to be very close together so that the beam layers between them may be assumed to deflect to the arcs of concentric circles.

In Fig. 307 'NA' and 'LC' are straight lines. The beam layers are subjected to a shortening in length from length 'NL' to length 'AC' *in a proportionate manner* as we proceed upwards from 'NL.' The amount of shortening is *directly proportional* to the distance of the particular layer from the neutral layer.

Similarly, the amount of lengthening in layers below 'NL' will be directly proportional to their distance from 'NL.'

As all beam layers between sections 'AB' and 'CD' were the same length in the unbent beam, it will be clear that '*strain*' will be proportional to distance from 'NL.' The '*strain variation diagram*' will therefore be linear in character, as shown in Fig. 307. If we assume further that Hooke's law applies to beam fibres, i.e. that the stress in a beam fibre is proportional to its strain, we see that the '*stress variation diagram*' for a beam section will also be linear. These two diagrams are extremely important, and should be thoroughly understood before proceeding further with the theory.

The stress in any fibre in a beam cross section is proportional to its distance from the neutral axis of the section.

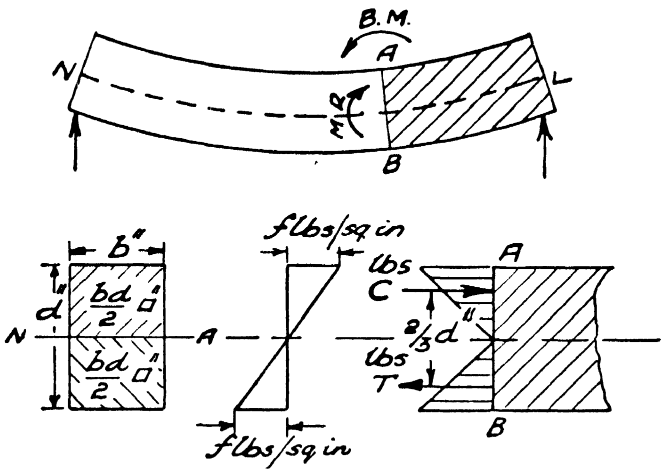


FIG. 308.—RECTANGULAR BEAM SECTION.

Moment of Resistance of a Rectangular Beam Section

The rectangular beam section (Fig. 308) has a breadth b'' and a depth d''. The applied bending moment at section 'AB' of the beam induces a maximum stress—in the extreme upper and lower fibres of the section—of 'f' lb. per sq. inch.

We may consider the beam section to be composed of a very large number of thin horizontal strips of equal width and depth. The stress acting upon any given strip will depend upon the posi-

tion of the strip in the cross section, with reference to the neutral axis of the section. The load carried by a strip will be 'stress × area.' The '*load variation diagram*' will therefore be similar in character to the 'stress variation diagram' as the elemental strips of cross section are equal in area. The system of loads acting on the top half-section of the beam will have a resultant 'C' lb., and a resultant pull 'T' lb. will act upon the bottom half-section.

'C' and 'T' are equal forces and they form a couple of moment 'C' (or 'T') × 'arm of couple.' The moment of this couple is the moment of resistance of the beam section.

Value of 'C' (or 'T')

The stress on the top half-section of the beam section varies uniformly from 'f' lb. per sq. inch to 'zero.' The total load 'C' will equal $f/2$ lb. per sq. inch × area of half-section.

$$C = \frac{f}{2} \text{ lb./in.}^2 \times \frac{bd}{2} \text{ sq. ins.} = \frac{fbd}{4} \text{ lb.}$$

$$C = T = \frac{fbd}{4} \text{ lb.}$$

Value of 'arm of couple'

'C' will act through the centre of gravity of the compression load-variation triangular diagram and 'T' will act through the C.G. of the tensile diagram.

Hence the distance between 'C' and 'T,' i.e. the '*arm of the couple*,' $= [d - (\frac{1}{3} \times d/2) - (\frac{1}{3} \times d/2)]$ ins. $= \frac{2}{3}d$ ins.

Moment of couple

$$\begin{aligned} \text{Moment of couple} &= \text{Force} \times \text{arm} \\ &= \text{C (or T) lb.} \times \tfrac{2}{3}d \text{ ins.} \\ &= \frac{fbd}{4} \text{ lb.} \times \tfrac{2}{3}d \text{ ins.} \\ &= \frac{fbd^2}{6} \text{ lb. ins.} \end{aligned}$$

As explained previously the moment of this couple balances the bending moment at every beam section. We may write 'M' to stand for 'B.M.' or 'M.R.' as required. The proof of the formula has been carried through with the units 'lb.' and 'inches'

in order to bring out the nature of the intermediate results. It is obvious that the formula is applicable to any system of units. Hence we may write in general terms :

$$M = \frac{fbd^2}{6}.$$

Allowance for Self-weight of Beams

The self-weight of a beam should always be considered. It is usually a uniform load producing a bending moment of '$Wl/8$' at the centre of span (in the case of simply supported beams). In *problems*, the self-weight of a beam is frequently omitted.

In the case of single timber beams or B.S.B.s, the beam's own weight is usually small compared with the load carried. Moreover, the choice of a practical beam section will normally leave a margin over the theoretical requirements and this should cover the effect of the weight.

In preliminary calculations the weight of a beam can only be estimated. The estimate should be checked when the beam has been designed and if sufficient allowance has not been made the calculations must be worked through again. Unless stated otherwise, the self-weight of a beam is neglected in the examples in this chapter.

EXAMPLES :

(i) *A timber beam, 2" wide × 6" deep, carries a uniformly distributed load of total value* 1000 *lb. The effective span of the beam is* 8 *ft. Calculate the maximum stress induced in the timber.*

$$M = \frac{fbd^2}{6}$$

$$M = \frac{Wl}{8} = \frac{1000 \times 8 \times 12}{8} \text{ lb. ins.} = 12000 \text{ lb. ins.}$$

[The span must always be in '*inches*' in beam stress problems, as the stress contains 'inch' units.]

$$\therefore 12000 = \frac{f \times 2 \times 6 \times 6}{6}$$

$$f = \frac{12000 \times 6}{2 \times 6 \times 6} = 1000 \text{ lb./ins.}^2$$

(ii) *Calculate the safe central-point load for a timber beam, 2"*

wide × *7″ deep and of 7′ 6″ effective span, if the maximum permissible stress in the timber is* 800 *lb. per sq. inch.*

Let 'W' lb. = central load.

$$\text{Max. B.M.} = \frac{Wl}{4}$$

$$M = \frac{fbd^2}{6}$$

$$\therefore \frac{Wl}{4} = \frac{fbd^2}{6}$$

$$\therefore \frac{W \times 7{\cdot}5 \times 12}{4} = \frac{800 \times 2 \times 7 \times 7}{6}$$

$$\therefore W = \frac{4 \times 800 \times 2 \times 49}{7{\cdot}5 \times 12 \times 6} = 580 \text{ lb.}$$

(iii) *A concrete beam, 6″ wide* × 12″ *deep, has an effective span of* 6 *ft. Assuming the density of the concrete to be* 130 *lb. per cu. foot, calculate the central-point load the beam can carry in addition to its own weight. The maximum tensile stress in the concrete is to be limited to* 60 *lb. per sq. inch.*

$$\text{Self-weight of beam} = \left[\left(\frac{6}{12} \times \frac{12}{12} \times 6\right) \times 130\right] = 390 \text{ lb.}$$

$$\text{B.M.}_{\text{max.}} \text{ due to weight of beam} = \frac{Wl}{8}$$

$$= \frac{390 \times 6 \times 12}{8} \text{ lb. ins.} = 3510 \text{ lb. ins.}$$

$$\text{Moment of resistance of beam section} = \frac{fbd^2}{6}$$

$$= \frac{60 \times 6 \times 12 \times 12}{6} \text{ lb. ins.} = 8640 \text{ lb. ins.}$$

∴ (8640 − 3510) lb. ins. = 5130 lb. ins. are available for the B.M. caused by the central load.

Let W lb. = value of the central load.

$$\therefore \frac{W \times l}{4} = 5130$$

$$W = \frac{4 \times 5130}{6 \times 12} \text{ lb.} = 285 \text{ lb.}$$

The working stress in tension for concrete is very much smaller than that in compression. To take advantage of the higher

compressive strength, the concrete is often relieved of all responsibility for tension and steel bars are embedded in suitable amount and position in order to provide the tensile force in the couple which resists bending.

Such a combination of steel and concrete is known as '*reinforced concrete.*'

(iv) *Design a suitable timber beam section given the following data:*

Effective span = 10 *ft.*

Loads carried: 11·6 *cwts., uniformly distributed.*

5·0 *cwts., central-point load.*

Working stress = 8 *cwts./in.*^{2}.

The maximum bending moment will occur at the centre of the span for both load systems.

$$\text{Hence B.M.}_{max} = \left[\frac{11\cdot6 \times 10 \times 12}{8} + \frac{5 \times 10 \times 12}{4}\right] \text{cwts. ins.}$$

$$= (174 + 150) \text{ cwts. ins.} = 324 \text{ cwts. ins.}$$

$$M = \frac{fbd^2}{6}.$$

$$\therefore 324 = \frac{8 \times bd^2}{6}. \quad \therefore bd^2 = \frac{6 \times 324}{8} = 243.$$

Theoretically, any beam section which makes 'bd^2' = 243 will be suitable. However, there is the necessity, in all beams, to limit the maximum deflection. Beams must be '*stiff*' as well as '*strong.*' The depth of a timber floor beam should be from '2' to '3' times its breadth.

Try 'b' = 3″. $\therefore 3d^2 = 243$ and $d^2 = 81$. $d = \sqrt{81} = 9''$.

A beam 3″ wide × 9″ deep would be suitable.

(v) *Timber beams, 2″ wide × 7″ deep, are to be used in a floor, the effective span of the beams being* 9 *ft. The total inclusive floor load is* 120 *lb. per sq. foot. Assuming the maximum permissible bending stress in the beams to be* 900 *lb. per sq. inch, calculate the maximum allowable spacing for the beams, centre to centre.*

Let W lb. = safe U.D. load for one beam.

$$\frac{W \times 9 \times 12}{8} = \frac{fbd^2}{6} = \frac{900 \times 2 \times 7 \times 7}{6}$$

$$W = \frac{8 \times 900 \times 2 \times 7 \times 7}{9 \times 12 \times 6} \text{ lb.}$$

$$= 1088\cdot9 \text{ lb.}$$

If 'x' ft. = spacing of beams centre to centre, $9x$ sq. ft. = area of floor supported by one beam.

$\therefore$ $(9x \times 120)$ lb. = U.D. load carried by one beam.

$$\therefore 9x \times 120 = 1088{\cdot}9$$

$$\therefore x = \text{say, } 1 \text{ ft.}$$

Maximum spacing for beams = 12".

(vi) *Fig.* 309 *shows a portion of the footing of a retaining wall. Assuming the maximum bending moment to occur at the toe of the upper wall, calculate the bending stresses induced in the footing, given a net upward pressure of 2 tons per sq. foot.*

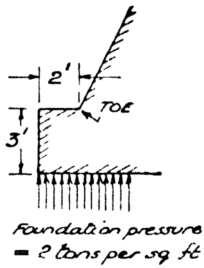


FIG. 309.—FOOTING STRESSES

Consider 1-ft. run of footing in a direction perpendicular to the plane of the diagram. We may regard this portion of the footing as a cantilever projecting 2 ft. from its support, of section 12" wide × 36" deep and carrying a uniform load of total value = $1 \times 2 \times 2$ tons (i.e. 1 ft. × 2 ft. × 2 tons/ft.²) = 4 tons.

Max. B.M. = 4 tons × 1 ft. = 4 tons ft. = 48 tons ins.

$$M = \frac{fbd^2}{6}$$

$$48 = \frac{f \times 12 \times 36 \times 36}{6} \quad \therefore f = \frac{48 \times 6}{12 \times 36 \times 36} \text{ tons/in.}^2$$

$$\therefore f = \frac{48 \times 6 \times 2240}{12 \times 36 \times 36} \text{ lb./in.}^2$$

$$= 41{\cdot}5 \text{ lb./in.}^2.$$

Beams with Non-rectangular Sections

It is clear that the formula $M = \frac{fbd^2}{6}$ cannot be used for sections which are not rectangular, because the properties of a rectangle were used in its derivation. We cannot employ this formula, for example, to determine the moment of resistance of a standard steel beam section.

The part of the expression $\frac{fbd^2}{6}$ which particularly depends

upon the assumption of a rectangular section is '$\frac{bd^2}{6}$'. For beams with other than rectangular sections we have to substitute another expression for $\frac{bd^2}{6}$.

General Theory of Bending

Fig. 310 shows a portion of a beam in the unbent and in the bent conditions. AB and CD are two vertical cross sections, assumed so close together that the portion of beam between them may be regarded as bending to the arc of a circle.

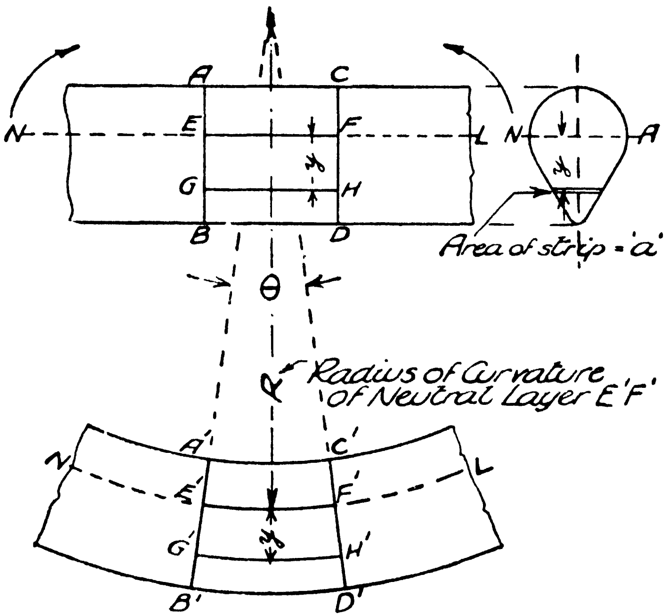


FIG. 310.—DIAGRAM ILLUSTRATING THE THEORY OF BENDING.

EF is the part of the neutral layer intercepted between the sections. GH represents a typical layer of material at a distance y from the neutral axis. R is the radius of curvature of the portion of the neutral layer, in the bent beam. The following are the steps in the development of the theory :

(1) Determination of the strain in layer G'H' by principles of geometry.

(2) Evaluation of the stress in this layer by means of Young's modulus.

(3) Determination of the load carried by the little strip of cross section at distance y from the N.A.

(4) Computation of the moment this load has about the N.A. and, by summation, the total moment of all such strip loads.

Step 1. Extension in layer G′H′ = G′H′ − GH.

$$\text{Strain in layer G'H'} = \frac{\text{Extension}}{\text{Original length}}$$
$$= \frac{\text{G'H'} - \text{GH}}{\text{GH}}.$$

But GH = EF and EF = E′F′ (being on the unstrained layer).

$$\therefore \text{Strain in layer G'H'} = \frac{\text{G'H'} - \text{E'F'}}{\text{E'F'}}.$$

Expressing these distances in terms of R and θ (the angle in radians contained by B′A′ and D′C′) we have:

$$\text{Strain in layer G'H'} = \frac{(R + y)\theta - R\theta}{R\theta} = \frac{y}{R}.$$

Step 2.
$$\frac{\text{Stress in G'H'}}{\text{Strain in G'H'}} = E.$$

$$\therefore \text{Stress in layer} = E \times \text{strain} = \frac{Ey}{R}.$$

$$\text{If } f = \text{the stress, } f = \frac{Ey}{R}.$$

Step 3. If a = the area of the cross-sectional strip, the load carried = stress × area

$$= \frac{Ey}{R} \times a = \frac{E}{R} \times ay.$$

Step 4. Moment of the load on this strip about NA

$$= \text{Load} \times \text{distance}$$
$$= \left(\frac{E}{R} \times ay\right) \times y = \frac{E}{R} \times ay^2.$$

The total 'moment of resistance' of the beam section is made up of all such moments as this.

$$\text{Total Moment of Resistance} = \Sigma\frac{E}{R} \times ay^2$$
$$= \frac{E}{R} \times \Sigma ay^2.$$

Σay^2 (*sigma* ay^2) is a geometrical property of the beam section, with reference to the axis NA. It is termed the ***moment of inertia*** of the beam section, and is denoted by the letter I.

Writing M for 'moment of resistance,'

$$M = \frac{E}{R} \times I.$$

$$\text{But } f = \frac{Ey}{R} \text{ or } \frac{E}{R} = \frac{f}{y} \text{ (step 2).}$$

$$\therefore M = \frac{fI}{y}.$$

This is the important formula for finding the moment of resistance of a beam section. Writing the results found as a continued ratio we get the complete expression for the theory.

$$\frac{E}{R} = \frac{M}{I} = \frac{f}{y}.$$

In using this expression f will normally represent a maximum stress, so that y, in that case, will be the distance from the neutral axis to an extreme fibre, top or bottom of the section, as the case may be.

M will usually be a 'bending moment,' such as $\frac{Wl}{4}$, $\frac{Wl}{8}$, etc.

Position of the Neutral Axis.—In the theory we found that the strip load was given by the expression $\frac{E}{R} \times ay$.

As long as y is measured downwards all these strip loads will represent tension. If we put y negative, i.e. measured the distance upwards from NA, the load would be compression.

$\Sigma\frac{E}{R} \times ay$ or $\frac{E}{R}\Sigma ay$ (since E and R are constants) will therefore represent a summation of a large number of positive and negative quantities. But, as the total compressive force = the total tensile force (being forces in a couple), $\frac{E}{R}\Sigma ay$ must = 0.

This, i.e. $\Sigma ay = 0$, means that the axis, from which y is measured, passes through the centre of gravity of the section. **The neutral axis of a beam section therefore passes through its centre of gravity.**

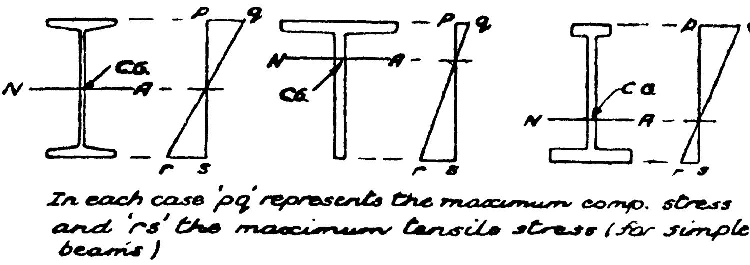


FIG. 311.—POSITION OF NEUTRAL AXIS.

Fig. 311 shows the position of the neutral axis for certain structural sections.

Moment of Inertia.—Fig. 312 shows beam sections divided up into a very large number of thin strips parallel to the neutral axis. If we multiply each strip area by the *square* of its distance from NA and sum up all the quantities obtained, we will obtain the value of ' I_{NA} ' for the given beam section.

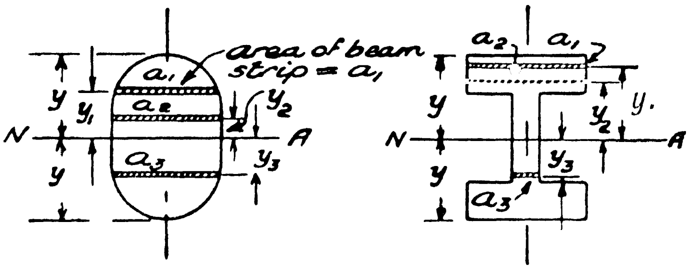


FIG. 312.

$$I_{NA} = (a_1 \times y_1^2) + (a_2 \times y_2^2) + (a_3 \times y_3^2) + \text{etc.}$$

The values for certain structural sections will be found in Fig. 313.

The proofs of ' I ' values for geometrical sections such as rectangles and circles involve the use of the calculus.

It will be observed that the value of the moment of inertia of a beam section has nothing to do with the material of the beam. It is a pure geometrical property and is a ' property of section ' of the given beam.

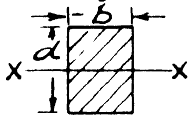
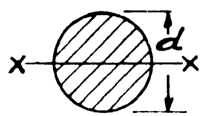
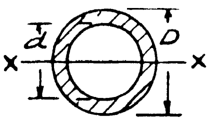
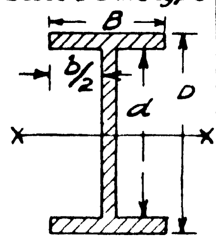
Section	Moment of Inertia	Section Modulus
Rectangular 	$I_{xx} = \frac{bd^3}{12}$	$Z_{xx} = \frac{bd^2}{6}$
Solid circular 	$I_{xx} = \frac{\pi d^4}{64}$	$Z_{xx} = \frac{\pi d^3}{32}$
Hollow circular 	$I_{xx} = \frac{\pi}{64}\left[D^4 - d^4\right]$	$Z_{xx} = \frac{\pi/64\left[D^4 - d^4\right]}{D/2}$
Steel beam type 	$I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12}$ $b = B -$ web thickness	$Z_{xx} = \frac{\frac{BD^3}{12} - \frac{bd^3}{12}}{D/2}$

FIG. 313.—PROPERTIES OF SECTION.

Units for 'I'

$$I = \Sigma ay^2.$$

Each term in this summation represents an 'area × a distance squared,' e.g. in.^{2} × in.^{2}. As inch units are nearly always used for expressing 'moment of inertia' values, it will be clear that the usual unit to employ is *ins.*^{4}

EXAMPLES :

(i) *A steel beam section has the following dimensions: flange width* = 6″, *flange thickness* = 1″, *overall depth* = 12″, *web thickness* = ½″. *Calculate the safe uniformly distributed load for this beam if the effective span* = 16 *ft., and the maximum permissible stress* = 10 *tons/in.*^{2}.

The expression for 'I_{NA}' for the beam section given is $\frac{BD^3}{12} - \frac{bd^3}{12}$ (shown as 'I_{XX}' in Fig. 313).

$$\therefore I_{NA} = \left(\frac{6 \times 12^3}{12} - \frac{5 \cdot 5 \times 10^3}{12}\right) \text{ ins.}^4.$$

$$= (864 - 458 \cdot 33) \text{ ins.}^4 = 405 \cdot 67 \text{ ins.}^4.$$

Let 'W' tons be the safe U.D. load, inclusive of self-weight of beam.

$$\frac{W \times 16 \times 12}{8} = f\frac{I}{y} = 10 \times \frac{405 \cdot 67}{6}$$

$$\left(y = \text{distance of extreme fibre from NA} = \frac{12''}{2} = 6''.\right)$$

$$W = \frac{8 \times 10 \times 405 \cdot 67}{16 \times 12 \times 6} \text{ tons} = 28 \cdot 17 \text{ tons.}$$

(ii) *A solid round steel bar, 2" diameter, rests freely on supports 12" apart. Calculate the safe central-point load the bar could support if the stress in the steel were limited to* 10,000 *lb./in.*^{2}. [*Neglect self-weight of bar.*]

'I' for section of bar $= \frac{\pi d^4}{64} = \frac{\pi \times 2^4}{64}$ ins.^{4} $= \cdot 785$ ins.^{4}.

Moment of resistance $= f\frac{I}{y} = 10{,}000 \times \frac{\cdot 785}{1}$.

$$\left[y = \frac{d}{2} = \frac{2''}{2} = 1''\right].$$

$$\therefore \text{M.R.} = 7850 \text{ lb. ins.}$$

$$\frac{Wl}{4} = 7850$$

$$\frac{W \times 12}{4} = 7850$$

$$W = 2616 \text{ lb.}$$

(iii) *Fig.* 314 *shows a steel chimney subjected to wind pressure. Estimate the stress which will be produced in the steel due to the wind pressure.*

'I' of chimney section $= \frac{\pi}{64}(D^4 - d^4)$

$$= \frac{\pi}{64}(30^4 - 28^4) \text{ ins.}^4 = 9592 \text{ ins.}^4.$$

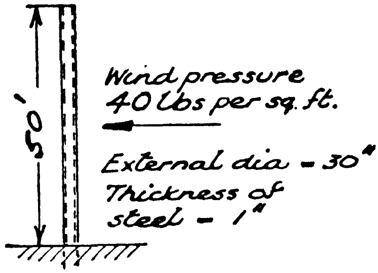


FIG. 314.—STEEL CHIMNEY.

The area of chimney elevation at right angles to direction of wind pressure $= (50 \times \frac{30}{12})$ sq. ft. The wind is not, however, acting on a flat, but on a curved, surface. It is usual to employ a reduction coefficient to obtain the *effective* area for wind pressure. A coefficient commonly employed for circular chimneys is ·6.

The resultant wind thrust $= \cdot 6 \times 50 \times \frac{30}{12} \times 40$ lb.
$= 3000$ lb.

Max. bending moment on chimney (at base)
$= (3000 \times \frac{50}{2})$ lb. ft. $= (3000 \times 25 \times 12)$ lb. ins.
$= 900000$ lb. ins.

$$M = f\frac{I}{y}. \quad \left[y = \frac{30''}{2} = 15''\right].$$

$$900000 = f\frac{I}{y}$$

$$= \frac{f \times 9592}{15}$$

$$\therefore f = \frac{15 \times 900000}{9592} \text{ lb./in.}^2$$

$$= 1408 \text{ lb./in.}^2$$

British Standard Beams

A British Standard (B.S.4) gives a list of beam sections which are known as '*British Standard Beams*' (B.S.B.). These sections are also listed in the '*section books*' which are published by steel firms. The various beam properties are evaluated and tabulated for reference purposes. Specimen pages from a section book are given on pages 258–265. All properties are given in 'inch' units.

Moments of Inertia.— I_{XX} or $I_{max.}$ The 'XX' axis of a beam is always at right angles to the web. It is for this axis that the B.S.B. section has the highest I-value, hence the term 'I_{max}.' The 'XX' axis is the neutral axis when the beam is used in the normal way, so that the column headed '$I_{max.}$' is the important one in beam problems.

I_{YY} *or* I_{min}. The 'YY' axis is always parallel to the web of a steel beam. It is for this axis that the section has the minimum moment of inertia. This property is important in the case of a B.S.B. being used as a column or in the unusual case of a beam being used with the 'YY' axis horizontal.

Description of a 'B.S.B.'—A B.S.B. is described as follows: '*overall depth*' × '*breadth of flange*' × '*weight of section in lb. per foot run*.' Thus a '9″ × 4″ × 21 lb.' B.S.B. has a depth of 9″, a flange breadth of 4″, and weighs 21 lb. for every foot of length.

EXAMPLE.—*A 9″ × 4″ × 21 lb. B.S.B. is used to carry a total inclusive load of 15 tons, uniformly spread over an effective span of 8 ft. Calculate the maximum stress in the steel. Look up the necessary properties on page 265.*

'I_{XX}' is given in the tables as 81·13 ins.[4].

$$M = f\frac{I}{y}$$

$$M = \frac{Wl}{8} = \frac{15 \times 8 \times 12}{8} \text{ tons ins.} = 180 \text{ tons ins.}$$

y = distance of extreme fibre from neutral axis

$= \frac{1}{2}$ overall depth $= 9''/2 = 4{\cdot}5''$.

$$\therefore 180 = \frac{f \times 81{\cdot}13}{4{\cdot}5}$$

$$\therefore f = 10 \text{ tons/in.}^2$$

Section Modulus.—Both 'I' and 'y' in the expression 'I/y' are properties of a given beam section. It is convenient to amalgamate them into a single property. To this new property the term '*section modulus*' is applied. The usual symbol for section modulus is 'Z.'

$$Z = \frac{I_{NA}}{y} = \frac{\text{Moment of inertia of beam section about NA}}{\text{Distance of extreme fibre from NA}}.$$

In most cases (i.e. in all cases in which the N.A. of the beam section is an axis of symmetry) y = half the overall depth of the beam section. In such cases we may write :

$$Z = \frac{I_{NA}}{\frac{1}{2}\text{ overall depth}}.$$

The formula for the moment of resistance of a beam section may now be expressed in the form most convenient for practical design.

$$\mathbf{M = fZ.}$$

The symbols ' M ' and ' f ' may have a variety of special meanings according to the type of beam problem in which they are involved. Usually ' M ' will stand for the ' maximum bending moment ' in the beam (e.g. ' $Wl/8$ ' or ' $Wl/4$,' etc.) and f will be the permissible working stress.

$$\mathbf{Z = M/f.}$$

$$\textbf{Necessary section modulus} = \frac{\textbf{Maximum bending moment}}{\textbf{Working stress in bending}}.$$

Values of Section Modulus.—The values of the ' section modulus ' for certain geometrical sections are tabulated in Fig. 313. They have been derived from the basic ' I ' value by the formula, $Z = I/y$. Thus, for a rectangular section, the section modulus $= \dfrac{bd^3}{12} \Big/ \dfrac{d}{2} = \dfrac{bd^2}{6}$.

Section modulus values for B.S.B. sections are given on pages 258 to 265. The important column is that headed ' *Axis x – x.Max.*'

Units for section modulus :

$$Z = \frac{I}{y} = \frac{\text{ins.}^4}{\text{ins.}} = \text{ins.}^3.$$

Thus a ' 6″ × 3″ × 12 lb. B.S.B.' has a section modulus of 7·00 *ins.*^{3}.

Extract from Steel Section Safe Load Tables.—The Safe Load Tables on pages 262 to 265 inclusive are reproduced by permission of the British Constructional Steelwork Association and the British Steel Makers.

The tables on pages 258 to 261 are for a bending stress of 8 tons per sq. in. and are retained in the present edition so that students can note the effect of the increase of permissible bending stress to 10 tons per sq. in. in B.S. 449 : 48.

EXAMPLES:

(i) *Given* 'I_{XX}' $= 204\cdot80$ *ins.*^{4} *and* 'I_{YY}' $= 21\cdot76$ *ins.*^{4} *for a* $10'' \times 6'' \times 40$ *lb. B.S.B., calculate* 'Z_{XX}' *and* 'Z_{YY}.'

$$Z_{XX} = \frac{I_{XX}}{y} = \frac{204\cdot80}{5} \text{ ins.}^3 = 40\cdot96 \text{ ins.}^3.$$

$$Z_{YY} = \frac{I_{YY}}{y} = \frac{21\cdot76}{3} \text{ ins.}^3 = 7\cdot25 \text{ ins.}^3.$$

These are the values given in the section table (page 261).

(ii) *If* 'Z_{XX}' *for a* $6'' \times 5'' \times 25$ *lb. B.S.B.* $= 14\cdot56$ *ins.*^{3}, *calculate* 'I_{XX}.'

$$Z_{XX} = \frac{I_{XX}}{y}. \quad \text{'}y\text{' in this case} = \frac{6''}{2} = 3''.$$

$$\therefore 14\cdot56 = \frac{I_{XX}}{3}. \quad \therefore I_{XX} = 43\cdot68 \text{ ins.}^4.$$

(iii) *A freely supported steel beam of 10-ft. effective span carries a uniform load of total value = 8 tons. Calculate the minimum permissible section modulus for the beam, assuming a working stress of 10 tons per sq. inch. Select a suitable B.S.B. section from the tables.*

$$\text{B.M.}_{\text{max.}} = \frac{Wl}{8} = \frac{8 \times 10 \times 12}{8} = 120 \text{ tons ins.}$$

$$M = fZ$$

$$120 = 10 \times Z$$

$$Z = \frac{120}{10} = 12 \text{ ins.}^3.$$

We must look down the column headed '*Moduli of Section, Axis x – x*,' until we arrive at a value either equal to 12 ins.^{3} or slightly in excess. The nearest value is 13·91 ins.^{3}, which is the section modulus of a $8'' \times 4'' \times 18$ lb. B.S.B.

(iv) *Select the most economical beam section from the section tables to carry, for an effective span of 12 ft., a central-point load of* 4·5 *tons, together with a uniform load of total value* 9 *tons.* $f = 10$ *tons/in.*^{2}.

B.M.$_{max}$:

$$\text{Due to central load} = \frac{Wl}{4} = \frac{4\cdot5 \times 12 \times 12}{4} = 162 \text{ tons ins.}$$

$$\text{,, ,, uniform ,,} = \frac{Wl}{8} = \frac{9 \times 12 \times 12}{8} = 162 \text{ ,, ,,}$$

$$\text{Total max. B.M.} = \underline{\underline{324 \text{ tons ins.}}}$$

[*Example continued on page 266.*]

JOISTS

Safe Distributed Loads, in Tons

8 Tons/Inch²

Size d × b Inches	SPANS IN FEET													
	10	12	14	16	18	20	22	24	26	28	30	32	36	40
24 × 7½	112	93·8	80·4	70·3	62·5	56·2	51·1	46·9	43·2	40·2	37·5	35·1	31·2	28·1
22 × 7	81·2	67·7	58·0	50·8	45·1	40·6	36·9	33·8	31·2	29·0	27·0	25·4	22·5	20·3
20 × 7½	89·2	74·3	63·7	55·7	49·5	44·6	40·5	37·1	34·3	31·8	29·7	27·8	24·7	22·3
20 × 6½	65·3	54·4	46·7	40·8	36·3	32·6	29·7	27·2	25·1	23·3	21·7	20·4	18·1	16·3
18 × 8	76·5	63·8	54·6	47·8	42·5	38·2	34·8	31·9	29·4	27·3	25·5	23·9	21·2	17·2
18 × 7	68·2	56·8	48·7	42·6	37·8	34·1	31·0	28·4	26·2	24·3	22·7	21·3	18·9	15·3
18 × 6	49·8	41·5	35·6	31·1	27·7	24·9	22·6	20·7	19·1	17·8	16·6	15·5	13·8	11·2
16 × 8	64·9	54·1	46·3	40·5	36·0	32·4	29·5	27·0	24·9	23·1	21·6	20·2	16·0	12·9
16 × 6H	48·3	40·2	34·5	30·2	26·8	24·1	21·9	20·1	18·5	17·2	16·1	15·1	11·9	9·6
16 × 6L	41·2	34·3	29·4	25·7	22·8	20·6	18·7	17·1	15·8	14·7	13·7	12·8	10·1	8·2
15 × 6	34·9	29·1	24·9	21·8	19·4	17·4	15·9	14·5	13·4	12·4	11·6	10·2	8·0	
15 × 5	30·4	25·3	21·7	19·0	16·9	15·2	13·8	12·6	11·7	10·8	10·1	8·9	7·0	
14 × 8	53·7	44·7	38·3	33·5	29·8	26·8	24·4	22·3	20·6	19·1	16·7	14·7		
14 × 6H	40·6	33·8	29·0	25·3	22·5	20·3	18·4	16·9	15·6	14·5	12·6	11·1		
14 × 6L	33·7	28·0	24·0	21·0	18·7	16·8	15·3	14·0	12·9	12·0	10·4	9·2		
13 × 5	23·2	19·3	16·6	14·5	12·9	11·6	10·5	9·6	8·9	7·7	6·7			
12 × 8	43·3	36·1	30·9	27·0	24·0	21·6	19·7	18·0	15·3	13·2				
12 × 6H	33·4	27·8	23·8	20·8	18·5	16·7	15·1	13·9	11·8	10·2				
12 × 6L	28·1	23·4	20·1	17·5	15·6	14·0	12·7	11·7	9·9	8·6				
12 × 5	19·6	16·3	14·0	12·2	10·9	9·8	8·9	8·1	6·9	6·0				

These tables were originally reproduced by permission and courtesy

NOTE —The British Constructional Steelwork Association and the British Steel Makers have revised the tables on pages 258 to 261 The new tables are shown on pages 262 to 265 and are based on a stress of 10 tons/in²

8
Tons/Inch²

JOISTS
Dimensions and Properties

Size d × b inches	Weight per ft in lbs	Area in sq ins	Standard Thicknesses		Moments of Inertia		Moduli of Section		Safe Distributed Load on 1 foot Span	Deflection Coefficient
			Web	Flange	Axis x—x Max	Axis y—y Min	Axis x—x Max	Axis y—y Min		
24 × 7½	95	27 94	57	1 011	2533 04	62 54	211 09	16 68	1125 8	000769
22 × 7	75	22 06	50	834	1676 80	41 07	152 44	11 73	813 0	000839
20 × 7½	89	26 19	60	1 010	1672 85	62 54	167 29	16 68	892 2	000923
20 × 6½	65	19 12	45	820	1226 17	32 56	122 62	10 02	654 0	000923
18 × 8	80	23 53	50	950	1292 07	69 43	143 56	17 36	765 7	001026
18 × 7	75	22 09	55	928	1151 18	46 56	127 91	13 30	682 2	001026
18 × 6	55	16 18	42	757	841 76	23 64	93 53	7 88	498 8	001026
16 × 8	75	22 06	48	938	973 91	68 30	121 74	17 08	649 3	001154
16 × 6	62	18 21	55	847	725 05	27 14	90 63	9 05	483 4	001154
16 × 6	50	14 71	40	726	618 09	22 47	77 26	7 49	412 1	001154
15 × 6	45	13 24	38	655	491 91	19 87	65 59	6 62	349 8	001231
15 × 5	42	12 36	42	647	428 49	11 81	57 13	4 72	304 7	001231
14 × 8	70	20 59	46	920	705 58	66 67	100 80	16 67	537 6	001319
14 × 6	57	16 78	50	873	533 34	27 94	76 19	9 31	406 3	001319
14 × 6	46	13 59	40	698	442 57	21 45	63 22	7 15	337 2	001319
13 × 5	35	10 30	35	604	283 51	10 82	43 62	4 33	232 6	001420
12 × 8	65	19 12	43	904	487 77	65 18	81 30	16 30	433 6	001538
12 × 6	54	15 89	50	883	375 77	28 28	62 63	9 43	334 0	001538
12 × 6	44	13 00	40	717	316 76	22 12	52 79	7 37	281 5	001538
12 × 5	32	9 45	35	550	221 07	9 69	36 84	3 88	196 5	001538

of the British Constructional Steelwork Association (*see page* 262)

JOISTS

Safe Distributed Loads, in Tons

8 Tons/Inch²

Size d × b inches	SPANS IN FEET 3	4	5	6	7	8	9	10	11	12	14	16	18	20
10 × 8							34.2	30.7	27.9	25.6	21.9	19.2	17.1	15.3
10 × 6					31.2	27.3	24.2	21.8	19.8	18.2	15.6	13.6	12.1	10.9
10 × 5			31.2	26.0	22.2	19.5	17.3	15.6	14.1	12.9	11.1	9.7	8.6	7.8
10 × 4½			26.1	21.7	18.6	16.3	14.4	13.0	11.8	10.8	9.3	8.1	7.2	6.5
9 × 7						30.8	27.4	24.6	22.4	20.5	17.6	15.4	13.7	11.1
9 × 4		24.0	19.2	16.0	13.7	12.0	10.6	9.6	8.7	8.0	6.8	6.0	5.3	4.3
8 × 6				25.5	21.9	19.1	17.0	15.3	13.9	12.7	10.9	9.5	7.5	6.1
8 × 5			23.9	19.9	17.0	14.9	13.2	11.9	10.8	9.9	8.5	7.4	5.9	4.7
8 × 4		18.5	14.8	12.3	10.5	9.2	8.2	7.4	6.7	6.1	5.2	4.6	3.6	2.9
7 × 4		15.0	12.0	10.0	8.6	7.5	6.6	6.0	5.4	5.0	4.3	3.2	2.6	
6 × 5		19.4	15.5	12.9	11.0	9.7	8.6	7.7	7.0	6.4	4.7	3.6		
6 × 4½	20.5	15.4	12.3	10.2	8.8	7.7	6.8	6.1	5.6	5.1	3.7	2.8		
6 × 3	12.4	9.3	7.4	6.2	5.3	4.6	4.1	3.7	3.3	3.1	2.2	1.7		
5 × 4½		13.3	10.6	8.8	7.6	6.6	5.9	5.3	4.4	3.7				
5 × 3	9.7	7.2	5.8	4.8	4.1	3.6	3.2	2.9	2.4	2.0				
4¾ × 1¾	5.0	3.7	3.0	2.5	2.1	1.8	1.6	1.4	1.1					
4 × 3	6.9	5.1	4.1	3.4	2.9	2.5	2.0	1.6						
4 × 1¾	3.2	2.4	1.9	1.6	1.3	1.2	96	78						
3 × 3	4.5	3.3	2.7	2.2	1.6	1.2								
3 × 1½	1.9	1.4	1.1	98	72	55								

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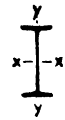
8 Tons/inch²

JOISTS

Dimensions and Properties



Size d × b inches	Weight per ft. in lbs.	Area in sq. ins.	Standard Thicknesses		Moments of Inertia		Moduli of Section		Safe Distributed Load on 1 foot Span	Deflection Coefficient
			Web	Flange	Axis x—x Max.	Axis y—y Min.	Axis x—x Max	Axis y—y Min.		
10 × 8	55	16·18	·40	·783	288·69	54·74	57·74	13·69	307·9	·001846
10 × 6	40	11·77	·36	·709	204·80	21·76	40·96	7·25	218·5	·001846
10 × 5	30	8·85	·36	·552	146·23	9·73	29·25	3·89	156·0	·001846
10 × 4½	25	7·35	·30	·506	122·34	6·49	24·47	2·88	130 5	·001846
9 × 7	50	14·71	·40	·825	208·13	40·17	46·25	11·48	246·7	·002051
9 × 4	21	6·18	·30	·457	81·13	4·15	18·03	2·07	96·2	·002051
8 × 6	35	10·30	·35	·648	115·06	19·54	28·76	6·51	153·4	·002308
8 × 5	28	8·28	·35	·575	89·69	10·19	22 42	4·08	119·6	·002308
8 × 4	18	5·30	·28	·398	55·63	3·51	13·91	1·75	74 2	·002308
7 × 4	16	4·75	·25	·387	39·51	3 37	11·29	1·69	60 2	·002637
6 × 5	25	7·37	·41	·520	43·69	9·10	14·56	3 64	77·7	·003077
6 × 4½	20	5·89	·37	·431	34·71	6·40	11·57	2·40	61 7	·003077
6 × 3	12	3·53	·23	·377	20·99	1·46	7·00	·97	37·3	·003077
5 × 4½	20	5·88	·29	·513	25·03	6·59	10·01	2·93	53·4	·003692
5 × 3	11	3·26	·22	·376	13·68	1·45	5·47	·97	29·2	·003692
4¾ × 1¾	6·5	1·91	·18	·325	6·73	·26	2·83	·30	15 1	·003887
4 × 3	10	2·94	·24	·347	7·79	1·33	3·89	·88	20·7	·004615
4 × 1¾	5	1·47	·17	·239	3·66	·19	1·83	·21	9 76	·004615
3 × 3	8·5	2·52	·20	·332	3·81	1·25	2·54	·83	13·5	·006154
3 × 1½	4	1·18	·16	·249	1·66	·13	1·11	·17	5·92	·006154

of the British Constructional Steelwork Association (see page 262).

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JOISTS
Safe Loads

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Reference Mark	Size $d \times b$ inches	SAFE DISTRIBUTED LOADS IN TONS FOR SPANS IN FEET													
		10	12	14	16	18	20	22	24	26	28	30	32	36	40
BSB 140	$24\times7\frac{1}{2}$	140	117	100	87·9	78·1	70·3	63·9	58·6	54·1	50·2	46·9	43·9	39·0	33·7
BSB 139	22×7	101	84·6	72·5	63·5	56·4	50·8	46·1	42·3	39·0	36·2	33·8	31·7	27·6	22·3
BSB 138	$20\times7\frac{1}{2}$	111	92·9	79·6	69·7	61·9	55·7	50·6	46·4	42·8	39·8	37·1	34·8	27·5	22·3
BSB 137	$20\times6\frac{1}{2}$	81·7	68·1	58·3	51·0	45·4	40·8	37·1	34·0	31·4	29·1	27·2	25·5	20·1	16·3
BSB 136	18×8	95·7	79·7	68·3	59·8	53·1	47·8	43·5	39·8	36·8	34·1	30·6	26·9	21·2	17·2
BSB 135	18×7	85·2	71·0	60·9	53·2	47·3	42·6	38·7	35·5	32·7	30·4	27·2	23·9	18·9	15·3
BSB 134	18×6	62·3	51·9	44·5	38·9	34·6	31·1	28·3	25·9	23·9	22·2	19·9	17·5	13·8	11·2
BSB 133	16×8	81·1	67·6	57·9	50·7	45·0	40·5	36·8	33·8	30·7	26·5	23·0	20·2		
BSB 132	$16\times6_H$	60·4	50·3	43·1	37·7	33·5	30·2	27·4	25·1	22·8	19·7	17·1	15·1		
BSB 131	$16\times6_L$	51·5	42·9	36·7	32·1	28·6	25·7	23·4	21·4	19·5	16·8	14·6	12·8		
BSB 130	15×6	43·7	36·4	31·2	27·3	24·2	21·8	19·8	18·2	15·5	13·3	11·6	10·2		
BSB 129	15×5	38·0	31·7	27·2	23·8	21·1	19·0	17·3	15·8	13·5	11·6	10·1	8·9		
BSB 128	14×8	67·2	56·0	48·0	42·0	37·3	33·6	30·5	26·1	22·2	19·2	16·7			
BSB 127	$14\times6_H$	50·7	42·3	36·2	31·7	28·2	25·3	23·0	19·7	16·8	14·5	12·6			
BSB 126	$14\times6_L$	42·1	35·1	30·1	26·3	23·4	21·0	19·1	16·3	13·9	12·0	10·4			
BSB 125	13×5	29·0	24·2	20·7	18·1	16·1	14·5	12·4	10·5	8·9	7·7				
BSB 124	12×8	54·2	45·1	38·7	33·8	30·1	26·0	21·5	18·0	15·3					
BSB 123	$12\times6_H$	41·7	34·7	29·8	26·0	23·1	20·0	16·5	13·9	11·8					
BSB 122	$12\times6_L$	35·1	29·3	25·1	21·9	19·5	16·8	13·9	11·7	9·9					
BSB 121	12×5	24·5	20·4	17·5	15·3	13·6	11·7	9·7	8·1	6·9					

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Bending stress in this table = 10 tons per sq. in.

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JOISTS
Dimensions and Properties

Size $d \times b$ inches	Weight per foot in pounds	Area in square inches	Standard Thicknesses		Moments of Inertia			Gross Moduli of Section		Bending Stress Coefficients	
			Web	Flange	Axis $x-x$ Gross	Axis $x-x$ Net	Axis $y-y$ Gross	Axis $x-x$ Max	Axis $y-y$ Min	Axis $x-x$	Axis $y-y$
24 × 7½	95	27 94	57	1 011	2533 04	2289 84	62 54	211 09	16 68	125 0	793 3
22 × 7	75	22 06	50	834	1676 80	1504 98	41 07	152 44	11 73	113 3	726 6
20 7½	89	26 19	60	1 010	1672 85	1506 77	62 54	167 29	16 68	129 1	665 8
20 × 6½	65	19 12	45	820	1226 17	1087 74	32 56	122 62	10 02	109 1	667 5
18 × 8	80	23 53	50	950	1292 07	1166 77	69 43	143 56	17 36	168 1	617 5
18 × 7	75	22 09	55	928	1151 18	1026 21	46 56	127 91	13 30	121 4	601 6
18 × 6	55	16 18	42	757	841 76	752 52	23 64	93 53	7 88	100 8	600 8
16 × 8	75	22 06	48	938	973 91	877 40	68 30	121 74	17 08	191 6	553 3
16 × 6	62	18 21	55	847	725 05	647 28	27 14	90 63	9 05	101 6	525 8
16 × 6	50	14 71	40	726	618 09	551 05	22 47	77 26	7 49	103 3	540 0
15 × 6	45	13 24	38	655	491 91	438 78	19 87	65 59	6 62	103 5	508 3
15 × 5	42	12 36	42	647	428 49	374 52	11 81	57 13	4 72	81 6	490 8
14 × 8	70	20 59	46	920	705 58	634 23	66 67	100 80	16 67	215 6	487 5
14 × 6	57	16 78	50	873	533 34	473 21	27 94	76 19	9 31	124 3	470 0
14 × 6	46	13 59	40	698	442 57	393 71	21 45	63 22	7 15	117 2	475 8
13 × 5	35	10 30	35	604	283 51	246 07	10 82	43 62	4 33	85 8	437 5
12 × 8	65	19 12	43	904	487 77	437 33	65 18	81 30	16 30	231 2	420 8
12 × 6	54	15 89	50	883	375 77	332 09	28 28	62 63	9 43	148 1	405 0
12 × 6	44	13 00	40	717	316 76	280 59	22 12	52 79	7 37	140 8	411 6
12 × 5	32	9 45	35	550	221 07	192 01	9 69	36 84	3 88	88 5	403 3

Constructional Steelwork Association and the British Steel Makers

JOISTS
Safe Loads

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Reference Mark	Size $d \times b$ inches	SAFE DISTRIBUTED LOADS IN TONS FOR SPANS IN FEET													
		3	4	5	6	7	8	9	10	11	12	14	16	18	20
BSB 120	10 × 8							42 7	38 4	34 9	32 0	27 4	24 0	19 0	15 3
BSB 119	10 × 6					39 0	34 1	30 3	27 3	24 8	22 7	19 5	17 0	13 4	10 9
BSB 118	10 × 5			39 0	32 5	27 8	24 3	21 6	19 5	17 7	16 2	13 9	12 1	9 6	7 8
BSB 117	10 × 4½			32 6	27 1	23 3	20 3	18 1	16 3	14 8	13 5	11 6	10 1	8 0	6 5
BSB 116	9 × 7						38 5	34 2	30 8	28 0	25 6	22 0	17 3	13 7	11 1
BSB 115	9 × 4		30 0	24 0	20 0	17 1	15 0	13 3	12 0	10 9	10 0	8 5	6 7	5 3	4 3
BSB 114	8 × 6				31 9	27 3	23 9	21 3	19 1	17 4	15 9	12 5	9 5	7 5	6 1
BSB 113	8 × 5			29 8	24 9	21 3	18 6	16 6	14 9	13 5	12 4	9 7	7 4	5 9	4 7
BSB 112	8 × 4		23 1	18 5	15 4	13 2	11 5	10 3	9 2	8 4	7 7	6 0	4 6	3 6	2 9
BSB 111	7 × 4		18 8	15 0	12 5	10 7	9 4	8 3	7 5	6 8	5 8	4 3	3 2	2 6	
BSB 110	6 × 5		24 2	19 4	16 1	13 8	12 1	10 7	9 3	7 7	6 4	4 7			
BSB 109	6 × 4½	25 7	19 2	15 4	12 8	11 0	9 6	8 5	7 4	6 1	5 1	3 7			
BSB 108	6 × 3	15 5	11 6	9 3	7 7	6 6	5 8	5 1	4 4	3 7	3 1	2 2			
BSB 107	5 × 4½		16 6	13 3	11 1	9 5	8 3	6 5	5 3	4 4	3 7				
BSB 106	5 × 3	12 1	9 1	7 2	6 0	5 2	4 5	3 6	2 9	2 4	2 0				
BSB 105	4¾ × 1¾	6 2	4 7	3 7	3 1	2 6	2 2	1 7	1 4	1 1					
BSB 104	4 × 3	8 6	6 4	5 1	4 3	3 3	2 5	2 0	1 6						
BSB 103	4 × 1¾	4 0	3 0	2 4	2 0	1 5	1 2	0 9	0 7						
BSB 102	3 × 3	5 6	4 2	3 2	2 2	1 6	1 2								
BSB 101	3 × 1½	2 4	1 8	1 4	0 9	0 7	0 5								

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Bending stress in this table = 10 tons per sq in

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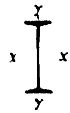
Dimensions and Properties



Size $d \times b$ inches	Weight per foot in pounds	Area in square inches	Standard Thicknesses: Web	Standard Thicknesses: Flange	Moments of Inertia: Axis x–x Gross	Moments of Inertia: Axis x–x Net	Moments of Inertia: Axis y y Gross	Gross Moduli of Section: Axis x x Max	Gross Moduli of Section: Axis y y Min	Bending Stress Coefficients: Axis x x	Bending Stress Coefficients: Axis y y
10 × 8	55	16 18	40	783	**288 69**	258 74	54 74	**57 74**	13 69	**230 0**	351 6
10 × 6	40	11 77	36	709	**204 80**	180 56	21 76	**40 96**	7 25	**168 1**	347 5
10 × 5	30	8 85	36	552	**146 23**	126 35	9 73	**29 25**	3 89	**112 2**	338 3
10 × 4½	25	7 35	30	505	**122 34**	104 06	6 49	**24 47**	2 88	**91 2**	340 0
9 × 7	50	14 71	40	825	**208 13**	182 73	40 17	**46 25**	11 48	**206 2**	313 3
9 × 4	21	6 18	30	457	**81 13**	69 81	4 15	**18 03**	2 07	**78 3**	301 6
8 × 6	35	10 30	35	648	**115 06**	101 21	19 54	**28 76**	6 51	**172 5**	278 3
8 × 5	28	8 28	35	575	**89 69**	76 89	10 19	**22 42**	4 08	**138 7**	274 1
8 × 4	18	5 30	28	398	**55 63**	47 85	3 51	**13 91**	1 75	**84 3**	270 0
7 × 4	16	4 75	25	387	**39 51**	33 80	3 37	**11 29**	1 69	**97 2**	240 8
6 × 5	25	7 37	41	520	**43 69**	37 36	9 10	**14 56**	3 64	**138 7**	203 3
6 × 4½	20	5 89	37	431	**34 71**	29 32	5 40	**11 57**	2 40	**120 0**	202 5
6 × 3	12	3 53	23	377	**20 99**	17 57	1 46	**7 00**	97	**69 1**	203 3
5 × 4½	20	5 88	29	513	**25 03**	20 86	6 59	**10 01**	2 93	**13[illegible] 5**	171 6
5 × 3	11	3 26	22	376	**13 68**	11 37	1 45	**5 47**	97	**82 9**	170 8
4¾ × 1¾	6 5	1 91	18	325	**6 73**	5 72	26	**2 83**	30	**30 8**	156 6
4 × 3	10	2 94	24	347	**7 79**	6 46	1 33	**3 89**	88	**83 7**	135 8
4 × 1¾	5	1 47	17	239	**3 66**	3 12	19	**1 83**	21	**34 5**	131 6
3 × 3	8 5	2 52	20	332	**3 81**	3 13	1 25	**2 54**	83	**87 5**	102 5
3 × 1½	4	1 18	16	249	**1 66**	1 36	13	**1 11**	17	**37 0**	99 1

Constructional Steelwork Association and the British Steel Makers

$$M = fZ$$
$$Z = M/f = \tfrac{324}{10} \text{ ins.}^3 = 32{\cdot}4 \text{ ins.}^3.$$

A 12″ × 5″ × 32 lb. B.S.B. has a section modulus of 36·84 ins.^{3}. The reader should check, from the section tables, that there is no other section, having a Z-value even higher than 36·84 ins.^{3}, which weighs less than 32 lb. per foot.

(v) *Find the necessary section modulus for the simply supported beam shown in Fig. 315. $f = 8$ tons/ins.^{2} in this case.*

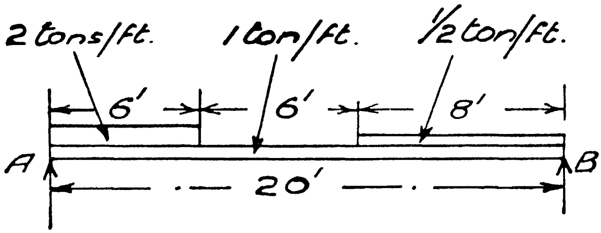


FIG. 315.

$$R_A \times 20 = (12 \times 17) + (4 \times 4) + (20 \times 10)$$
$$= 204 + 16 + 200 = 420$$
$$R_A = 21 \text{ tons.}$$
$$R_B \times 20 = (12 \times 3) + (4 \times 16) + (20 \times 10)$$
$$= 36 + 64 + 200 = 300$$
$$R_B = 15 \text{ tons.}$$

Position of max. B.M.

If we proceed along the beam from the left reaction up to a point 9 ft. from the left we take up [(2 × 6) + (1 × 9)] tons = 21 tons of load. This equals the left-end reaction, hence the point of max. bending moment is 9′ from the left support.

$$\text{B.M.}_{\text{max.}} = \text{Reaction moment} - \text{load moments}$$
$$= (21 \times 9) - (12 \times 6) - (9 \times 4{\cdot}5) \text{ tons ft.}$$
$$= (189 - 72 - 40{\cdot}5) \text{ tons ft.} = 76{\cdot}5 \text{ tons ft.}$$
$$M = fZ$$
$$\therefore Z = \frac{76{\cdot}5 \times 12}{8} \text{ ins.}^3 = 114{\cdot}75 \text{ ins.}^3.$$

The B.S B. with the smallest poundage which has the necessary

section modulus is a '20″ × 6½″ × 65 lb.' B.S.B. (Z = 122·62 ins.^{3}).

(vi) *Calculate suitable filler joists for the cantilevered floor example shown in Fig. 316. The super. load for the floor = 150 lb. per sq. foot. The density of the floor = 140 lb. per cu. foot. f = 10 tons/in.* .

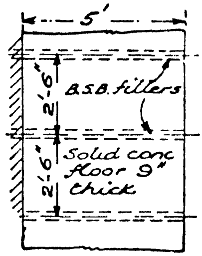


FIG. 316.

For one sq. foot of 9″ floor the self-weight will be $\frac{9''}{12''} \times 140 = 105$ lb.

$$\begin{aligned} \text{Dead load} &= 105 \text{ lb./ft.}^2 \\ \text{Super. ,,} &= 150 \text{ ,, ,,} \\ \text{Total load} &= 255 \text{ lb./ft.}^2. \end{aligned}$$

Load carried by one filler joist = (2·5 × 5 × 255) lb.
= 3187·5 lb. = 1·423 tons

$$\text{B.M.}_{\text{max.}} = \frac{Wl}{2} = \frac{1{\cdot}423 \times 5 \times 12}{2} \text{ tons ins.} = 42{\cdot}69 \text{ tons ins.}$$

$$M = fZ$$

$$Z = \frac{M}{f} = \frac{42{\cdot}69}{10} \text{ ins.}^3 = 4{\cdot}269 \text{ ins.}^3$$

A 5″ × 3″ × 11 lb. section will be suitable as it provides a section modulus of 5·47 ins.^{3}.

(vii) *A timber beam of 9-ft. span has a section 3″ wide × 9″ deep. It carries two equal concentrated loads, each 6¾ cwts. The loads are symmetrically placed 'x' ft. from either support. Calculate the maximum value of 'x' if the stress in the beam is not to exceed 6 cwts./in.^{2}.*

Each reaction = 6·75 cwts.

$$\therefore \text{B.M.}_{\text{max.}} = (6{\cdot}75 \times x) \text{ cwts. ft.}$$
$$= (6{\cdot}75 \times x \times 12) \text{ cwts. ins.} = 81x \text{ cwts. ins.}$$

$$M = fZ$$

$$Z = \frac{bd^2}{6} = \frac{3 \times 9 \times 9}{6} = 40{\cdot}5 \text{ ins.}^3.$$

$$\therefore 81x = 6 \times 40{\cdot}5$$

$$x = \frac{6 \times 40{\cdot}5}{81} \text{ ft.} = 3 \text{ ft.}$$

Note.—$M = fZ = \frac{fbd^2}{6}$ for rectangular beam sections. This formula was previously obtained by first principles (page 242).

(viii) *A* 12″ × 8″ × 65 *lb. B.S.B., used as a freely supported beam with an effective span of* 18 *ft., carries a uniform load of total value* 24 *tons. Calculate the stress in the beam at a point* 3 *ins. beneath the top of the compression flange, at a beam section* 3 *ft. from the left support. Draw the stress variation diagram for the beam at the given section.* Z_{XX} *for given B.S.B.* = 81·3 *ins.*³.

$R_L = R_R = 12$ tons.

Load on beam up to given section $= (3/18 \times 24)$ tons $= 4$ tons.

B.M. at given section = Reaction moment − load moment

$$= [(12 \times 3) - (4 \times 3/2)] \text{ tons ft.}$$

$$= (36 - 6) \text{ tons ft.} = 30 \text{ tons ft.}$$

$$M = fZ$$

$$f = \frac{30 \times 12}{81 \cdot 3} \text{ tons/in.}^2 = 4 \cdot 43 \text{ tons/in.}^2$$

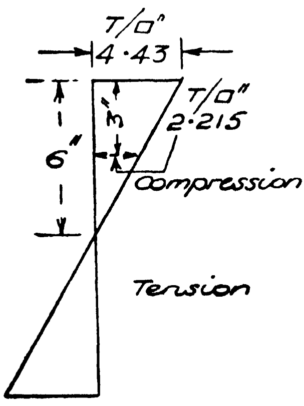


FIG. 317.—STRESS VARIATION.

The stress varies from 4·43 tons/in.² *at the top of the section* to zero *at the neutral axis*, i.e. 6″ below (see Fig. 317).

Therefore at 3″ below top of beam the stress will be 3/6 × 4·43 = 2·215 tons/in.² (compression).

Example of Steel and Timber Floor

A floor, 30 *ft.* × 28 *ft., is to be constructed of timber beams supported by steel beams. The beams are to be arranged in the manner indicated in Fig.* 318. *The total floor load, inclusive of estimated self-weights, is* 240 *lb. per sq. foot. A special allowance of* 3000 *lb. is to be made for the self-weight of the compound girder. Design suitable timber and steel beams, giving the necessary section modulus in the case of the compound girder.*

Working stresses: Timber, 900 *lb./in.*². *Steel,* 10 *tons/in.*².

Timber beams:

All the timber beams have 10-ft. span and are spaced at 15″ centres.

Area of floor supported by one beam

$$= (\tfrac{15}{12} \times 10) \text{ sq. ft.} = 12{\cdot}5 \text{ sq. ft.}$$

Uniform load carried by one beam

$$= (240 \times 12{\cdot}5) \text{ lb.} = 3000 \text{ lb.}$$

$$\text{B.M.}_{max.} = \frac{Wl}{8} = \frac{3000 \times 10 \times 12}{8} \text{ lb. ins.} = 45{,}000 \text{ lb. ins.}$$

$$M = \frac{fbd^2}{6}$$

$$45{,}000 = \frac{900 \times bd^2}{6}$$

$$bd^2 = \frac{6 \times 45000}{900} = 300.$$

If $b = 3''$, $d^2 = 100$. $\therefore d = 10''$.

Suitable dimensions for timber beams would be: breadth $= 3''$, depth $= 10''$.

B.S.B.s:

Effective span = 18 ft.

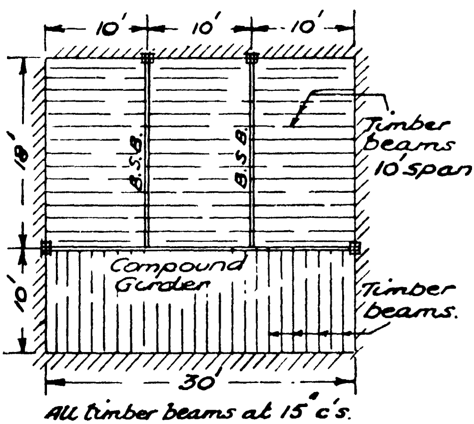


FIG 318.—FLOOR EXAMPLE.

The B.S.B.s carry the reaction loads of the timber beams. As the distance between the timber beams is small, relatively to the effective span of a B.S.B., we may assume the load carried by a B.S.B. to be uniformly distributed.

In this case the uniform load to be taken for each B.S.B. is that corresponding to a floor area 18 ft. $\times \left(\frac{10 \text{ ft.}}{2} + \frac{10 \text{ ft.}}{2}\right)$

$$= (18 \times 10) \text{ sq. ft.} = 180 \text{ sq. ft.}$$

Total uniform load = (180 × 240) lb. = 19·3 tons.

$$M = fZ$$

$$\frac{19{\cdot}3 \times 18 \times 12}{8} = 10 \times Z$$

$$Z = \frac{19{\cdot}3 \times 18 \times 12}{8 \times 10} \text{ ins.}^3$$

$$= 52{\cdot}11 \text{ ins.}^3.$$

A 12″ × 6″ × 44 lb. B.S.B. has a section modulus of 52·79 ins.^{3}.

A 15″ × 5″ × 42 lb. B.S.B. has $Z_{XX} = 57{\cdot}13$ ins.^{3}. Either of these beams will be suitable from strength point of view.

[If no allowance has been made in the floor loading for self-weight of B.S.B.s in any given case, the excess of supplied section modulus over required section modulus should be checked to ensure the necessary margin.]

Compound girder :

The compound girder carries two reaction loads (from the respective B.S.B.s) and a uniform load due to the fact that it directly supports the ends of the timber beams in the lower bay. Each B.S.B. carries a uniform load of 19·3 tons, and hence transmits a load of 9·65 tons to the compound girder.

The uniform load carried by the compound

$$= [(30 \times 5) \text{ sq. ft.} \times (240 \text{ lb. per sq. ft.})] = 36{,}000 \text{ lb.}$$

Adding in the self-weight allowance of 3000 lb., the total uniform load carried = 39,000 lb. = 17·42 tons.

B.M.$_{max.}$:

The maximum bending moment occurs at the centre of the beam for the uniform load system. For the concentrated load system it remains at the maximum value for the middle 10 ft. of the beam. B.M.$_{max.}$ therefore occurs at the centre of the beam.

B.M.$_{max.}$ = Reaction moment − load moments.

The total left-end reaction $= \left(9{\cdot}65 + \frac{17{\cdot}42}{2}\right)$ tons = 18·36 tons.

$$\text{B.M.}_{max.} = (18{\cdot}36 \times 15) - (9{\cdot}65 \times 5) - (8{\cdot}71 \times \tfrac{15}{2}) \text{ tons ft.}$$

$$= 275{\cdot}4 - 48{\cdot}25 - 65{\cdot}325 = 161{\cdot}83 \text{ tons ft.}$$

$$Z = \frac{M}{f} = \frac{161 \cdot 83 \times 12}{10} = 194 \cdot 196 \text{ ins.}^3.$$

Necessary section modulus for the compound girder = 194·196 ins.^{3}. A list of properties of compounds will suggest a suitable section to employ.

Shear Stress in Beam Webs

The question of shear stress in beam webs becomes important when B.S.B.s are fully loaded for small spans. As the span decreases the carrying capacity of a beam in tons increases, and hence the support reactions increase. This results in increasing shear stress in the beam web. A point will arise when the shear stress becomes unsafe and the webs will require stiffening. Section books usually indicate on safe-load tables the spans below which, if the beam be fully loaded, the webs will require strengthening.

Limitation of Deflection in Beams

The maximum deflection of a beam has to be limited to an amount which depends upon the particular employment of the beam. For steel beams in steel-framed buildings the limit is usually taken as 1/325th part of the span.

The theoretical treatment of beam deflection is beyond the scope of this book, but the following may be found useful.

The *zigzag black line* marked on the tabular safe-load tables (e.g. on page 262) is a safeguard from the deflection point of view. Beams which carry the tabular loads given to the left of the black line will not have excessive deflection. Such loads are therefore satisfactory from both strength and '*stiffness*' points of view. The loads given to the right of the demarcation line will not cause excessive deflection, but do not stress the steel to the safe working stress and are therefore not economical.

In the case of 8 tons/in.^{2} working stress the maximum span permissible will be found to be 24 times the overall depth of the beam.

For 10 tons/in.^{2} working stress the maximum corresponding span/depth ratio is 19·2.

Section and Safe Load Tables.—The 'section and safe load

tables ' for joists, published in the various handbooks of steel firms prior to 1948, were based on a maximum fibre stress in bending of 8 tons/in.^{2}. The columns of figures were usually set out under such headings as those shown on page 258. In 1948, the revised ' British Standard 449 ' appeared, and ' f ' was changed from 8 tons/in.^{2} to 10 tons/in.^{2}, under certain conditions. Other changes introduced into B.S. 449 : 1948 made necessary some alterations in the headings of the tables dealing with ' Dimensions and Properties ' (pages 262–265). The scope of this book only requires the use of the revised tables for examples of simple beams—with assumed equal safe bending stresses in compression and tension flanges.

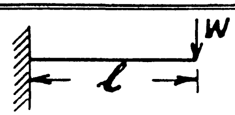
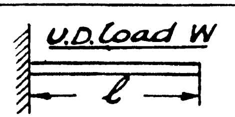
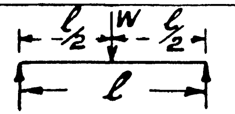
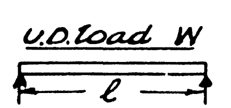
Type of beam & nature of load	Maximum B.M.	Maximum deflection
	Wl.	$\frac{1}{3}\frac{Wl^3}{EI}$
	$\frac{Wl}{2}$	$\frac{1}{8}\frac{Wl^3}{EI}$
	$\frac{Wl}{4}$	$\frac{1}{48}\frac{Wl^3}{EI}$
	$\frac{Wl}{8}$	$\frac{5}{384}\frac{Wl^3}{EI}$

FIG. 319.—DEFLECTION FORMULÆ.

Deflection formulæ.—Fig. 319 gives four important deflection formulæ. The proofs of these will have to be considered by the reader in his more advanced studies.

EXAMPLE (i).—*Calculate the maximum deflection of a* 12″ × 8″ × 65 *lb. B.S.B. which carries a U.D. load of* 18 *tons. The effective span is* 24 *ft.* $E = 13{,}000$ *tons/in.*^{2}. I_{XX} *for given beam section* = 487·77 *ins.*^{4}.

(i) *By formula :*

$$\text{Maximum deflection} = \frac{5}{384}\,\frac{Wl^3}{EI}$$

$$= \frac{5}{384} \times \frac{18 \times (24 \times 12)^3}{13{,}000 \times 487{\cdot}77}\text{ ins.} = {\cdot}89\text{ ins.}$$

* (ii) *By deflection coefficient :*

Deflection coefficient = ·001538 (see page 259)

[The load 18 tons corresponds to a maximum fibre stress of 8 tons/in.².]

∴ Maximum deflection = ·001538 × 24² ins. = ·89 ins.

(iii) *Check by tabular safe loads* (8 *tons/in.*²)*:*

For the given section, the load of 18 tons is the tabular load for a span of 24 ft., the span given in the question and which is adjacent to the zigzag black line. In this case the deflection will be :

$$\frac{\text{Span}}{325} = \frac{24 \times 12}{325}\text{ ins.} = {\cdot}89\text{ ins.}$$

EXAMPLE (ii).—*Calculate the maximum deflection of a timber beam,* 3″ *wide* × 9″ *deep, which carries a central-point load. The effective span is* 12 *ft. and the load produces a maximum beam stress of* 1000 *lb./in.*² *E* = 1,200,000 *lb./in.*².

[*Neglect self-weight of beam.*]

Let W lb. = central load

$$\frac{Wl}{4} = \frac{fbd^2}{6}$$

$$\frac{W \times 12 \times 12}{4} = \frac{1000 \times 3 \times 9 \times 9}{6}$$

$$W = 1125\text{ lb.}$$

$$\text{Max. deflection} = \frac{1}{48}\,\frac{Wl^3}{EI}$$

$$I = \frac{bd^3}{12} = \frac{3 \times 9 \times 9 \times 9}{12} = 182{\cdot}25\text{ ins.}^4$$

$$\therefore \text{Max. deflection} = \frac{1}{48} \times \frac{1125 \times 144^3}{1{,}200{,}000 \times 182{\cdot}25}$$

$$= {\cdot}32\text{ ins.}$$

Timber beams carrying plastered ceilings should not deflect more than about $\frac{1}{30}$″ per foot of span.

* These coefficients are not now given in section books.

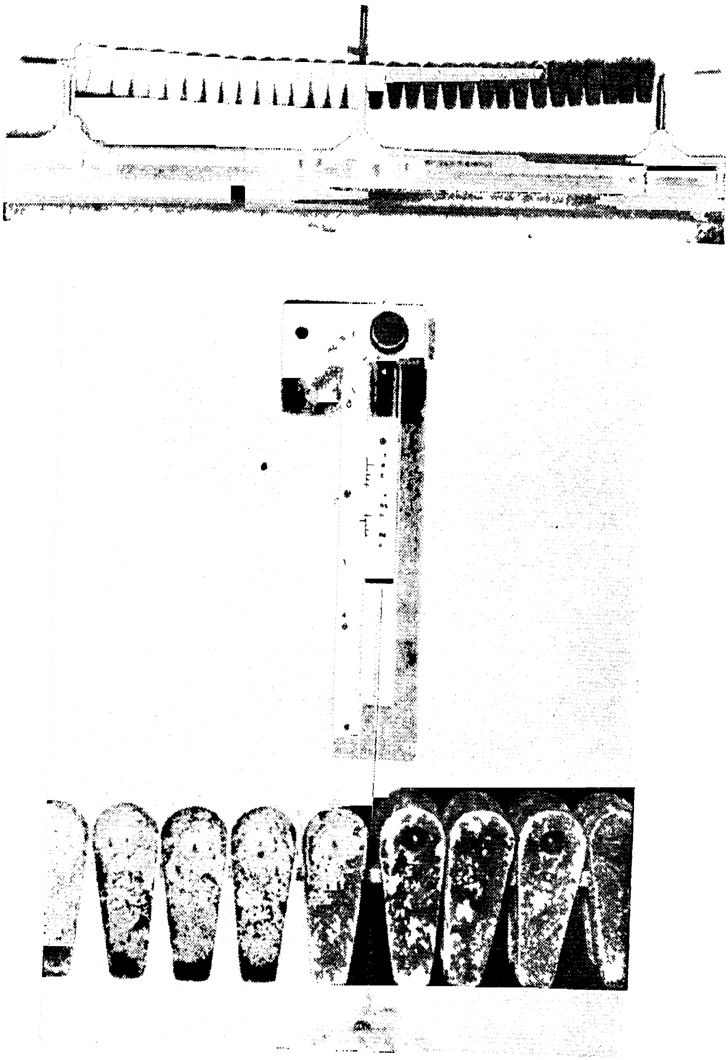


FIG 320—BEAM DEFLECTION EXPERIMENT

Experiments on Beam Deflection

The experimental laws of deflection may be verified by the apparatus given in Fig. 320. The first photograph shows a simply supported beam with a uniform load. The deflection is being measured in this experiment by means of a vernier scale.

The second photograph is an enlargement of the vernier scale. The vernier is attached to a vertical pin which rests upon the beam at the centre, or at any desired point.

Safe Distributed Load on 1-ft. Span

This column in the section tables (pages 258–261) is useful when the load is required for spans intermediate between those given in safe-load tables.

$$M = fZ. \quad \therefore \frac{Wl}{8} = fZ. \quad \therefore W = \frac{8fZ}{l}.$$

The safe uniform load is therefore inversely proportional to the span.

Taking, for example, a 10″ × 5″ × 30 lb. B.S.B., and assuming $l = 1$ ft. (i.e. 12″) and $f = 8$ tons/in.^{2},

$$W = \frac{8 \times 8 \times 29\cdot25}{12} \text{ tons} = 156 \text{ tons}.$$

This is the value given in the table. The safe load for a span, say 10′ 6″, would be $\frac{156}{10\cdot5} = 14\cdot85$ tons.

If section tables are available, the safe uniform load for *any* span may thus be quickly obtained. Modern section tables, however, do not contain 'safe loads per foot of span' columns.

Modulus of Rupture

Beam specimens of timber are tested to destruction in testing machines in order to determine the relative values of the different varieties of timber for use as beams. The standard beam specimens are 2″ × 2″ in section and are supported and loaded in the manner laid down in B.S. 373. If the maximum bending moment the beam can support before fracture be divided by the section modulus of the beam section, a value of '*f*' is determined, which is a guide to the value of the particular variety of timber as a beam material. The value of '*f*' thus fixed is termed the

'*modulus of rupture*' of the given timber. The modulus of rupture must be divided by a '*factor of safety*' in order to arrive at a suitable working stress for beam-design calculations. The factor of safety is of the order of '10.' This high 'factor' is a safeguard against excessive beam deflection.

If the modulus of rupture of a given timber be assumed to be 11,000 lb./in.^{2} and a factor of safety of '10' be adopted, the working stress in bending would be $\frac{11{,}000 \text{ lb./in.}^2}{10} = 1100$ lb./in.^{2}. L.C.C. regs. give details of working stresses to be used in timber design (see page 299).

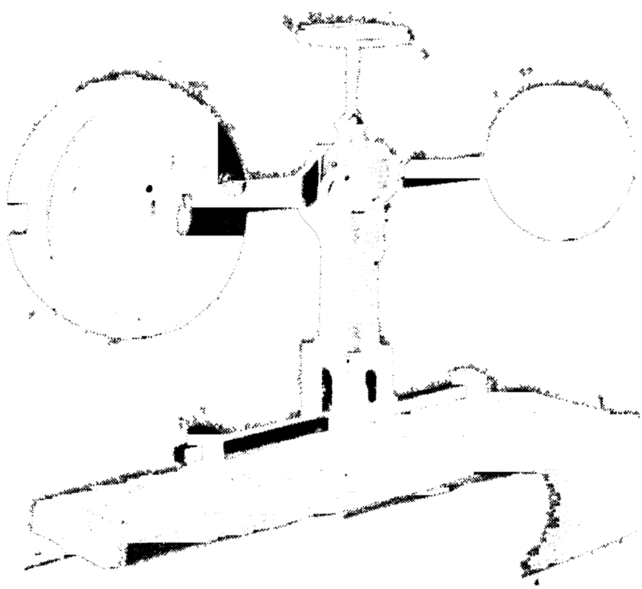


FIG 321.—2-TON TRANSVERSE BENDING MACHINE.

Fig. 321 shows a specimen of timber being tested in a transverse bending machine. The specimen is centrally loaded in this case. An alternative method is to have two symmetrical and equal point loads, thus subjecting the specimen to a uniform bending moment over the central portion of its length.

EXERCISES 12

[The self-weights of beams are to be neglected unless otherwise stated.]

(1) A timber beam has a section 3″ wide × 6″ deep. It is subjected to a bending moment of 180 cwts. ins.

Calculate (i) the maximum fibre stress produced, (ii) the resultant compressive and tensile forces which constitute the couple resisting bending, (iii) the arm of the resistance moment.

(2) Calculate the safe central-point load for a timber beam, 2″ wide × 6″ deep × 9 ft. effective span, if the maximum permissible bending stress is 900 lb. per sq. inch.

(3) What would be the safe point load for the beam of exercise (2), if the load were placed at 3 ft. from the left end ?

(4) Calculate suitable dimensions for a timber beam of 10-ft. effective span which has to carry a uniform load of 980 lb. The working stress is 900 lb./in.^{2}.

(5) Calculate the maximum stress induced in the timber beam given in Fig. 322. The beam is 2″ wide and 6″ deep.

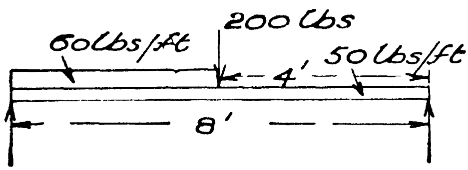


FIG. 322.

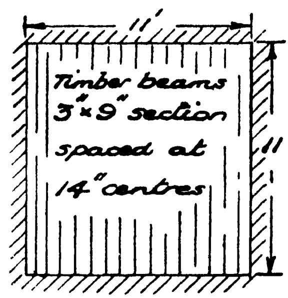


FIG. 323.

(6) Obtain the safe load per sq. foot of floor (inclusive of self-weight of floor) for the timber floor shown in Fig. 323. The maximum stress in the timber is not to exceed 1200 lb. per sq. inch.

(7) The cantilever (Fig. 324) is composed of timber weighing 40 lb. per cu. ft. It carries a concentrated load of 2000 lb. at 2 ft. from the support and a uniform load of 30 lb. per foot run. Assuming a working permissible stress of 800 lb. per sq. in.,

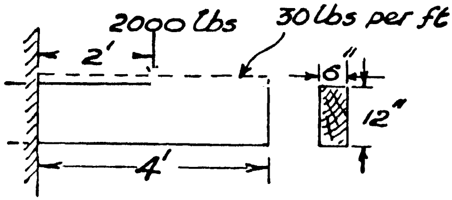


FIG. 324.

find the additional concentrated load the cantilever could safely carry at its free end.

(8) A 9" × 4" × 21 lb. B.S.B. (Z_{XX} = 18 in.^{3}) has an effective span of 8 ft. Calculate the safe uniform load this beam could carry if the working stress be 10 tons/in.^{2}. What would be the safe central load in this case?

(9) (i) I_{XX} for an 8" × 5" × 28 lb. B.S.B. section is 89·69 ins.^{4}. Calculate Z_{XX}.

(ii) Z_{XX} for a 14" × 8" × 70 lb. B.S.B. section is 100·8 ins.^{3}. Calculate I_{XX}.

(iii) If I_{XX} for a B.S.B. is 618 ins.^{4} and Z_{XX} = 77·25 ins.^{3}, what is the overall depth of the beam?

(10) Calculate the necessary section modulus for the B.S.B. shown in Fig. 325. f = 8 tons/in.^{2}.

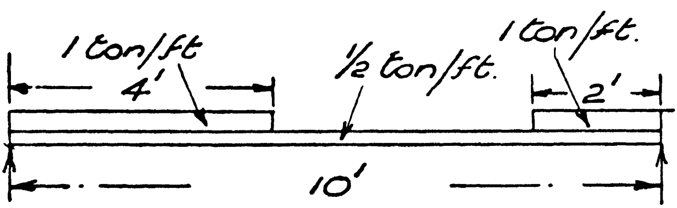


FIG. 325.

(11) Assuming a permissible working stress of 10 tons/in.^{2}, obtain the necessary section modulus for the steel beam given in Fig. 326.

(12) Select a suitable B.S.B. section, from the tables given on pages 258 to 261, given the following data:

Effective span = 6 ft.

Loads: 5 tons U.D. together with a point load of 2½ tons at mid-span.

Working stress = 8 tons/in.^{2}.

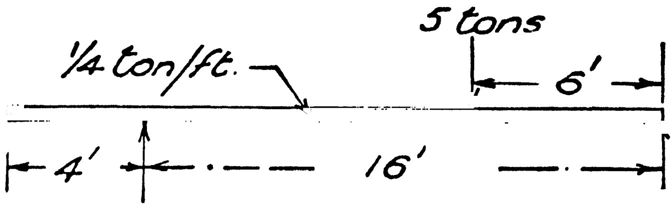


FIG 326.

(13) A balcony floor is composed of concrete with steel filler joists at 2′ 6″ centres. The total thickness of the floor is 9″ and its average density is 140 lb. per cu. foot. The filler joists are 5″ × 3″ × 11 lb. B.S.B.s [$Z_{xx} = 5{\cdot}47$ ins.^{3}] and the floor projects 4 ft. from a wall. Assuming a super. load of 200 lb. per sq. foot on the floor, calculate the maximum stress in the steel.

(14) Fig. 327 shows a compound grillage which is composed of three B.S.B.s. Calculate the section modulus required in each

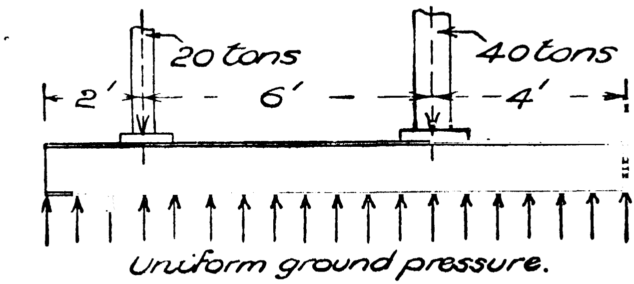


FIG. 327.

B.S.B., assuming a working stress of 12 tons/in.^{2}. Neglect self-weight of grillage.

[Invert the diagram and treat as an overhanging beam with a uniformly distributed load of 5 tons per foot run.]

(15) Fig. 328 is a plan view of a floor with overhanging timber beams resting on B.S.B.s. Assuming the floor to weigh 15 lb. per sq. foot and the live load to be 156 lb.

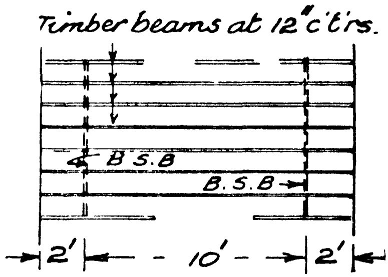


FIG. 328.

per sq. foot, find suitable dimensions for the timber beams. 'f' = 1200 lb./in.^{2}.

(16) Calculate the safe inclusive load per sq. foot for the floor given in Fig. 329.

Working stresses: Steel = 10 tons/in.^{2}.
Timber = 1200 lb./in.^{2}.

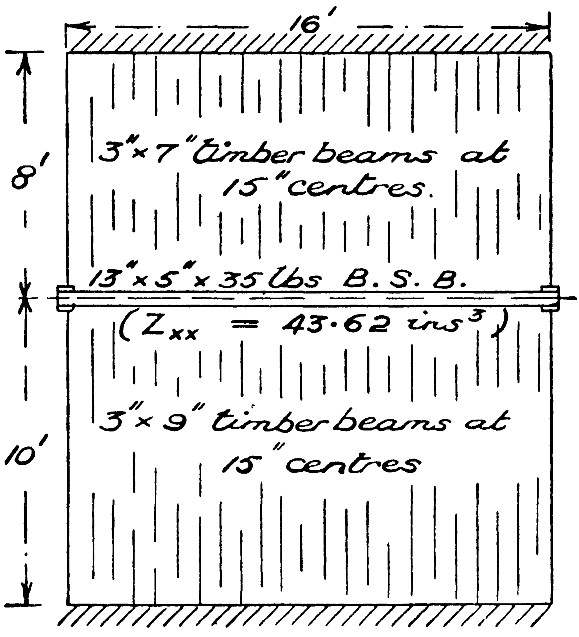


FIG. 329.

(17) Calculate the stress caused by *bending* in the case of the solid mild steel column given in Fig. 330.

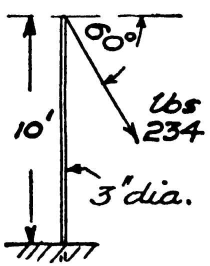


FIG. 330.

(18) A steel tube, 4" external diameter and $\frac{1}{8}$" thick, projects 1' 3" horizontally from a wall. It carries a vertical load of 'W' lb. at the free end. Calculate the maximum permissible value of 'W,' if the bending stress it produces must not exceed 6000 lb./in.^{2}.

(19) A steel beam has the following dimensions:

Flange width $= 5''$
Flange thickness $= 1''$
Overall depth $= 10''$
Web thickness $= \frac{1}{2}''$

Calculate the safe uniform load the beam can carry, in addition to a point load of 10 tons at the centre, if the effective span = 10 ft.

The working stress for the steel = 10 tons per sq. inch.

(20) The effective span of timber floor beams, $3''$ wide $\times$ $6''$ deep, is to be 9 ft. Assuming a total inclusive floor load of 178 lb. per sq. foot, calculate the maximum permissible spacing for the beams. The modulus of rupture of the timber is 12,000 lb. per sq. inch. A factor of safety of 10 must be used.

CHAPTER XIII

INTRODUCTION TO THE PRINCIPLES OF COLUMN CALCULATIONS

It is intended to outline, in as simple a manner as possible, the fundamental principles underlying the calculation of the strength of a compression member.*

Two factors have to be taken into consideration when dealing with compression members which usually do not enter into the corresponding tension-member problem.

In the case of compression members, (i) the length of the member is very important, (ii) the manner in which the ends are fixed has an important bearing on the safe load the member can carry.

A compression member has a two-fold tendency to failure. Firstly, the applied load tends to crush the fibres of the member. In a '*short*' member this is the important effect produced. Secondly, the load may tend to produce failure by the side-bending or '*buckling*' of the member (Fig. 331). This is an important consideration in a '*long*' compression member. The interpretations of '*short*' and '*long*' will be given a little later.

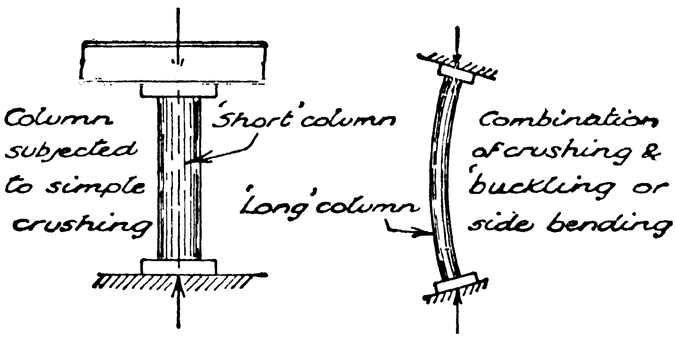


FIG. 331.– SHORT AND LONG COLUMNS.

If you stand your 12″-scale rule on end and subject it to an increasing compressive load, it soon begins to '*buckle*' sideways. A 6″-scale rule of the same cross-section would stand up to a greater load, whilst a 24″ rule would buckle more easily than the

* **For a more advanced treatment of column design, based on B.S. 449, the reader should consult** *Structural Steelwork*, by Reynolds and Kent.

12″ rule. It is clear, therefore, that, other things being equal, the 'buckling' tendency becomes of greater relative importance as the length of the member increases.

Consider two scale rules, both 12″ long and of the same sectional area. Assume one of the rules to be of the normal section and the other to be of triangular section, such as is found in the special rules having full-length multiple scales. If the simple compression test previously mentioned were applied to these two rules, the one with triangular section would stand up to a higher load before buckling.

It appears, therefore, that length and 'cross-sectional area' are not the only factors involved in the strength of an axially loaded column of a given material. The *shape* of the section has an important bearing on the value of the safe column load. The precise manner in which the cross-sectional dimensions enter into calculations of column strength introduces another '*property of section*' which involves the '*moment of inertia*' of the column section and its '*sectional area*.' This new property is termed the '*radius of gyration*' of the section. It is denoted by the letter 'r' in the subsequent calculations.

Sometimes the letters 'k' and 'g' are used for denoting 'radius of gyration.'

Radius of Gyration

The radius of gyration of a column section *about* a given axis (i.e. *with reference to* a given axis) is the square root of the result obtained by dividing the moment of inertia of the section, about the axis, by the area of the section.

$$r = \sqrt{\frac{I}{A}}.$$

In Fig. 332, $r_{XX} = \sqrt{\frac{I_{XX}}{A}}$

$$r_{YY} = \sqrt{\frac{I_{YY}}{A}}$$

EXAMPLE.—*Calculate* 'r_{XX}' *and* 'r_{YY}' *for a* 12″ × 8″ × 65 *lb. B.S.B. section given* $I_{XX} = 487{\cdot}77$ *ins.*^{4}, $I_{YY} = 65{\cdot}18$ *ins.*^{4} *and sectional area* $= 19{\cdot}12$ *ins.*^{2}.

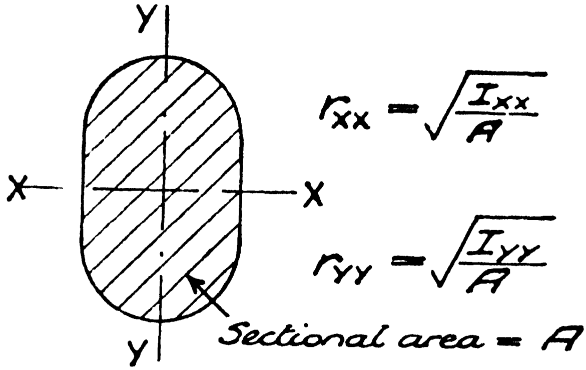


FIG. 332.—RADIUS OF GYRATION.

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}} = \sqrt{\frac{487 \cdot 77}{19 \cdot 12}} = 5 \cdot 05 \text{ ins.}$$

$$r_{YY} = \sqrt{\frac{I_{YY}}{A}} = \sqrt{\frac{65 \cdot 18}{19 \cdot 12}} = 1 \cdot 85 \text{ ins.}$$

[Note that the *unit* in which to express radius of gyration is the '*inch.*']

These values may be checked by the tables given on page 291.

Least Radius of Gyration.—In the beam calculations of the last chapter, '$I_{max.}$,' i.e. 'I' about the 'XX' axis, was the important value of the moment of inertia. When a B.S.B. is used as a column, it will clearly tend to side-bend (under concentric loading) so that 'YY' is the neutral axis, because for this axis the resistance to bending is much smaller. The 'YY' axis is therefore the important axis in column calculations, in the majority of cases. As 'I_{YY}' is '*least I*,' the value of '*least r*' will be associated with the 'YY' axis.

$$\text{Least } r = \sqrt{\frac{\text{least I}}{A}} = \sqrt{\frac{I_{YY}}{A}}.$$

The 'YY' axis, for all B.S.B. sections and for most compound column sections, is the one which is associated with 'least r.' In other cases, if there is any doubt, evaluate both 'I_{XX}' and 'I_{YY}.'

Formulæ for Radius of Gyration Values

In Fig. 333 certain geometrical sections are shown, the least 'r' values for which should be memorised.

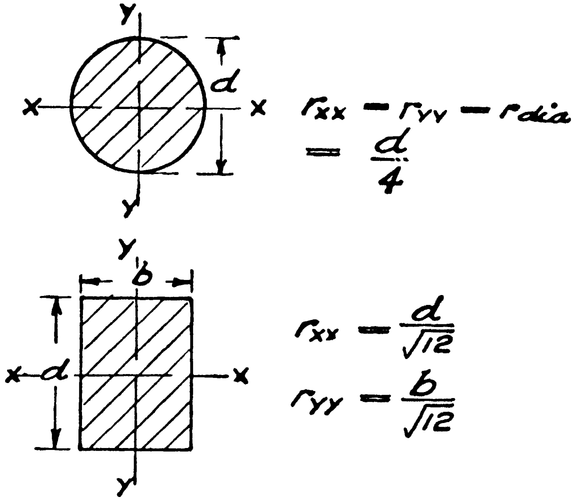


FIG. 333.—CIRCULAR AND RECTANGULAR SECTIONS.

Solid circular section:

$$r_{XX} = r_{YY} = \sqrt{\frac{I}{A}} = \sqrt{\frac{\pi d^4}{64} \div \frac{\pi d^2}{4}} = \sqrt{\frac{d^2}{16}} = \frac{d}{4}.$$

Least 'r' in this case is *one-quarter the diameter.*

Solid rectangular section:

$$r_{XX} = \sqrt{\frac{I_{XX}}{A}} = \sqrt{\frac{bd^3}{12} \div bd} = \sqrt{\frac{d^2}{12}} = \frac{d}{\sqrt{12}}.$$

$$r_{YY} = \sqrt{\frac{I_{YY}}{A}} = \sqrt{\frac{db^3}{12} \div db} = \sqrt{\frac{b^2}{12}} = \frac{b}{\sqrt{12}}.$$

$$\text{Least } r = \frac{\text{Lesser transverse dimension}}{\sqrt{12}}.$$

Steel-beam type section (*Fig.* 334) *:*

In the example given, 'least r' will be associated with the 'YY' axis. Taking the symbols shown in diagram and treating the section as consisting of three component rectangles,

$$I_{YY} = \frac{af^3}{12} + \frac{bw^3}{12} + \frac{af^3}{12}.$$

$$A = af + bw + af.$$

$$\text{Least } r = \sqrt{\frac{I_{YY}}{A}}.$$

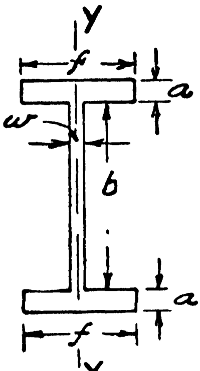


FIG. 334.—JOIST COLUMN SECTION.

EXAMPLES :

(i) *Calculate 'least r' for a solid circular column section 6" diameter.*

$$\text{Least } r = \frac{d}{4} = \frac{6''}{4} = 1{\cdot}5 \text{ ins.}$$

(ii) *Find the least radius of gyration for a rectangular section, 4" × 6".*

$$\text{Least } r = \frac{4''}{\sqrt{12}} = 1{\cdot}15''.$$

(iii) *Obtain the least radius of gyration for the steel column section given in Fig. 335.*

$$\text{Least } r = \sqrt{\frac{I_{YY}}{A}}$$

$$I_{YY} = 2\left(\frac{1 \times 8^3}{12}\right) + \frac{10 \times \frac{1}{2}^3}{12}$$

$$= 85{\cdot}33 + {\cdot}1 = 85{\cdot}43 \text{ ins.}^4$$

$$A = 2(1 \times 8) + (10 \times \tfrac{1}{2}) = 21 \text{ sq. ins.}$$

$$\text{Least } r = \sqrt{\frac{85{\cdot}43}{21}} = \sqrt{4{\cdot}07} = 2{\cdot}02 \text{ ins.}$$

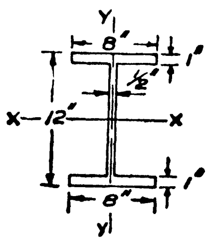


FIG. 335.—12" × 8" STEEL JOIST COLUMN.

(iv) *A hollow rectangular section is 12" × 6", overall dimensions. The material of the section is 1" thick. Calculate the least radius of gyration of the section.*

Let the YY-axis be parallel to the 12" side.

$$I_{\text{least}} = I_{YY} = \frac{BD^3}{12} - \frac{bd^3}{12} = \left(\frac{12 \times 6^3}{12} - \frac{10 \times 4^3}{12}\right) \text{ins.}^4$$

$$= (216 - 53{\cdot}33) = 162{\cdot}67 \text{ ins.}^4$$

Sectional area = 32 in.².

$$\therefore \text{ Least } r = \sqrt{\frac{162 \cdot 67}{32}} = 2 \cdot 25 \text{ ins.}$$

End Fixture of Columns

Before explaining the application of the section property 'radius of gyration' in column-strength calculations we must consider the effect on compression members of the manner in which the ends are fixed.

If you place your 12″-scale rule uprightly on a rough table and press vertically with the tip of the finger, the rule will deflect from end to end in one curve. The ends are merely held in position, and there is no restraint at the ends tending to prevent free bending. Columns fixed in this manner are said to be '*position fixed only.*'

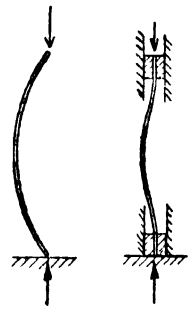


FIG 336—EFFECT OF END FIXTURE.

Now imagine the rule to be held in two grips as shown in Fig. 336. The bending tendency is not now in one simple curve. The end restraint causes a complicated form of bending. A column held in the manner demonstrated is said to be '*direction fixed*' at its ends (see also Fig. 337).

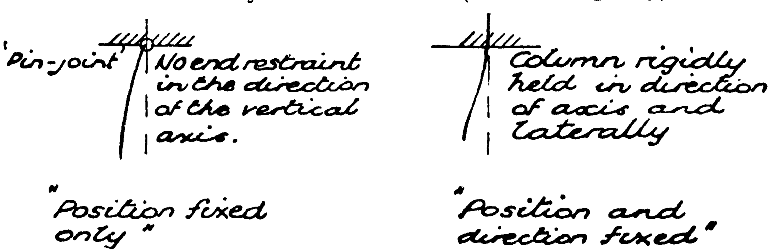


FIG 337—END FIXTURE OF COLUMNS.

Effective Column Length.—A compression member whose ends are '*position fixed only*' behaves, as has been stated, as if its ends were '*pin-jointed.*' Even when the ends are more rigidly held, a portion of a compression member tends to deflect as if this portion had pin-joints at its ends. The length of the member which is thus equivalent to a pin-jointed strut is known as its '*effective length.*' It will be clear that, other things being equal, the less the 'effective length' the stronger the member.

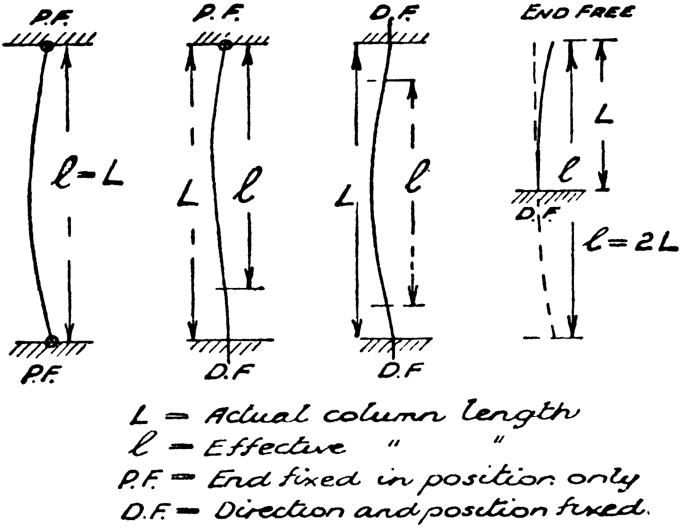


FIG. 338.—EFFECTIVE COLUMN LENGTH.

Various building regulations give methods to be adopted in deciding upon the effective length of a column in any given practical case. The diagrams given in Fig. 338 indicate that the 'effective length' may be as low as 0·70 of the actual length and as high as twice that length.

Slenderness Ratio.—The actual safe concentric load for a column depends upon a combination of its 'effective length' and the 'least radius of gyration' of its section. The '*slenderness ratio*' of a column is the ratio of its 'effective length' to its 'least radius of gyration.'

$$\text{Slenderness ratio} = \frac{\text{Effective length in inches}}{\text{Least } r \text{ in inches}} = \frac{l''}{r''} = \frac{l}{r}.$$

EXAMPLES :

(i) *A mild steel column of solid circular section, 5″ dia., has an effective length of 12 ft. Calculate its slenderness ratio.*

Least radius of gyration of column section $= \frac{\text{Diameter}}{4} = \frac{5''}{4}$
$= 1{\cdot}25$ ins.

$$\text{Slenderness ratio} = \frac{l}{r} = \frac{144''}{1{\cdot}25''} = 115{\cdot}2.$$

(ii) *A steel strut in a truss has a length of 6 ft. Its ends are to be*

regarded as 'position fixed only.' Assuming 'r' (least) from section tables to be given as 0·68", *calculate the slenderness ratio of the strut.*

Effective length of strut = actual length = 72".

$$\text{Slenderness ratio} = \frac{l}{r} = \frac{72''}{\cdot 68''} = 105\cdot 9.$$

A '*short*' compression member is one with a low slenderness ratio value. The higher the value of '*l/r*,' the '*longer*' the member in this general classification.

The precise importance of the 'slenderness ratio' value in any given case is that it determines the working compressive stress permissible in the member.

Calculation of Column Strength

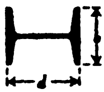
The value of the estimated safe load for a given column will depend upon the particular '*column formula*' we accept. There are several such formulæ in common use. The reader is referred to text-books on the 'theory of structures' for descriptions of formulæ known as '*Eulers*,' '*Rankines*,' etc. It is possible, in the limits set for this chapter, to refer in detail to one method of calculation only. In this method a permissible working stress is given for each '$\frac{l}{r}$' value. Lists of such stresses are given in L.C.C. regulations, B.S. No. 449 and other building regulations.

The table shown in Fig. 339 gives the values of permissible axial stresses as laid down in B.S. 449: 1948, and in the London Building (Constructional) By-Laws, 1952.

MILD STEEL COLUMNS

Slenderness Ratio $\frac{\text{Effective Column Length}}{\text{Least Radius of Gyration}}$ $\frac{l}{r}$	Working Stress Tons per sq. inch of Section for Axial Loading F_a	Slenderness Ratio $\frac{\text{Effective Column Length}}{\text{Least Radius of Gyration}}$ $\frac{l}{r}$	Working Stress Tons per sq. inch of Section for Axial Loading. F_a
20	8·0	90	4·6
30	7·5	100	4·1
40	7·1	110	3·7
50	6·6	120	3·3
60	6·1	130	2·9
70	5·6	140	2·6
80	5·1	150	2·3

FIG. 339.—WORKING STRESSES IN MILD STEEL COLUMNS.



JOIST STANCHIONS

Safe Loads

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Reference Mark	Size $d \times b$ inches	SAFE CONCENTRIC LOADS IN TONS FOR EFFECTIVE HEIGHTS IN FEET													
		6	7	8	9	10	11	12	13	14	15	16	18	20	22
BSB 140	24×7½	186	175	164	153	142	131	120	110	100	91 0	82 7	68 6	57 5	48 7
BSB 139	22×7	141	132	122	113	103	94 3	85 1	76 6	68 9	62 0	55 9	46 0	38 3	
BSB 138	20×7½	176	166	157	147	137	127	117	107	98 0	89 4	81 4	67 8	57 0	48 4
BSB 137	20×6½	121	112	104	95 5	86 9	78 3	70 3	62 9	56 4	50 6	45 6	37 4		
BSB 136	18×8	163	156	148	140	132	124	116	107	99 9	92 0	84 8	71 6	60 8	52 0
BSB 135	18×7	145	136	127	118	110	101	92 0	83 5	75 6	68 5	62 0	51 3	42 9	
BSB 134	18×6	98 8	91 1	83 3	75 3	67 5	60 0	53 3	47 4	42 2	37 6	33 8	27 5		
BSB 133	16×8	154	147	140	132	125	118	110	103	96 0	88 8	81 9	69 5	59 1	50 6
BSB 132	16×6H	111	102	94 3	85 5	76 7	68 3	60 7	53 9	48 1	43 0	38 5	31 4		
BSB 131	16×6L	90 9	84 0	77 1	70 1	63 0	56 3	50 2	44 7	39 8	35 6	32 0	26 1		
BSB 130	15×6	81 5	75 2	69 0	62 6	56 2	50 2	44 6	39 7	35 4	31 7	28 4	23 2		
BSB 129	15×5	67 1	59 7	52 2	45 3	39 1	33 8	29 4	25 6	22 6					
BSB 128	14×8	145	138	132	125	118	112	105	98 5	91 8	85 1	78 6	67 0	57 2	49 0
BSB 127	14×6H	105	97 9	90 4	82 8	75 1	67 5	60 4	54 0	48 4	43 4	39 0	31 9		
BSB 126	14×6L	84 6	78 3	72 0	65 7	59 3	53 1	47 4	42 3	37 7	33 8	30 4	24 8		
BSB 125	13×5	57 7	51 9	46 0	40 2	35 0	30 4	26 5	23 2	20 5	18 2				
BSB 124	12×8	135	129	123	117	111	105	99 8	93 8	87 6	81 5	75 6	64 7	55 4	47 6
BSB 123	12×6H	101	94 2	87 3	80 5	73 3	66 3	59 5	53 4	48 0	43 1	38 8	31 9		
BSB 122	12×6	82 0	76 2	70 4	64 5	58 6	52 8	47 3	42 3	37 9	34 0	30 6	25 0		
BSB 121	12×5	52 3	46 8	41 3	36 0	31 2	27 1	23 6	20 6	18 2	16 1				

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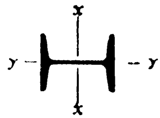
JOIST STANCHIONS

Dimensions and Properties



Size $d \times b$ inches	Weight per foot in pounds	Area in square inches	Standard Thicknesses		Radii of Gyration		Moduli of Section		Bending Stress Coefficients	
			Web	Flange	Axis y-y	Axis x-x	Axis y-y	Axis x-x	Axis y-y	Axis x-x
24 7½	95	27 94	57	1 011	**1 50**	9 52	**16 68**	211 09	**793 3**	125 0
22 7	75	22 06	50	834	**1 36**	8 72	**11 73**	152 44	**726 6**	113 3
20 7½	89	26 19	60	1 010	**1 55**	7 99	**16 68**	167 29	**665 8**	129 1
20 × 6½	65	19 12	45	820	**1 31**	8 01	**10 02**	122 62	**667 5**	109 1
18 × 8	80	23 53	50	950	**1 72**	7 41	**17 36**	1'3 56	**617 5**	168 1
18 7	75	22 09	55	928	**1 45**	7 22	**13 30**	127 91	**601 6**	121 4
18 6	55	16 18	42	757	**1 21**	7 21	**7 88**	93 53	**600 8**	100 8
16 8	75	22 06	48	938	**1 76**	6 64	**17 08**	121 74	**553 3**	191 6
16 6	62	18 21	55	847	**1 22**	6 31	**9 05**	90 63	**525 8**	101 6
16 6	50	14 71	40	726	**1 24**	6 48	**7 49**	77 26	**540 0**	103 3
15 6	45	13 24	38	655	**1 23**	6 10	**6 62**	65 59	**508 3**	103 5
15 5	42	12 36	42	647	**98**	5 89	**4 72**	57 13	**490 8**	81 6
14 8	70	20 59	46	920	**1 80**	5 85	**16 67**	100'80	**487 5**	215 6
14 × 6	57	16 78	50	873	**1 29**	5 64	**9 31**	76 19	**470 0**	124 3
14 6	46	13 59	40	698	**1 26**	5 71	**7 15**	63 22	**475 8**	117 2
13 5	35	10 30	35	604	**1 03**	5 25	**4 33**	43 62	**437 5**	85 8
12 8	65	19 12	43	904	**1 85**	5 05	**16 30**	81 30	**420 8**	231 2
12 6	54	15 89	50	883	**1 33**	4 86	**9 43**	62 63	**405 0**	148 1
12 6	44	13 00	40	717	**1 30**	4 94	**7 37**	52 79	**411 6**	140 8
12 × 5	32	9 45	35	550	**1 01**	4 84	**3 88**	36 84	**403 3**	88 5

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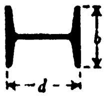

JOIST STANCHIONS

Safe Loads

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Reference Mark	Size d×b inches	SAFE CONCENTRIC LOADS IN TONS FOR EFFECTIVE HEIGHTS IN FEET 3	4	5	6	7	8	9	10	11	12	13	14	16	18
BSB 120	10×8	130	125	120	114	109	104	99 5	94 4	89 3	84 1	78 9	73 7	63 6	54 3
BSB 119	10×6	90 7	85 7	80 7	75 7	70 6	65 5	60 5	55 4	50 3	45 4	40 8	36 7	29 8	24 5
BSB 118	10×5	64 9	60 0	55 1	50 1	45 2	40 3	35 3	30 8	26 9	23 5	20 6	18 2		
BSB 117	10×4½	52 4	47 9	43 3	38 8	34 2	29 6	25 4	21 8	18 8	16 3	14 2	12 4		
BSB 116	9×7	116	111	106	101	96 0	90 8	85 6	80 4	75 2	69 9	64 7	59 5	50 0	42 0
BSB 115	9×4	42 4	38 0	33 6	29 2	24 8	20 8	17 5	14 8	12 5	10 8				
BSB 114	8×6	79 6	75 3	70 9	66 6	62 2	57 9	53 5	49 1	44 7	40 5	36 5	32 8	26 7	22 0
BSB 113	8×5	61 4	57 1	52 7	48 4	44 0	39 7	35 3	31 1	27 3	24 0	21 1	18 7	14 9	
BSB 112	8×4	36 2	32 4	28 6	24 7	20 9	17 5	14 7	12 4	10 5	9 0				
BSB 111	7×4	32 8	29 5	26 2	22 9	19 6	16 5	13 9	11 8	10 0	8 6				
BSB 110	6×5	54 7	50 8	46 9	43 1	39 2	35 3	31 4	27 7	24 3	21 3	18 8	16 6	13 2	
BSB 109	6×4½	42 2	38 7	35 1	31 5	27 9	24 3	21 0	18 0	15 5	13 5	11 8	10 3		
BSB 108	6×3	22 1	18 9	15 6	12 5	10 0	8 1	6 6							
BSB 107	5×4½	43 2	39 9	36 7	33 5	30 3	27 0	23 7	20 7	18 1	15 8	13 9	12 3		
BSB 106	5×3	20 8	18 0	15 1	12 3	9 9	8 0	6 6	5 5						
BSB 105	4¾×1¾	8 1	5 5	3 8											
BSB 104	4×3	18 7	16 2	13 6	11 1	8 9	7 2	5 9	4 9						
BSB 103	4×1¾	6 0	4 0	2 8											
BSB 102	3×3	16 3	14 2	12 1	10 0	8 2	6 7	5 5	4 6						
BSB 101	3×1½	4 3	2 8												

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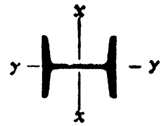
Dimensions and Properties



Size $d \times b$ inches	Weight per foot in pounds	Area in square inches	Standard Thicknesses		Radii of Gyration		Moduli of Section		Bending Stress Coefficients	
			Web	Flange	Axis y–y	Axis x–x	Axis y–y	Axis x–x	Axis y y	Axis x–x
10 × 8	55	16 18	40	783	**1 84**	4 22	**13 69**	57 74	**351 6**	230 0
10 × 6	40	11 77	36	709	**1 36**	4 17	**7 25**	40 96	**347 5**	168 1
10 × 5	30	8 85	36	552	**1 05**	4 06	**3 89**	29 25	**338 3**	112 2
10 × 4½	25	7 35	30	505	**94**	4 08	**2 88**	24 47	**340 0**	91 2
9 × 7	50	14 71	40	825	**1 65**	3 76	**11 48**	46 25	**313 3**	206 2
9 × 4	21	6 18	30	457	**82**	3 62	**2 07**	18 03	**301 6**	78 3
8 × 6	35	10 30	35	648	**1 38**	3 34	**6 51**	28 76	**278 3**	172 5
8 × 5	28	8 28	35	575	**1 11**	3 29	**4 08**	22 42	**274 1**	138 7
8 × 4	18	5 30	28	398	**81**	3 24	**1 75**	13 91	**270 0**	84·3
7 × 4	16	4 75	25	387	**84**	2 89	**1 69**	11 29	**240 8**	97 2
6 × 5	25	7 37	41	520	**1 11**	2 44	**3 64**	14 56	**203 3**	138 7
6 × 4½	20	5 89	37	431	**96**	2 43	**2 40**	11 57	**202 5**	120 0
6 × 3	12	3 53	23	377	**64**	2 44	**97**	7 00	**203 3**	69 1
5 × 4½	20	5 88	29	513	**1 06**	2 06	**2 93**	10 01	**171 6**	132 5
5 × 3	11	3 26	22	376	**67**	2 05	**97**	5 47	**170 8**	82 9
4¾ × 1¾	6 5	1 91	18	325	**37**	1 88	**·30**	2 83	**156 6**	30 8
4 × 3	10	2 94	24	347	**67**	1 63	**·88**	3 89	**135 8**	83 7
4 × 1¾	5	1 47	17	239	**36**	1 58	**·21**	1 83	**131 6**	34 5
3 × 3	8 5	2 52	20	332	**70**	1 23	**83**	2 54	**102 5**	87 5
3 × 1½	4	1 18	16	249	**33**	1 19	**17**	1 11	**99 1**	37 0

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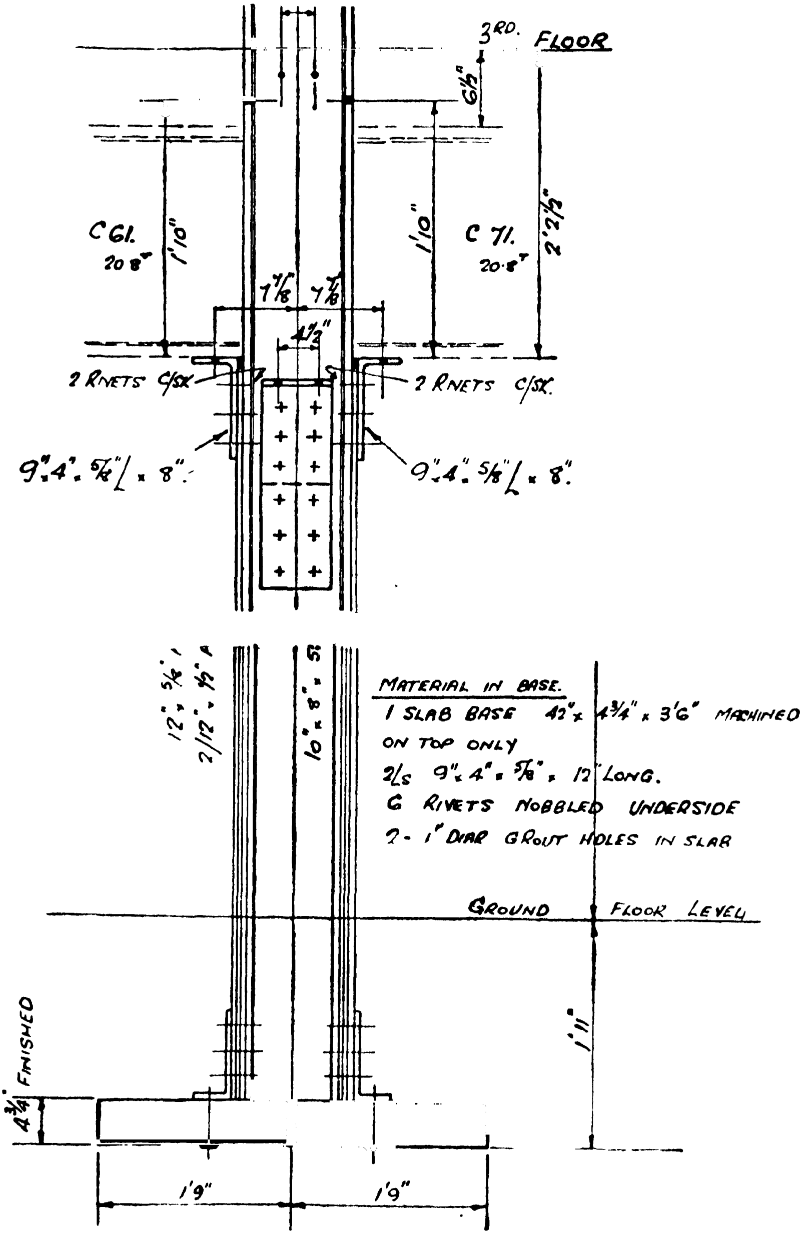
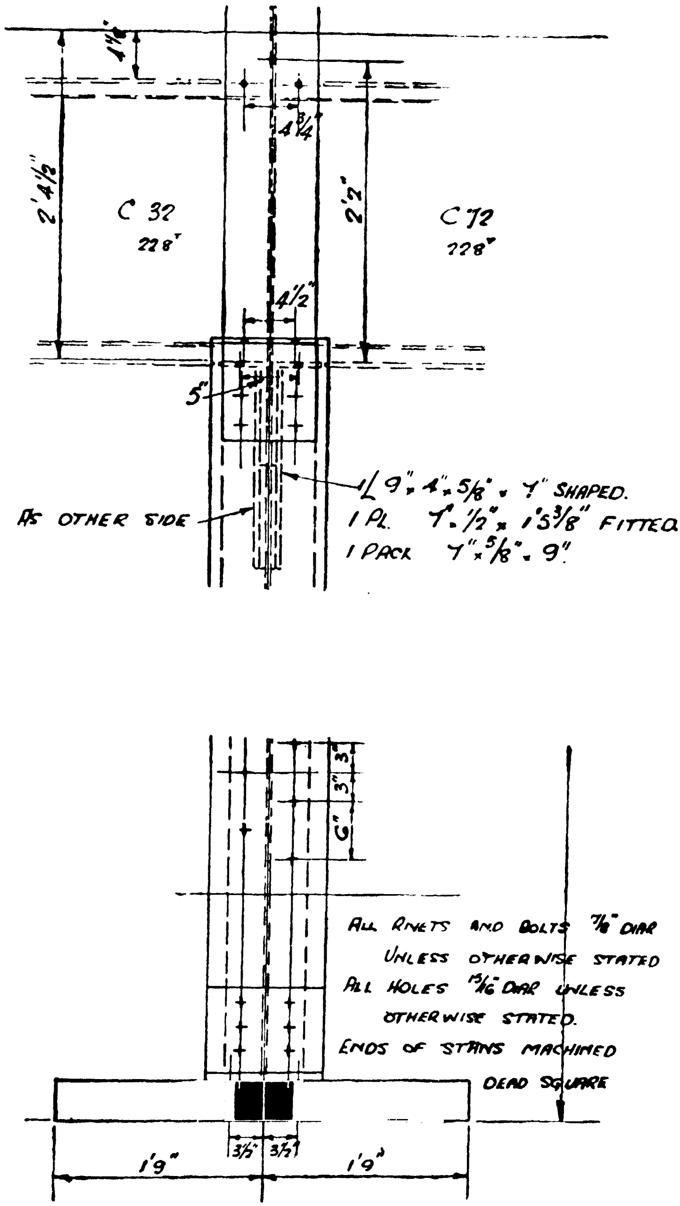


EXAMPLES OF EXTRACTS FROM TYPICAL WORKING



DRAWINGS FOR STEEL COLUMN FABRICATION.

Examples on Steel Columns

(i) *Calculate the safe axial load for a mild steel column of solid circular section,* 5 *ins. diameter, having an effective length of* 10 *ft.* 5 *ins.*

$$\text{Least } r = \frac{d}{4} = \frac{5''}{4} = 1{\cdot}25''$$
$$l = 10'\,5'' = 125''$$
$$\therefore \frac{l}{r} = \frac{125}{1{\cdot}25} = 100.$$

The working stress corresponding to this value of the slenderness ratio = 4·1 tons per sq. inch.

$$\text{Sectional area of column} = \frac{\pi d^2}{4} = \frac{\pi \times 5^2}{4} \text{ sq. ins.}$$
$$= 19{\cdot}64 \text{ sq. ins.}$$

$$\therefore \text{Safe axial load} = (4{\cdot}1 \times 19{\cdot}64) \text{ tons}$$
$$= 80{\cdot}52 \text{ tons.}$$

(ii) *The safe axial load for a joist column (i.e. a B.S.B. section used as a column)* 12″ × 8″ × 65 *lb., for an effective height of* 12 *ft., is given on page* 290 *as* 99·8 *tons. Taking the necessary properties from the table on page* 291, *check the safe load given.*

On page 291 we find the least radius of gyration for the column section (axis YY) to be 1·85 ins.

$$\therefore \frac{l}{r} = \frac{(12 \times 12)''}{1{\cdot}85''} = 77{\cdot}84.$$

This value lies between the tabular values '70' and '80.' We must therefore '*interpolate*' to find the working stress corresponding to the given '$\frac{l}{r}$' value.

	$\frac{l}{r}$		*Working Stress*
	70		5·6 tons/in.².
	80		5·1 ,,
Difference	10	Difference	·5
,,	7·84	,,	$= \frac{\cdot 5}{10} \times 7{\cdot}84 = \cdot 392$ tons/in.²

∴ The *rise* from 70 to 77·84 in $\frac{l}{r}$ value means a *drop* of ·392 tons/in.² below the 5·6 tons/in.² stress value.

NOTE.—All calculations are based on the table on page 289.

$\therefore$ If $\frac{l}{r} = 77{\cdot}84$, the working stress $= (5{\cdot}6 - {\cdot}392)$ tons/in.^{2}
$= 5{\cdot}21$ tons/in.^{2}.

The property tables (page 291) give 19·12 sq. ins. as the area of the section.

$\therefore$ Safe axial load $= (5{\cdot}2 \times 19{\cdot}12)$ tons
$= 99{\cdot}6$ tons.

This agrees with the tabular value.

(iii) *A joist column,* 10″ × 5″ × 30 *lb., has an actual length of* 11 *ft. The end fixture is such that the effective length is* ·875 *times the actual length. The least radius of gyration for the column section is* 1·05 *ins. and the area of section* = 8·9 *in.*^{2} *Calculate the safe axial load for the column.*

Effective column length $= (11 \times 12)'' \times {\cdot}875$
$= 115{\cdot}5$ ins.

$$\frac{l}{r} = \frac{115{\cdot}5''}{1{\cdot}05''} = 110.$$

For this value of $\frac{l}{r}$, the working stress $= 3{\cdot}7$ tons/in.^{2}.

$\therefore$ Safe axial load $= (3{\cdot}7 \times 8{\cdot}85)$ tons
$= 32{\cdot}75$ tons.

(iv) *A column of effective length* 7′ 8″ *has to support an axial load of* 100 *tons. Select a suitable joist-column section from the list given below.*

Size	*Sectional Area*	*Least Radius of Gyration*
(1) 12″ × 5″ × 32 lb.	9·45 sq. ins.	1·01 ins.
(2) 10″ × 8″ × 55 ,,	16·18 ,,	1·84 ,,
(3) 10″ × 6″ × 40 ,,	11·77 ,,	1·36 ,,

Try section No. 1 *:*

$$\frac{l}{r} = \frac{92''}{1{\cdot}01''} = 91{\cdot}1.$$

By interpolation :

$\frac{l}{r}$	F_a *tons/in.*^{2}
90	4·6
100	4·1
Difference = 10	Difference = ·5

Difference $= 1 \cdot 1$ Difference $= \frac{\cdot 5}{10} \times 1 \cdot 1 = \cdot 055$ tons/in.^{2}.

$\therefore$ for $\frac{l}{r} = 91 \cdot 1$, $F_a = (4 \cdot 6 - \cdot 055)$ tons/in.^{2} $= 4 \cdot 545$ tons/in.^{2}.

Safe axial load $= (4 \cdot 545 \times 9 \cdot 45)$ tons $= 42 \cdot 95$ tons.

This section is therefore unsuitable.

Try section No. 2 :

$$\frac{l}{r} = \frac{92''}{1 \cdot 84''} = 50.$$

$$\therefore F_a = 6 \cdot 6 \text{ tons/in.}^2.$$

Safe axial load $= (6 \cdot 6 \times 16 \cdot 18)$ tons $= 106 \cdot 8$ tons.

This section is suitable.

It will be found that section No. 3 is not suitable.

Timber Posts and Struts *

The method of calculation of the safe axial load for a timber post or strut is given in the London Building (Constructional) By-laws, 1952. The tables given are published by kind permission of the L.C.C.

In the by-laws the maximum permissible compressive stress, in lb. per sq. inch, for axially loaded posts and struts is dependent upon the lesser of (i) the ratio of the effective length of the member to the least radius of gyration (l/k) or (ii) the ratio of the effective length to the least lateral dimension (l/b). Part (ii) above applies only to posts and struts of solid rectangular cross section. The following limits are laid down :

The value of 'l/k' must not exceed 200 *and that of 'l/b' must not exceed* 58.

As will be seen in the table given (Table XXVIII in the L.C.C. By-laws), there are two classes of timber referred to : Class A and Class B.

Structural timber shall be either :

(*a*) Douglas fir (coast) (*Pseudotsuga taxifolia* Brit.), Longleaf pitch pine (*Pinus palustris* Mill.), or Shortleaf pitch pine (*Pinus echinata* Mill.) which for the purpose of these by-laws shall be known as class A timbers ; or

(*b*) Canadian spruce (*Picea glauca* Voss.), European larch (*Larix decidua* Mill.), Red pine (*Pinus resinosa* Alt.), Western

* Reproduced, by permission and courtesy of L.C.C., from London Building (Constructional) By-laws, 1952.

hemlock (*Tsuga heterophylla* Sarg.) or Whitewood (*Picea abies* Karst.), which for the purpose of these by-laws shall be known as Class B timbers.

TABLE XXVIII

Maximum permissible compressive stress in posts and struts in lb. per sq. inch.

Ratio of Effective Length to		Class of Timber.		Ratio of Effective Length to		Class of Timber.	
Least radius of gyration. (l/k)	Least lateral dimension. (l/b)	A	B	Least radius of gyration. (l/k)	Least lateral dimension. (l/b)	A	B
0	0	1,000	800	80	23	700	560
10	3	980	785	90	26	610	490
20	6	960	770	100	29	530	420
30	9	940	750	120	35	400	320
40	11	910	730	140	40	310	250
50	14	870	700	160	46	240	190
60	17	830	660	180	52	200	160
70	20	770	620	200	58	160	130

The maximum permissible compressive stress for intermediate values of l/k or l/b shall be obtained by interpolation between the two nearest stresses for the class of timber used.

FIG. 340.

Effective Length of Posts and Struts (Timber)

The importance of end fixture on the 'effective length' of steel columns has been considered earlier in the chapter. Table XXIX below is extracted from the London Building (Constructional) By-laws, 1952, by permission and courtesy of the L.C.C., and deals with timber posts and struts.

TABLE XXIX

Effective length of posts and struts.

Type of post or strut	Effective length, where L is the length of the post or strut between centres of restraining members.
Properly restrained at both ends in position and direction	0·7L
Properly restrained at both ends in position and at one end in direction . . .	0·85L
Properly restrained at both ends in position but not in direction	L
Properly restrained at one end in position and direction and at the other end partially restrained in direction but not in position	1·5L
Properly restrained at one end in position and direction but not restrained at the other end	2·0L

Where a post or strut is of a type not specified in this Table, the effective length of that post or strut shall be determined to the satisfaction of the district surveyor.

FIG. 341.

It will be noted that ' L ' in Table XXIX is, in each case, given as the length of the post or strut, *between centres of restraining members.*

Introduction to the Principles of Eccentric Loading

An eccentric load is one which does not act at the centre of gravity of the section of the member. As the increase in stress due to the fact of eccentricity is usually very marked, eccentric loads are avoided as far as is possible. If unavoidable, the actual amount of eccentricity is kept as low as practical requirements permit.

In foundation work it is desirable to have uniform ground pressure under the base slab. If several columns are supported by one combined grillage, the resultant of the column loads should pass through the centre of gravity of the plan outline of the grillage.

In riveted joints, the resultant load carried does not, sometimes, pass through the centre of gravity of the rivets in the group. In such a case some of the rivets will have to take more than their share of the total load.

In a steel frame, with columns continuing through several floors, a floor beam has to be connected either to the web or to the flange of the column. The beam reaction therefore constitutes an '*eccentric load.*'

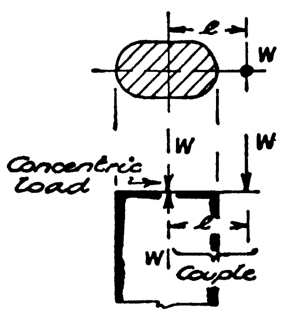


FIG. 342.—ECCENTRIC LOAD.

Theory of Eccentricity

Fig. 342 shows a short column carrying a load ' W ' which acts at a distance '*e*' from the centre of gravity of the section. If we introduce, at the C.G. of the section, two vertical forces each equal to 'W' in magnitude but acting in opposite directions to one another, we do not affect the equilibrium conditions, as the two forces introduced merely cancel one another. This procedure, however, enables us to get a clear picture of the real effect

of the given eccentric load. The two forces bracketed together (see Fig. 342) form a *couple* which tends to bend the column. The remaining force, acting at the C.G. of the section, tends to crush the column.

We thus have the important result that *an eccentric load is equivalent to a concentric load together with a bending moment.*

The magnitude of the hypothetical concentric load = ' W ' and the magnitude of the bending moment = ' W $\times$ *e*,' i.e. ' *eccentric load* $\times$ *arm of eccentricity.*'

Stress Produced by an Eccentric Load

The concentric load portion of the equivalent system will produce a direct stress (compressive), uniform in value all over the section.

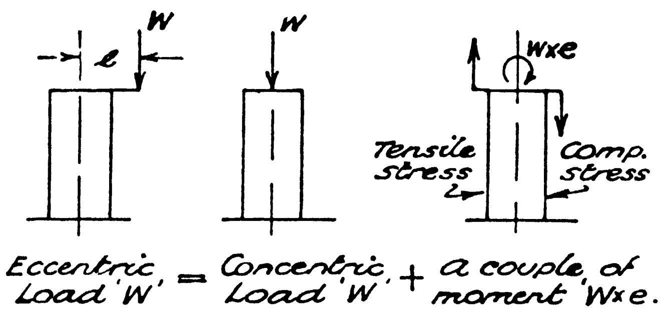


FIG. 343.—EFFECT OF ECCENTRIC LOADING.

The bending moment will produce ' bending stresses,' compressive and tensile (Fig. 343). The variation of stress over the column section due to bending will be similar to that produced in a simple beam, i.e. the stress-variation diagram will be a *straight line.*

It should be noted that the neutral axis for *bending* passes through the centre of gravity of the column section and is at *right angles* to the direction of eccentricity. The compressive stress due to the bending moment will occur in the portion of section which lies to the same side of the neutral axis as the eccentric load. The *net* stress distribution over the column section is found by adding the direct and bending stresses in the manner indicated in the following examples.

EXAMPLES :

(i) *Calculate the safe axial load for a timber post of solid rectangular section, 4″ × 6″, if the length of the post between the centres of the restraining members = 8 ft. The post is properly restrained at both ends in position and direction and the timber is ' Class A.'*

Least lateral dimension = 4″.
Value of ' L ' in inches = 8″ × 12″ = 96″.
Effective length of post = 0·7 × L″.
= ·07 × 96″ = 67·2″.

Ratio of effective length to least lateral dimension

$$= \frac{67{\cdot}2''}{4''} = 16{\cdot}8$$

Maximum permissible compressive stress.
From Table XXVIII :

When ' l/b '	= 14,	max. stress	= 870	lb. per sq. in.
,, ,,	= 17	,, ,,	= 830	,, ,,
Increase	= 3	Decrease	= 40	,, ,,

For increase in ' l/b ' of (16·8 − 14) i.e. 2·8 the decrease in max. permissible stress $= \left(\frac{40}{3} \times 2{\cdot}8\right)$ lb. per sq. inch = 37·33 lb. per sq. inch.

Hence for l/b = 16·8, max. stress = (870 − 37·33)
= 832·67 lb. per sq. inch.
Sectional area of post = 4″ × 6″ = 24 sq. ins.
∴ Safe axial load = (832·67 × 24) lb.
= 19984 lb.

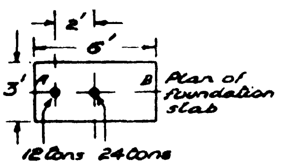


FIG. 344.—FOUNDATION PRESSURE.

(ii) *A concrete foundation slab carries the loads indicated in Fig. 344. Calculate the maximum pressure exerted on the subsoil. Test whether there will be any tendency for uplift of the slab.*

$$\text{Direct stress} = \frac{\text{Total load}}{\text{Area of base}}.$$

If there is an actual *concentric* load in addition to the eccentric load, add the two loads together in computing the direct stress.

$$\text{Direct stress} = \frac{(12 + 24)\text{ tons}}{18\text{ sq. ft.}} = \frac{36}{18}\text{ tons per sq. foot}$$

$$= 2\text{ tons per sq. ft.}$$

$$\text{Bending moment} = 12\text{ tons} \times 2\text{ ft.} = 24\text{ tons feet}$$

$$\text{Section modulus of slab} = \frac{bd^2}{6} = \frac{3 \times 6 \times 6}{6}\text{ ft.}^3$$

$$= 18\text{ ft.}^3.$$

$$\text{Bending stresses}\quad \frac{M}{Z} = \frac{24}{18} = 1\tfrac{1}{3}\text{ tons per sq. foot.}$$

Max. stress (i.e. pressure) on foundation at 'A' $= (2 + 1\frac{1}{3})$ tons per sq. foot $= 3\frac{1}{3}$ tons per sq. foot.

Net stress or pressure at 'B' $= (2 - 1\frac{1}{3})$ tons per sq. foot $= \frac{2}{3}$ tons per sq. foot. The slab is therefore under no tilting tendency as would have occurred if the '*bending tension stress*' had exceeded the '*direct compression stress.*'

(iii) *Determine whether the given eccentric load of* 10 *tons will produce tension in the short steel column given in Fig.* 345. *Write down the maximum compressive stress. Look up the necessary section properties.*

As the eccentricity is at right angles to axis 'YY,' we require to look up the value of 'Z_{YY}' (page 293).

$Z_{YY} = 3{\cdot}89$ ins.^{3}. Sectional area $= 8{\cdot}85$ sq. ins.

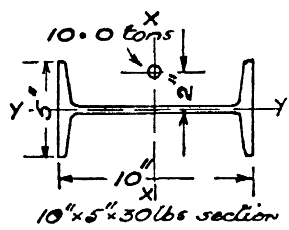


FIG. 345.

$$\text{Direct compressive stress} = \frac{\text{Load}}{\text{Area}} = \frac{10}{8{\cdot}85} = 1{\cdot}14\text{ tons/in.}^2.$$

Bending moment about 'YY' axis = Eccentric load × Eccentric arm

$$= 10\text{ tons} \times 2\text{ ins.} = 20\text{ tons/ins.}$$

$$M = fZ$$

$$20 = f \times 3{\cdot}89. \quad \therefore f = \frac{20}{3{\cdot}89} = 5{\cdot}13\text{ tons/in.}^2.$$

This is the maximum tensile stress produced by the bending

moment. Hence there will be a *net* tensile stress of (5·13 − 1·14) tons/in.² = 3·99 tons/in.².

The maximum compressive stress will be

$$(1{\cdot}14 + 5{\cdot}13)\ \text{tons/in.}^2 = 6{\cdot}27\ \text{tons/in.}^2.$$

This example illustrates the marked effect of eccentricity of loading on the value of the maximum compressive stress when the eccentricity is with respect to the 'YY' axis in joist columns.

Further Application of the Theory of Eccentricity

The problem of rectangular sections under eccentric loading is very important in masonry work. Formulæ may be derived to obtain the maximum stresses without all the detailed working given in the foregoing examples. Such formulæ are considered in Chapter XV.

EXERCISES 13

All calculations to be made, where necessary, by use of the section tables on pages 290 to 293.

(1) Calculate the value of the least radius of gyration in each of the following cases: (i) a solid circular section, 4″ dia., (ii) a rectangular section, 4″ × 3″, (iii) a B.S.B. section, 10″ × 8″ × 55 lb. (I_{XX} = 288·69 ins.⁴, I_{YY} = 54·74 ins.⁴, A = 16·18 ins.²), (iv) a B.S.B.-type section, flanges 6″ wide × 1″ thick, web thickness = ½″, overall depth = 12″.

(2) Calculate the slenderness ratio of a column of 9 ft. effective length if the section of the column were (i) a circle 6″ dia., (ii) a B.S.B. section 14″ × 8″ × 70 lb. (r_{XX} = 5·85 ins., r_{YY} = 1·80 ins.).

(3) Obtain the safe axial load for a joist pillar section, 16″ × 8″ × 75 lb., if the effective height is 12 ft. Look up the necessary section properties and 'F_a' value.

(4) Find the maximum permissible effective length for a 12″ × 6″ × 44 lb. joist pillar, which has to carry an axial load of 38 tons. Least 'r' for section = 1·30 ins., sectional area = 13 sq. ins. The following table is provided:

$\frac{l}{r}$	*Permissible Stress*
120	3·3 tons/in.².
130	2·9 ,,
140	2·6 ,,

(5) A column has to support an axial load of 100 tons. The actual height of the column is 14′ 8″. The effective length of the column is to be taken as ·75 the actual height. Find which

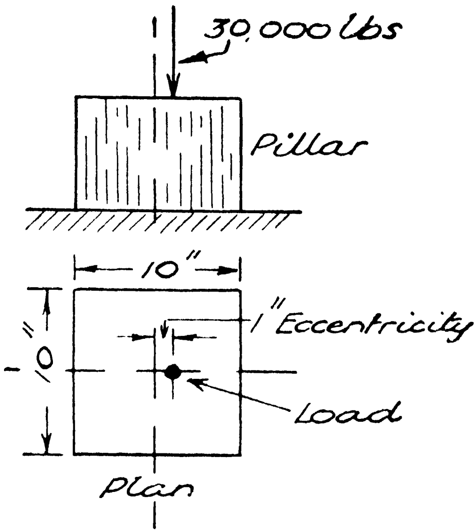


FIG. 346.

of the following sections would be most suitable: 12″ × 8″ × 65 lb., 14″ × 8″ × 70 lb., 18″ × 8″ × 80 lb. The necessary section properties and the appropriate working stress must be looked up (pages 291 and 289).

(6) Calculate the maximum and minimum compressive stresses in the eccentrically loaded short masonry pillar shown in Fig. 346 (Neglect self-weight of pillar.)

(7) In the case of the eccentrically loaded short joist column given in Fig. 347, show that the compressive stress varies from 4 tons/in.² to zero.

The column-section properties are:

$$Z_{XX} = 63{\cdot}2 \text{ ins.}^3,\ Z_{YY} = 7{\cdot}15 \text{ ins.}^3,\ A = 13{\cdot}59 \text{ sq. ins.}$$

Sketch the stress-variation diagram for the column section.

(8) Fig. 348 shows a short masonry pillar with foundation slab. Taking the conditions given in the figure, calculate the foundation

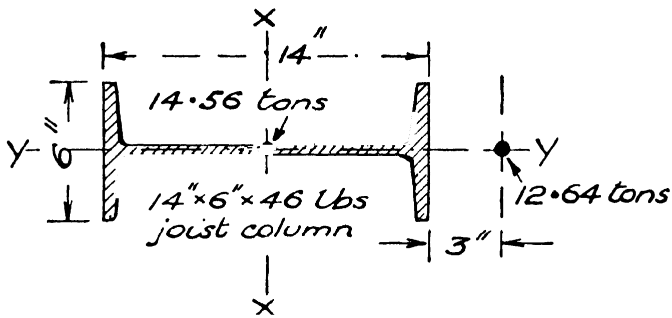


FIG. 347.

pressures at ' A ' and ' B ' respectively. The density of the masonry is 130 lb. per cu. foot.

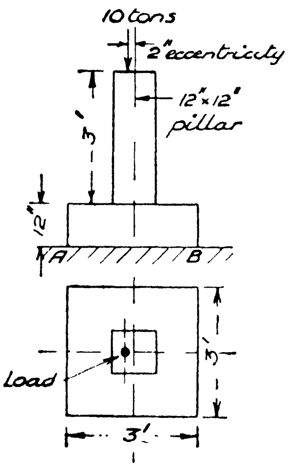


FIG. 348.

(Treat the self-weight of the pillar and base as a concentric load on the foundation.)

(9) A 10" × 8" × 55 lb. joist column has the following properties of section :

Sectional area = 16·18 sq. ins.

Z_{XX} = 57·74 ins.³.

A load is carried on the ' YY ' axis of the section at an eccentricity of x" with respect to the ' XX ' axis. Calculate the maximum value of ' x ' if no tensile stress is to be developed in the column.

(10) Calculate the necessary diameter for a short, circular concrete column which has to support an axial load of 21 tons. The working compressive stress in the concrete is 600 lb./in.².

(i) Assuming ' E ' for the concrete to be 2,000,000 lb./in.², find how much the designed column will shorten on a length of 12".

(ii) Compute the safe load for the column if the load were placed at a distance of 1" from the centre of the column section.

(11) Fig. 349 shows a joint between two adjacent voussoirs in an arch ring. The resultant thrust between the voussoirs is 2000 lb. and is positioned as shown in the diagram. The joint

'AB' is 24″ long and the joint carries the given thrust for 12″ thickness of arch ring. Calculate the maximum compressive stress at the joint.

[Resolve the thrust normally to the joint 'AB.' Treat as an eccentric load, acting on an area of breadth 12″ and depth 24″.]

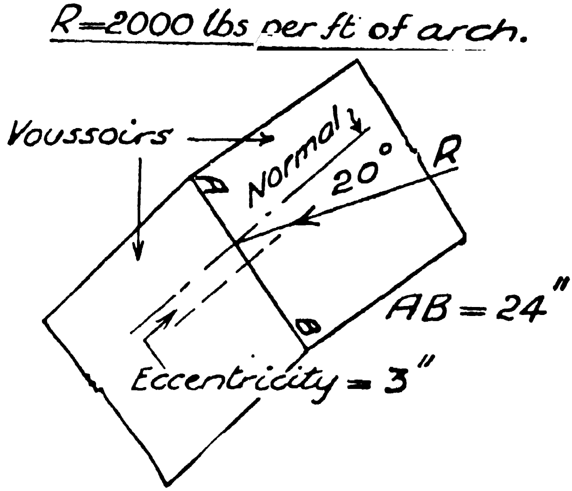


FIG. 349.

CALCULATION METHODS FOR LOADED FRAMES

IN this chapter an introduction will be made to the calculation of the member forces in a loaded frame. The calculation method is sometimes to be preferred to the construction of a stress diagram.

Method of Sections

Consider the truss shown in Fig. 350 to be cut through at the

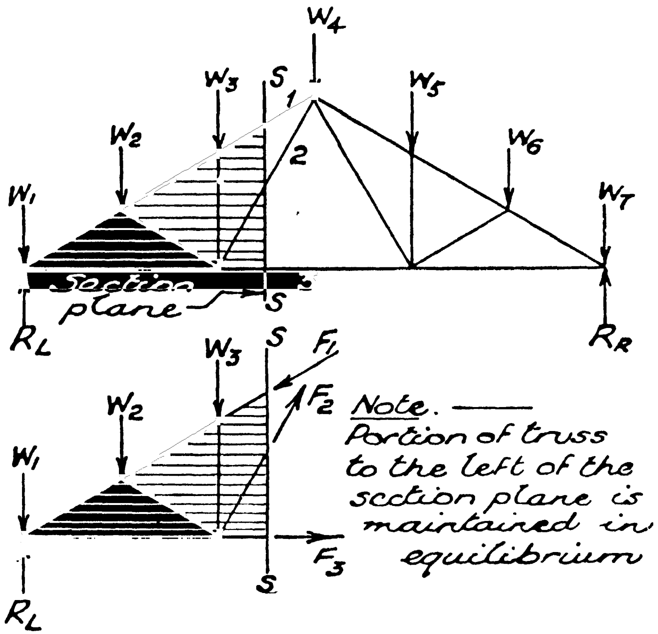


FIG. 350.—METHOD OF SECTIONS.

section plane 'SS,' and the portion to the right of the plane to be removed.

In order to prevent collapse of the truss certain forces 'F_1,' 'F_2,' and 'F_3' will have to be introduced. These forces are assumed to act in the direction of the lengths of the respective members.

In the uncut state the hatched portion of the truss was in equilibrium under the action of the external forces 'R_L,' 'W_1,' 'W_2,' 'W_3,' and forces exerted on it by bars (1), (2), and (3) at the imaginary section plane 'SS.' The forces 'F_1,' 'F_2,' and 'F_3' which we have to substitute for the respective member forces—to maintain equilibrium when the cut is actually made in the truss—will therefore be equal to these member forces.

We may regard the hatched portion of the truss as being a solid body acted upon by seven forces, viz. 'R_L,' 'W_1,' 'W_2,' 'W_3,' 'F_1,' 'F_2,' and 'F_3' in such a way as to maintain equilibrium. The laws of equilibrium may now be employed to deduce the magnitudes of the unknown forces.

The usual method of solution of an example such as that given in Fig. 350 is to employ the '*principle of moments.*' The method of calculation is then sometimes referred to as the '*method of moments.*'

Method of Moments

The section plane 'SS' is positioned to cut the member whose force is required and not more than two others. The plane need not be vertical—it merely indicates the member forces involved in the calculation. No measurements are actually made to any point in the section plane.

Rule.—*To find the force in any given member of the three cut by the plane, take moments about the intersection point of the other two members.*

By this procedure only one unknown member force will appear in the '*moments equation.*'

Struts and Ties.—The determination of '*struts*' and '*ties*' in the calculation method is effected as follows:

Place arrow heads, indicating the assumed sense of the respective member forces, on the force lines. These arrows should be placed on the side of the plane away from the portion of truss being considered, i.e. to the right of the plane in the example shown in Fig. 350. If the chosen arrow-head direction is incorrect in the case of any particular member, the numerical value of the force obtained for that member will be prefixed by a negative sign when the 'moments equation' is solved. The arrow head in such a case must be reversed. If (and after the possible reversal

just indicated) the arrow head point towards the section plane, a thrust is indicated and the member is a '*strut*.' If, on the other hand, the arrow head point away from the plane, thus indicating a 'pull,' the particular member is a '*tie*.' The numerical value of the force in any member is not affected by an incorrect estimation of the proper arrow-head sense.

Note.—Having drawn-in the section plane, and having decided to consider the portion of the truss, say, to the left of the plane, completely ignore all external loads on the truss to the right of the plane.

EXAMPLE (i).—*Calculate the force in each of the members marked* (1), (2), *and* (3) *respectively in the given roof truss* (*Fig.* 351). *State whether the respective members are struts or ties.*

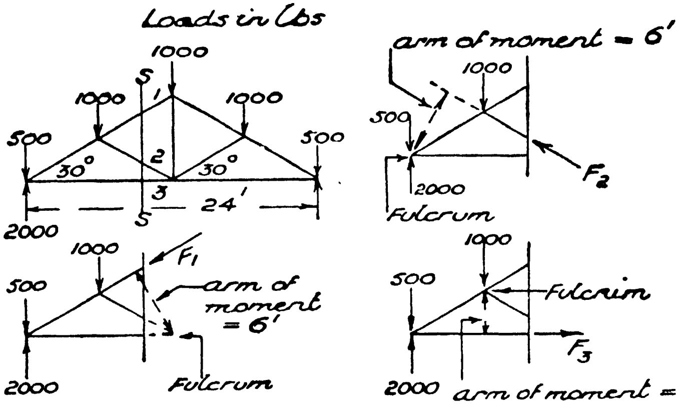


FIG. 351.—CALCULATION OF MEMBER FORCES.

The members given in the question may be solved by introducing a section plane 'SS.'

Member (1).—To find the force in member (1) we have to take moments about the intersection point of members (2) and (3), i.e. the mid-point of the bottom tie (see Fig. 351, bottom left diagram). Only the external loads which act on the portion of truss to the left of the section plane are considered.

$$(F_1 \times 6) + (1000 \times 6) + (500 \times 12) = 2000 \times 12$$
$$6F_1 + 6000 + 6000 = 24000$$
$$6F_1 = 12000. \quad \therefore F_1 = 2000 \text{ lb.}$$

The numerical answer is positive. This indicates that the

arrow-head direction chosen for member (1) is correct. As the member exerts a thrust *on the section plane,* it is a strut.

Member (2).—To find this member force, moments are taken about the intersection point of members (1) and (3), i.e. the left-end reaction point of the truss (see top right-hand diagram).

$$F_2 \times 6 = 1000 \times 6$$
$$\therefore F_2 = 1000 \text{ lb.}$$

The answer is positive, hence the arrow is correct and the member is a '*strut.*'

Member (3).—Moments are taken this time about the mid-point of the left rafter of the truss, this being the point in which member (1) cuts member (2) (see lower right-hand diagram in Fig. 351).

$$(F_3 \times 3{\cdot}46) + (500 \times 6) = 2000 \times 6$$
$$3{\cdot}46F_3 = 9000. \quad \therefore F_3 = 2600 \text{ lb.}$$

Member (3) is a tie.

The '*arms*' of the respective moments, in the foregoing calculations, may be obtained by direct measurement on a scale drawing of the truss or by trigonometrical calculation. The results which would be obtained by drawing a stress diagram for the truss would, of course, agree with those obtained by the calculation method shown.

The '*method of sections*' may be used to find the forces in certain specified members of a frame without the trouble of having to construct a stress diagram. The calculation method may be used to check and confirm a member-force value obtained by a graphical method.

In some cases it is not possible to complete a stress diagram by the usual procedure. Completion may often be effected by calculating one member of the frame, employing a suitable 'section plane' (see page 145).

EXAMPLE (ii).—*Calculate the force in the member marked by a cross in the truss given in Fig.* 352.

'SS' is a section plane cutting the given member and two others. The two latter members intersect at the left-end reaction point of the truss, hence this is the point about which moments must be taken.

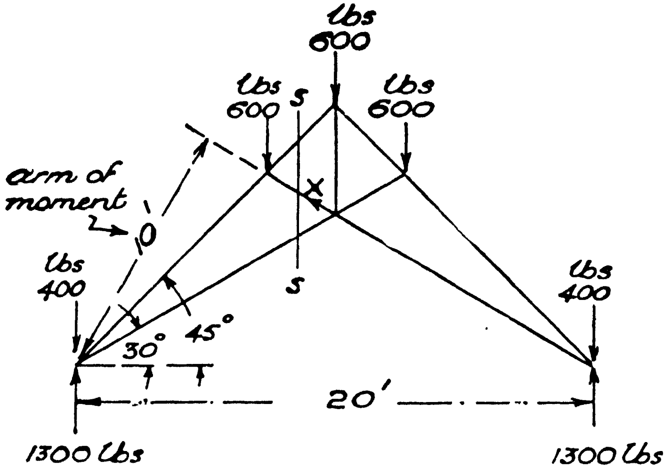


FIG. 352.—CALCULATION OF ONE MEMBER.

Let 'F' lb. = force in the given member.

The 'arm' of the moment = 10 ft.

$\therefore$ F $\times$ 10 = 600 $\times$ 7·32 ft.

[7·32 ft. is the horizontal distance from the left-end reaction point to the vertical line of action of the 600-lb. force.]

$\therefore$ F = 439·2 lb.

The arrow head being correct, the member is a strut.

The reader may check the accuracy of the answer by drawing a stress diagram for the truss.

EXAMPLE (iii).—*Obtain the forces in members marked 'C' and 'D' respectively in the cantilever-truss example given in Fig.* 353

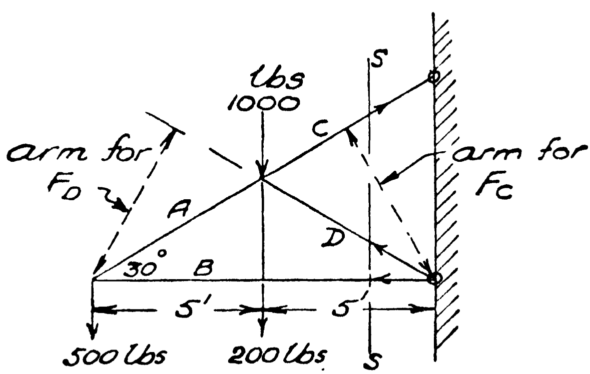


FIG. 353.—CANTILEVER TRUSS.

The 'method of sections' must be used. Find also, by any calculation method, the forces in members 'A' and 'B.'

Member 'C':

Section plane 'SS' cuts member 'C' and two others. The two other members intersect at the lower reaction point.

Taking moments about this point, the force in member 'C' has an 'arm' of (10 sin 30°) ft. = (10 × ½) ft. = 5 ft.

$$\therefore (F_C \times 5) = (1000 \times 5) + (200 \times 5) + (500 \times 10)$$
$$5F_C = 5000 + 1000 + 5000$$
$$= 11000$$
$$F_C = 2200 \text{ lb.}$$

The arrow head has been correctly placed on member 'C' as the answer is positive, hence member 'C' is a tie.

Member 'D':

In this case the two remaining members intersect at the extreme left point of the truss.

$$\text{The arm for '}F_D\text{'} = 10 \sin 30° = 5 \text{ ft.}$$
$$\therefore F_D \times 5 = (1000 \times 5) + (200 \times 5)$$
$$\therefore F_D = 1200 \text{ lb.}$$

Member 'D' is a strut.

Member 'A':

'F_A' is easily found by vertical resolution of the forces acting at the left end of the truss.

$$F_A \cos 60° = 500$$
$$\therefore F_A = \frac{500}{\cos 60°} = \frac{500}{\frac{1}{2}} = 1000 \text{ lb.}$$

Member 'A' is clearly a 'tie' as it must have an upward vertical component at the bottom end.

Member 'B.'—By horizontal resolution:

$$F_A \cos 30° = F_B$$
$$\therefore F_B = 1000 \times \cdot 866 = 866 \text{ lb.}$$

Member 'B' is a strut, as it clearly must push to the left to balance the effect 'F_A' has towards the right.

Member 'A.'—We could have found 'F_A' by a '*moments*' method. Imagine a vertical section plane cutting members 'A' and 'B.' If a 'fulcrum' be chosen on member 'B,' this 'mem-

ber force' will vanish from the 'moments equation.' A convenient point is the mid-point of the bottom member of the frame. The 'arm' of the moment for 'F_A' will be 2·5 ft. (= 5 sin 30°).

$$\therefore F_A \times 2{\cdot}5 = 500 \times 5$$
$$\therefore F_A = 1000 \text{ lb.}$$

Similarly 'F_B' could be found by taking moments about any convenient point on member 'A.'

Braced Girders with Parallel Flanges

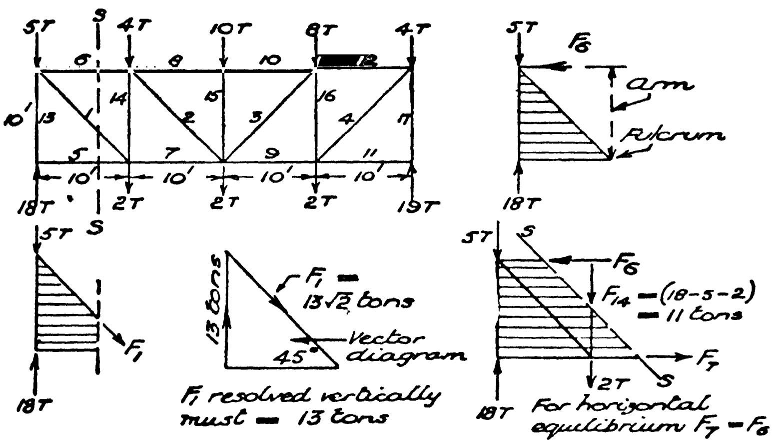


FIG. 354.—SOLUTION OF A BRACED GIRDER.

EXAMPLE.—*Calculate the force in each member of the braced girder shown in Fig.* 354.

The first step is to number the members in the order in which they are going to be solved. We will calculate the members in the following order: (i) *diagonal members,* (ii) *horizontal members,* (iii) *vertical members.*

$$R_L \times 40 = (5 \times 40) + (6 \times 30) + (12 \times 20) + (10 \times 10)$$
$$R_L = 18 \text{ tons.}$$
$$R_R \times 40 = (4 \times 40) + (10 \times 30) + (12 \times 20) + (6 \times 10)$$
$$R_R = 19 \text{ tons.}$$

Diagonal Bars

Bar (1).—Imagine a section plane 'SS' cutting bar (1). The portion of the girder to the left of the section plane is in *vertical*

equilibrium under the action of (i) the reaction (18 tons) vertically upwards, (ii) the external load (5 tons) vertically downwards, and (iii) the *vertical effect* of the force in bar (1) (see Fig. 354, bottom left diagrams). The total external load which bar (1), *when resolved vertically*, has to balance is (18 — 5) tons = 13 tons, i.e. the *shear force* in the panel in which bar (1) is the diagonal. The force in bar (1) must be more than '13 tons,' so that when resolved vertically it equals the necessary '13 tons.' From the vector diagram, $\frac{13 \text{ tons}}{F_1} = \sin 45°$

$$\therefore F_1 = \frac{13 \text{ tons}}{\sin 45°} = 13\sqrt{2} \text{ tons.} \quad \left(\text{Sin } 45° = \frac{1}{\sqrt{2}}.\right)$$

We may term '$\sqrt{2}$' in this case the '*reduction coefficient*' for reducing '*shear force values*' to corresponding '*inclined bar forces*.' The coefficient depends simply upon the slope of the inclined bar. If the inclination be 'θ' to the horizontal, the reduction coefficient $= \frac{1}{\sin \theta}$. If the girder dimensions are given, the reduction coefficient $= \frac{\text{Length of diagonal}}{\text{Height of truss}}$.

Rule.—*To find the force in any inclined member, multiply the shear force in the corresponding panel by the appropriate reduction coefficient.*

Bar 1. $F_1 = \text{S.F.} \times \sqrt{2} = (18 - 5)\sqrt{2}$ tons $= 13\sqrt{2}$ tons (tie).

„ 2. $F_2 = \text{S.F.} \times \sqrt{2} = (18 - 5 - 4 - 2)\sqrt{2}$ tons $= 7\sqrt{2}$ tons (tie).

„ 3. $F_3 = \text{S.F.} \times \sqrt{2} = (19 - 4 - 8 - 2)\sqrt{2}$ tons $= 5\sqrt{2}$ tons (tie).

„ 4. $F_4 = \text{S.F.} \times \sqrt{2} = (19 - 4)\sqrt{2}$ tons $= 15\sqrt{2}$ tons (tie).

Horizontal Bars

Fig. 354 (top right-hand diagram) shows what 'bar 6' does to prevent collapse of the truss. 'Bar 6' must exert sufficient thrust to prevent the girder failing by turning about the opposite joint in the lower flange. The joint concerned is the one in

which the diagonal in the panel meets the lower flange of the girder. For equilibrium of the hatched portion of truss :

$$(F_6 \times 10) + (5 \times 10) = 18 \times 10$$
$$F_6 + 5 = 18$$
$$F_6 = 13 \text{ tons.}$$

' Bar 6 ' is obviously a strut.

To solve ' *bar 7*,' moments are taken about the opposite joint in the top flange, i.e. where the ' 4 tons ' load acts. The reader must get accustomed to ' visualising ' the portion of truss concerned in the ' moments ' equation without having to draw a number of separate diagrams. Until familiarity with the method is acquired, the necessary sketch diagrams had better be constructed.

Bar 5. As there is no other horizontal force at the left-end reaction point, there cannot be any force in bar 5, i.e. $F_5 = 0$.

Bar 6. $F_6 = 13$ tons (as above). Strut.

Bar 7. $(F_7 \times 10) + (5 \times 10) = 18 \times 10$

$F_7 = 13$ tons. Tie. [' Bar 7 ' must *pull.*]

Bar 8. $(F_8 \times 10) + (4 \times 10) + (2 \times 10) + (5 \times 20) = 18 \times 20$

$$F_8 + 4 + 2 + 10 = 36$$
$$F_8 = 20 \text{ tons.}$$

' Bar 8 ' is a strut.

Bar 9. $(F_9 \times 10) + (4 \times 10) = 19 \times 10.$

[Note the ' 8 tons ' and ' 2 tons ' pass through the fulcrum.]

$F_9 + 4 = 19$

$\therefore F_9 = 15$ tons. ' Bar 9 ' is a tie.

Bar 10. $F_{10} = F_8$ by horizontal equilibrium requirement at top of ' bar 15.' Bar 10 is a strut. Force = 20 tons.

Bar 11. $F_{11} = 0$ (as for ' bar 5 ').

Bar 12. $(F_{12} \times 10) + (4 \times 10) = 19 \times 10$

$$F_{12} + 4 = 19$$

$F_{12} = 15$ tons. ' Bar 12 ' is a strut.

Vertical Members

The vertical members are solved by considering vertical equilibrium at one end of the member. Having decided which

end to take, ***do not pay any attention to any external loads which may act at the other end of the member.***

Bar 13. Consider bottom end.

$F_{13} = 18$ tons (strut).

Bar 14. Bottom end.

$F_1 = 13\sqrt{2}$ tons in the direction of the length of the bar. Its vertical upward component $= 13$ tons (i.e. the S.F. in the panel).

∴ Bar 14 must push downwards with a force of ' 11 tons,' so that with the ' 2 tons ' external load the total downward force = 13 tons.

$\therefore F_{14} = 11$ tons (strut).

Bar 15. Top end.

$F_{15} = 10$ tons (strut).

Bar 16. Bottom end.

$F_{16} + 2 = 15$ (the upward pull in ' bar 4 ').

$F_{16} = 13$ tons (strut).

Bar 17. Bottom end.

$F_{17} = 19$ tons (strut).

Some girder types lend themselves to simple solutions (of certain members) by taking special section planes.

Fig. 354 (bottom right-hand diagram) shows a section plane cutting bars ' 6,' ' 14,' and ' 7.'

Considering the horizontal equilibrium of the hatched portion, the only two horizontal forces are ' F_6' and ' F_7.' ' F_7' must therefore equal ' F_6' in magnitude. Note that if member ' 14 ' were not vertical we could not express this equality.

Acting vertically on the portion of truss considered, there are four forces.

Equating the force acting upwards to those acting downwards :

$$F_{14} + 5 + 2 = 18$$
$$\therefore F_{14} = 11 \text{ tons.}$$

Clearly, member ' 14 ' is a strut.

It is possible, thus, to write down the forces in several members of the frame by inspection. In examinations the candidate should clearly state the principle upon which the solution of a given member is based.

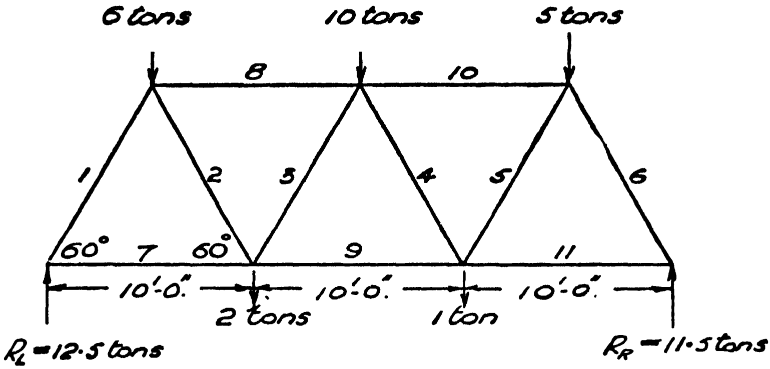


FIG. 355.—WARREN GIRDER.

EXAMPLE (ii).—*Calculate the force in each bar of the Warren girder (Fig. 355).*

$$R_L \times 30 = (6 \times 25) + (2 \times 20) + (10 \times 15) + (1 \times 10) + (5 \times 5)$$
$$= 375$$
$$R_L = 12{\cdot}5 \text{ tons.}$$
$$R_R \times 30 = (5 \times 25) + (1 \times 20) + (10 \times 15) + (2 \times 10) + (6 \times 5)$$
$$= 345$$
$$R_R = 11{\cdot}5 \text{ tons.}$$

Inclined Members

$$\text{Reduction coefficient} = \frac{\text{Length of inclined member}}{\text{Height of truss}}$$
$$= \frac{1}{\sin 60^\circ} = 1{\cdot}155.$$

Bar 1. $F_1 = \text{S.F.} \times 1{\cdot}155$
$= (12{\cdot}5 \times 1{\cdot}155)$ tons $= 14{\cdot}44$ tons (strut).

Bar 2. $F_2 = \text{S.F.} \times 1{\cdot}155$
$= (12{\cdot}5 - 6) \times 1{\cdot}155$ tons
$= 7{\cdot}51$ tons (tie).

Bar 3. $F_3 = \text{S.F.} \times 1{\cdot}155$
$= (12{\cdot}5 - 6 - 2) \times 1{\cdot}155$
$= (4{\cdot}5 \times 1{\cdot}155)$ tons
$= 5{\cdot}20$ tons (strut).

Bar 4. (Taking forces from right end.)

$F_4 = \text{S.F.} \times 1{\cdot}155$
$= (11{\cdot}5 - 5 - 1) \times 1{\cdot}155$
$= 6{\cdot}35$ tons (strut).

Bar 5. $F_5 = \text{S.F.} \times 1{\cdot}155$
$= (11{\cdot}5 - 5) \times 1{\cdot}155$
$= (6{\cdot}5 \times 1{\cdot}155)$ tons
$= 7{\cdot}51$ tons (tie).

Bar 6. $F_6 = \text{S.F.} \times 1{\cdot}155$
$= 11{\cdot}5 \times 1{\cdot}155$
$= 13{\cdot}28$ tons (strut).

Horizontal Bars

Height of truss $= 5 \tan 60° = 8{\cdot}66$ ft.

Bar 7. $F_7 \times 8{\cdot}66 = 12{\cdot}5 \times 5$ [Moments about top-flange joint where the 6 tons load acts.]
$F_7 = 7{\cdot}22$ tons (tie).

Bar 8. $(F_8 \times 8{\cdot}66) + (6 \times 5) = (12{\cdot}5 \times 10)$
$8{\cdot}66F_8 = 95$
$F_8 = 10{\cdot}97$ tons (strut).

Bar 9. $(F_9 \times 8{\cdot}66) + (2 \times 5) + (6 \times 10) = (12{\cdot}5 \times 15)$
$8{\cdot}66F_9 + 10 + 60 = 187{\cdot}5$
$F_9 = \frac{117{\cdot}5}{8{\cdot}66} = 13{\cdot}57$ tons (tie).

Bar 10. $(F_{10} \times 8{\cdot}66) + (5 \times 5) = 11{\cdot}5 \times 10$
$8{\cdot}66F_{10} = 90$
$F_{10} = 10{\cdot}4$ tons (strut).

Bar 11. $F_{11} \times 8{\cdot}66 = 11{\cdot}5 \times 5$
$F_{11} = 6{\cdot}64$ tons (tie).

EXAMPLE (iii).—*Calculate the force in each of the bars marked 'A,' 'B,' 'C,' and 'D' in the loaded frame given in Fig.* 356.

The inclination of the sloping members is not given in degrees in this example. The length of each diagonal member $= \sqrt{3^2 + 4^2} = 5$ ft.

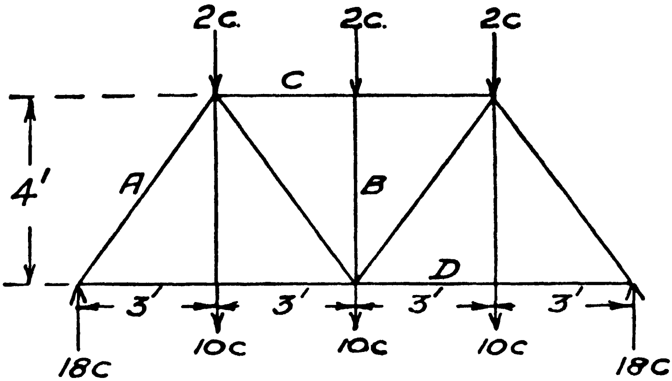


FIG. 356.—SYMMETRICALLY LOADED FRAME.

Bar A :

$$\text{Reduction coefficient} = \frac{\text{Length of diagonal}}{\text{Height of girder}}$$

$$= \frac{5 \text{ ft.}}{4 \text{ ft.}} = 1{\cdot}25.$$

$F_A = \text{S.F.} \times 1{\cdot}25 = (18 \times 1{\cdot}25) \text{ cwts.} = 22{\cdot}5 \text{ cwts.}$

If we imagine a section plane vertically through member 'A' and consider the equilibrium of the girder to left of the plane, it will be clear that member 'A' must be pushing on the section plane, hence it is a strut.

In the usual case of a braced girder, i.e. one in which the loads are not very unsymmetrically disposed, it will be found that diagonal members sloping downwards towards the centre of the girder are 'ties' and those upwards towards the centre are 'struts.' When a girder has to carry a live load which passes on to and off the girder, some diagonal members may be called upon to act as both '*struts*' and '*ties*'—according to the position of the load.

Bar B :

Considering equilibrium at the top end of the bar, $F_B = 2$ cwts. The bar is a strut. (Do not worry about the bottom end.)

Bar C :

Taking moments about the mid-point of bottom flange :

$$(F_C \times 4) + (2 \times 3) + (10 \times 3) = 18 \times 6$$
$$4F_C + 6 + 30 = 108$$
$$4F_C = 72$$
$$F_C = 18 \text{ cwts.}$$

In the above 'moments equation' bar 'C' was assumed to be exerting a thrust on the left portion of the frame. Hence this bar is a strut. When a truss is supported at its ends, as in this example, all the top flange members are struts.

Bar D:

To solve this bar, moments are taken about the top right-hand joint of the frame. Assuming bar 'D' to be pulling on the portion of the truss to the right:

$$F_D \times 4 = 18 \times 3$$

[The '2-cwt.' and '10-cwt.' loads pass through the chosen fulcrum.]

$$\therefore F_D = 54/4 = 13{\cdot}5 \text{ cwts.}$$

'Bar D' is a tie.

All bottom flange members will be ties when the frame is supported as in the example.

EXAMPLE (iv).—*Fig.* 357 *shows a vertical frame carrying wind loads. Determine the nature and the magnitude of the forces in the respective members* (i) *by calculation method,* (ii) *by the construction*

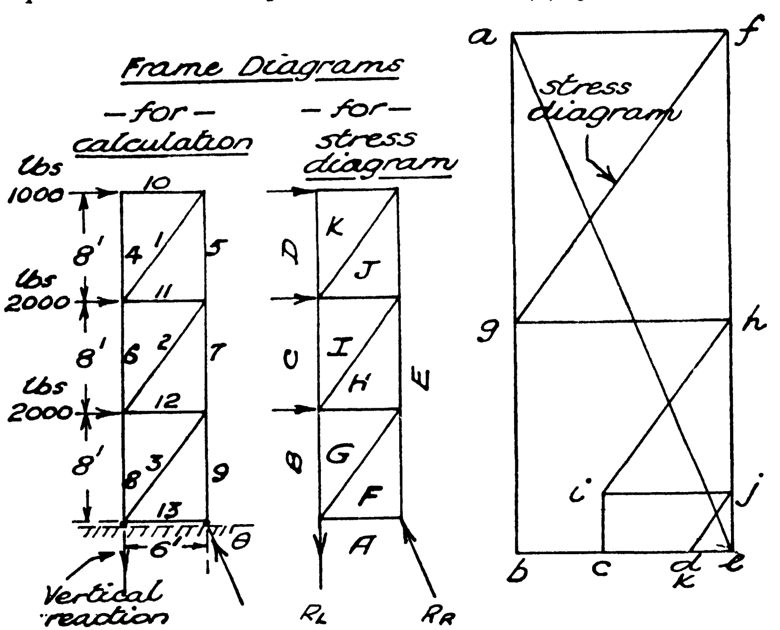


FIG. 357.—VERTICAL FRAME WITH WIND LOADS.

of a stress diagram. Assuming the left-end reaction of the frame to be vertical, determine the frame reactions.

Only a few members will be solved—the remainder are left as an exercise for the reader. The accuracy of the answers may be checked by means of the stress diagram.

Member 1 *:*

This is a 'diagonal' member and a shear-force 'reduction coefficient' is required.

$$\text{Reduction coefficient} = \frac{\text{Length of diagonal}}{6 \text{ ft.}}$$

$$= \frac{\sqrt{6^2 + 8^2}}{6} = \frac{10}{6} = 1\tfrac{2}{3}.$$

$$F_1 = \text{S.F.} \times \text{Reduction coefficient}$$

$$= (1000 \times 1\tfrac{2}{3}) \text{ lb.} = 1667 \text{ lb.}$$

Imagine a section plane to be taken horizontally through 'member 1.' Considering the portion of truss above the plane to be in equilibrium and acted upon by 'member 1' *from below the plane*, it is clear the member must be *pulling* on the plane so that it may have a component to the left to balance the '1000 lb.' external load. Hence 'F_1' is a tie.

Member 6 *:*

To solve this member moments are taken about the joint in which members '5' and '7' meet. Member '6' must pull on the top panel to prevent collapse about the stated fulcrum, hence it is a 'tie.'

$$F_6 \times 6 = 1000 \times 8$$

[The '2000-lb.' load passes through the fulcrum.]

$$\therefore F_6 = \frac{8000}{6} \text{ lb.} = 1333 \text{ lb.}$$

Member 12 *:*

This member could be solved by considering horizontal equilibrium at, say, its left end.

It is readily solved by considering a section plane cutting members '8,' '12' and '7' (inclined at about 60° to the horizontal).

Considering horizontal equilibrium of the portion of frame above the section plane :

$$F_{12} = (2000 + 2000 + 1000) \text{ lb.} = 5000 \text{ lb.}$$

'Member 12' is a strut as it has to *push* to the left against the section plane.

Member 3:

$$F_3 = \text{S.F.} \times \text{reduction coefficient}$$
$$= (2000 + 2000 + 1000) \times 1\tfrac{2}{3} \text{ lb.}$$
$$= (5000 \times 1\tfrac{2}{3}) \text{ lb.} = 8333 \text{ lb.}$$

'Member 3' is a tie.

Bar 13:

Consider horizontal equilibrium at the left end of the bar.

F_{13} = 'F_3' resolved horizontally, as the reaction and member '8' are vertical.

$$F_{13} = F_3 \times 6/10$$
$$= \frac{8333 \times 6}{10} \text{ lb.} = 5000 \text{ lb.}$$

'Member 13' is a strut.

Determination of Reactions

To find 'R_L' take moments about the right reaction point.

$$R_L \times 6 = (2000 \times 8) + (2000 \times 16) + (1000 \times 24)$$
$$= 16000 + 32000 + 24000 = 72000$$
$$\therefore R_L = 12000 \text{ lb.}$$

If 'V' and 'H' be the vertical and horizontal components respectively of the right-end reaction:

$$V = R_L = 12000 \text{ lb.}$$
$$H = (2000 + 2000 + 1000) \text{ lb.} = 5000 \text{ lb.}$$
$$R_R^2 = V^2 + H^2 = 12000^2 + 5000^2 = 169{,}000{,}000$$
$$\therefore R_R = 13000 \text{ lb.}$$
$$\tan\theta = V/H = \frac{12000}{5000} = 2{\cdot}4$$
$$\theta = 67^\circ\ 24'.$$

We may now check 'member 13' by considering equilibrium at its right end.

$$F_{13} = R_R \times \cos\theta$$
$$= (13000 \times \cos 67^\circ\ 24')$$
$$= 13000 \times {\cdot}3843$$
$$= 5000 \text{ lb.}$$

Construction of Stress Diagram

Having drawn the load line '*bcde,*' we note that member 'DK' has no force in it, hence points '*d*' and '*k*' are coincident. The stress diagram is then completed in the manner indicated on page 144 for the cantilever-truss example.

Three Concurrent Member Forces in Equilibrium

In the graphical method, the vector lines in a force diagram are scaled in order to obtain the required force values. Thus we scale the '*diagonal*' in a parallelogram of forces example. As explained on page 13, we could express the value of the diagonal in trigonometrical terms. Trigonometry may be usefully employed in problems involving three concurrent forces in equilibrium.

Fig. 358 shows a roof truss carrying a vertical load 'W' at the

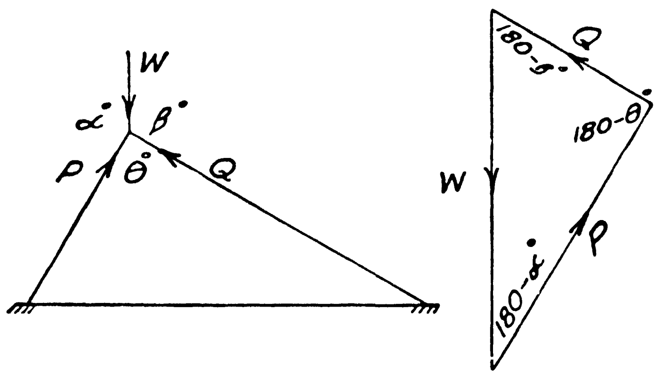


FIG. 358 —LAMI'S THEOREM.

apex. The corresponding rafter forces are 'P' and 'Q.' 'W,' 'P' and 'Q' form a '*triangle of forces.*' Assuming the angles 'α,' 'β' and 'θ' as indicated, the angles in the triangle of forces will be the supplements of these angles, at the respective corners shown. Expressing the *sine rule,* used in the trigonometrical solution of triangles, we have :

$$\frac{W}{\sin (180 - \theta)^\circ} = \frac{P}{\sin (180 - \beta)^\circ} = \frac{Q}{\sin (180 - \alpha)^\circ}.$$

But the sine of an angle = the sine of its supplement

$$\therefore \frac{W}{\sin \theta^\circ} = \frac{P}{\sin \beta^\circ} = \frac{Q}{\sin \alpha^\circ}.$$

Each force is therefore proportional to the sine of the angle between the other two. This theorem is known as '*Lami's theorem.*'

EXAMPLES :

(i) *A weight of* 100 *lb. is suspended by two chains in the manner shown in Fig.* 359. *Calculate the pull in each chain.*

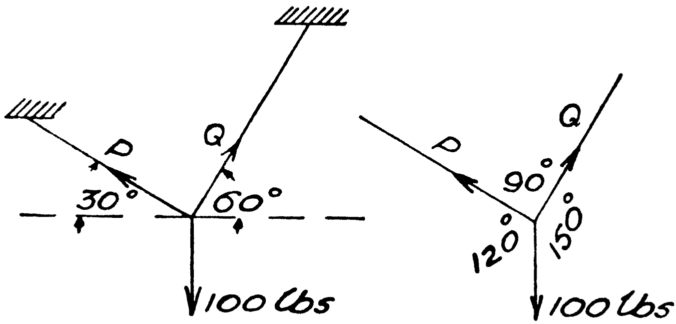


FIG. 359.—THREE CONCURRENT FORCES.

Let 'P' and 'Q' be the pulls in the left and right chains respectively.

$$\frac{100}{\sin 90^\circ} = \frac{P}{\sin 150^\circ} = \frac{Q}{\sin 120^\circ}$$

$$\therefore P = \frac{100 \times \sin 150^\circ}{\sin 90^\circ} = \frac{100 \times \sin 30^\circ}{1} = (100 \times \cdot 5) \text{ lb.} = 50 \text{ lb.}$$

$$Q = \frac{100 \times \sin 120^\circ}{\sin 90^\circ} = \frac{100 \sin 60^\circ}{1} = (100 \times \cdot 866) \text{ lb.} = 86\cdot 6 \text{ lb.}$$

[Note : $\sin 150^\circ = \sin (180^\circ - 150^\circ) = \sin 30^\circ$.]

(ii) *Calculate the pull in the tie and the thrust in the rafter in the example given in Fig.* 360.

Let 'S' = the thrust in the rafter and 'T' = the pull in the tie.

It will be helpful to visualise (or actually draw out) the three forces as either all acting towards, or all acting from, the common

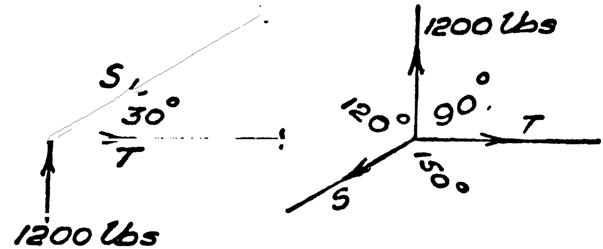


FIG. 360.—FORCES AT END OF TRUSS.

point of concurrence. An error in the assumed arrow-head directions will then be apparent.

$$\frac{1200}{\sin 150°} = \frac{T}{\sin 120°} = \frac{S}{\sin 90°}$$

$$\therefore T = \frac{1200 \sin 120°}{\sin 150°} = \frac{1200 \sin 60°}{\sin 30°} = \frac{1200 \times \cdot 866}{\cdot 5} \text{ lb.} = 2078 \text{ lb.}$$

$$S = \frac{1200 \sin 90°}{\sin 150°} = \frac{1200 \times 1}{\sin 30°} = \frac{1200}{\cdot 5} \text{ lb.} = 2400 \text{ lb.}$$

The two examples given could have been easily solved by resolution method.

Resolving vertically in the latter example:

$$S \cos 60° = 1200$$

$$\therefore S = \frac{1200}{\cos 60°} = \frac{1200}{\cdot 5} \text{ lb.} = 2400 \text{ lb.}$$

Resolving horizontally:

$$T = S \cos 30° = (2400 \times \cdot 866) \text{ lb.} = 2078 \text{ lb.}$$

Lami's theorem is useful when the resolution is not so simply effected as in this example.

(iii) *Calculate the pull in the tie and the thrust in the jib in the simple jib-crane example shown in Fig.* 361. *The resultant pull on the jib head is inclined to the vertical.*

Let 'S' = thrust in the jib and 'T' = pull in the tie.

$$\frac{S}{\sin 82\frac{1}{2}°} = \frac{T}{\sin 157\frac{1}{2}°} = \frac{2}{\sin 120°}$$

$$\therefore S = \frac{2 \sin 82\frac{1}{2}°}{\sin 120°} = \frac{2 \sin 82\frac{1}{2}°}{\sin 60°} = \frac{2 \times \cdot 9914}{\cdot 866} \text{ lb.} = 2 \cdot 29 \text{ cwts.}$$

$$T = \frac{2 \sin 157\frac{1}{2}°}{\sin 120°} = \frac{2 \sin 22\frac{1}{2}°}{\sin 60°} = \frac{2 \times \cdot 3827}{\cdot 866} \text{ lb.} = \cdot 884 \text{ cwts}$$

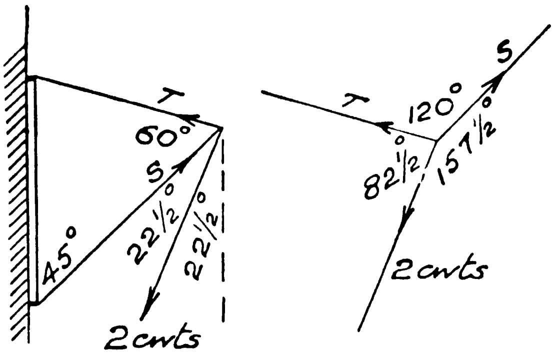


FIG 361.—SOLUTION BY LAMI'S THEOREM

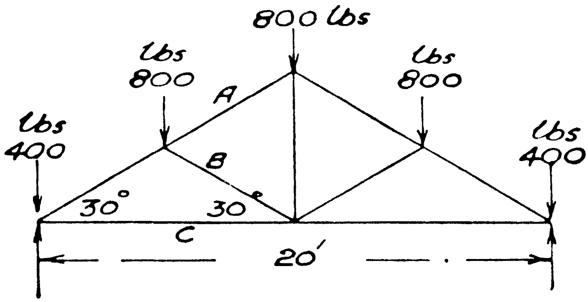


FIG 362.

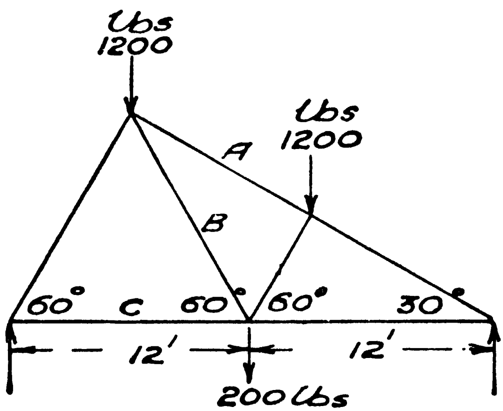


FIG. 363.

Exercises 14

(1) Using the method of sections obtain the force in each of the members marked 'A,' 'B,' and 'C' in the roof-truss example shown in Fig. 362. Distinguish between struts and ties.

(2) Draw a section plane cutting members 'A,' 'B,' and 'C' in the truss given in Fig. 363. Calculate the forces in these members respectively by considering the equilibrium of the portion of the truss: (i) to the left, (ii) to the right, of the section plane.

(3) Find the force in each member of the loaded frame shown

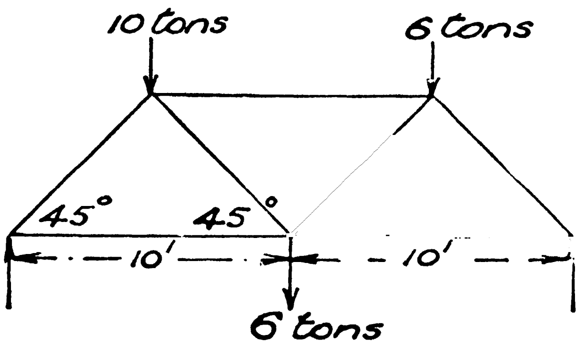


FIG. 364.

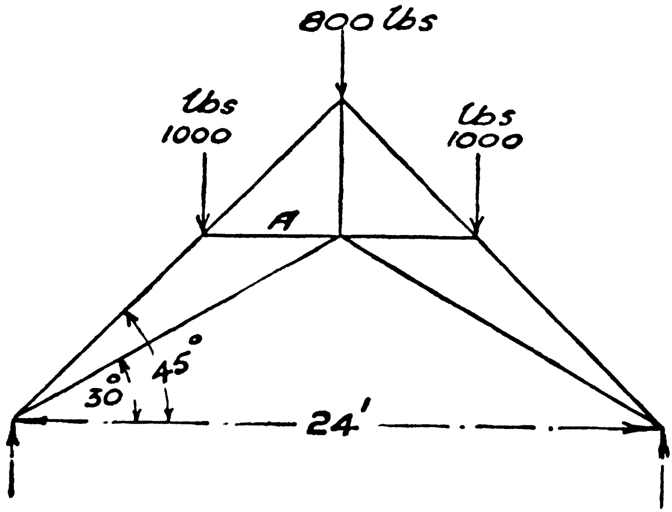


FIG. 365.

in Fig. 364. State whether the respective members are struts or ties. Check the calculated values by the construction of a stress diagram.

(4) Obtain, by the method of sections, the force in member 'A' of the truss given in Fig. 365. State whether the member is in tension or compression.

(5) Fig. 366 shows a braced girder with symmetrical loading. Calculate the force in each member and state whether 'strut' or 'tie.' Draw up a table of the results.

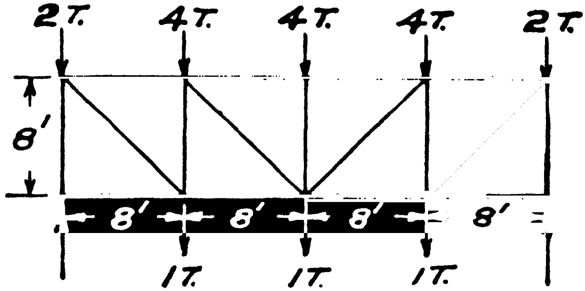


FIG. 366.

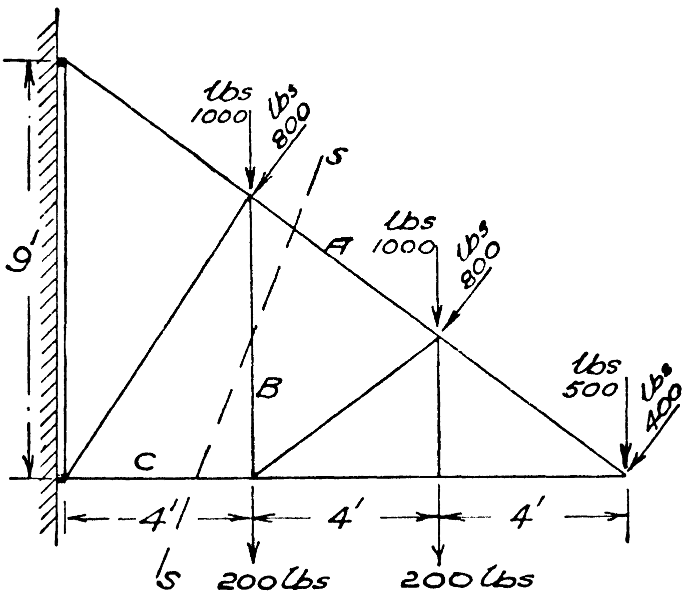


FIG. 367.

(6) The cantilever truss given in Fig. 367 carries wind and dead loads. Using the method of sections, find the forces in members ' A,' ' B,' and ' C ' respectively.

[' SS ' represents a suitable section plane. There will be no necessity to compound the dead and wind loads. The moment of each load (about the appropriate fulcrum) may be taken independently.]

(7) Using Lami's theorem, calculate the force in each of the rafters of the truss given in Fig. 368. Verify the results by drawing a triangle of forces.

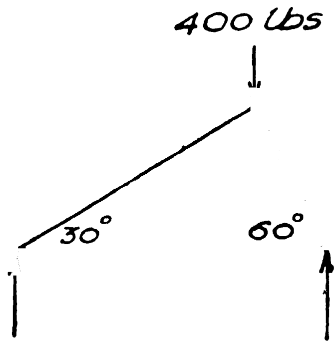


FIG. 368.

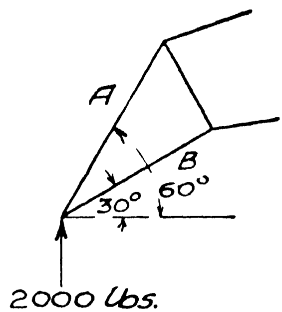


FIG. 369.

(8) Calculate the forces in members ' A ' and ' B ' respectively, in the example given in Fig. 369. Use Lami's theorem. Verify, by resolving vertically and horizontally, that the calculated values are correct.

CHAPTER XV

GRAVITY RETAINING WALLS

THE pressure which a wall has to sustain may be caused by wind, by retained liquid, or by retained earth or other granular material.

In calculations and formulæ concerned with the resultant thrust of retained material, it is usual to consider 1-ft. length (in plan) of the wall. All the forces involved in the stability of a typical retaining wall, i.e. one whose vertical section remains constant throughout its length, are directly proportional to the wall length. It is convenient, therefore, to take 1-ft. length as a basis for numerical computations.

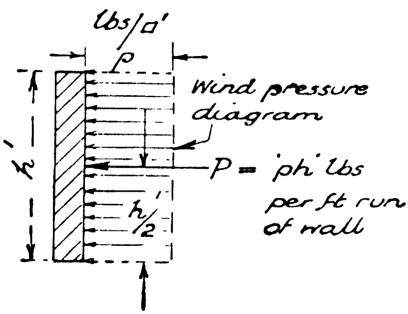


FIG. 370.—WIND PRESSURE.

Wind Pressure

Let 'h' ft. = Height of wall (Fig. 370).

'p' lb. per sq. ft. = Uniform wind pressure.

'P' lb. = Resultant thrust per foot length of wall.

$\therefore$ P $= p \times (h \times 1) = ph$ lb.

The wind-pressure diagram being a rectangle, the resultant thrust will act at $\frac{h}{2}$ ft. from the bottom of the wall.

Overturning moment about base of wall = Force $\times$ arm

$$= \mathrm{P} \times \frac{h}{2} = \frac{ph^2}{2} \text{ lb. ft.}$$

EXAMPLE.—*A vertical wall,* 6 *ft. high, is subjected to a wind pressure of uniform intensity,* 30 *lb. per sq. foot. Calculate, per*

foot run of wall, (i) *the resultant wind thrust,* (ii) *the overturning moment about wall base.*

(i) $P = ph = 30 \times 6 = 180$ lb.

(ii) Overturning moment $= P \times \frac{h}{2} = 180 \text{ lb.} \times \frac{6}{2} \text{ ft.}$

$= 180 \text{ lb.} \times 3 \text{ ft.} = 540 \text{ lb. ft.}$

Liquid Pressure

The following is a brief summary of some of the properties of liquid pressure which will be involved in the type of problem considered in this chapter. The liquid in each case is assumed to be at rest.

(i) The intensity of pressure at a given point in a liquid is the same in all directions. For example, at a given depth the pressure is the same horizontally as it is vertically.

(ii) The intensity of pressure increases uniformly with the depth. The pressure-variation diagram will therefore be a straight line (see Fig. 371).

(iii) The value of the pressure intensity at a given point in a liquid will be given by the formula :

Pressure = Density of liquid × Depth of point.

Let 'p' lb. per sq. foot = Intensity of pressure.
'w' lb. per cu. foot = Density of the liquid.
'h' ft. = Depth of point considered.

$\therefore p = wh$ lb. per sq. foot.

Thus at a point 10 ft. deep in water of density 62·5 lb. per cu. foot the pressure intensity would be $(62{\cdot}5 \times 10) = 625$ lb. per sq. foot.

(iv) The direction of pressure of a retained liquid against the retaining surface is at right angles to that surface. If a retaining wall have a vertical back the resultant liquid thrust will be horizontal.

Magnitude of Resultant Liquid Thrust

Case (i).—**Retaining Wall with Vertical Back.**—In Fig. 371, the area against which the liquid is pressing per foot run of wall = h ft. × 1 ft. = h sq. ft. The pressure intensity varies from zero, at the free surface of the liquid, to a value 'wh' lb. per sq. foot

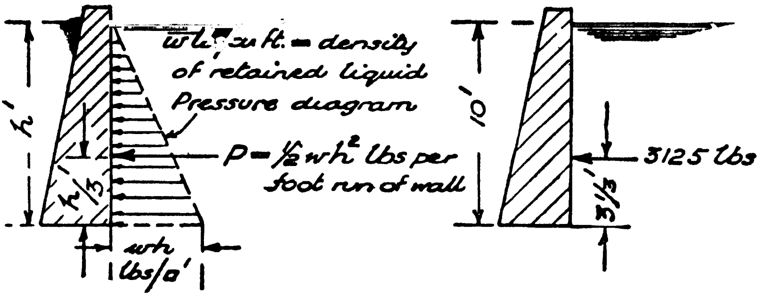


FIG. 371 —LIQUID PRESSURE.

at the bottom of the wall. The pressure varies uniformly and the average pressure over the total area ' h ' sq. ft. will be $\frac{wh}{2}$ lb. per sq. foot.

The total thrust per foot length of wall

$$= \text{Average pressure} \times \text{Area under pressure}$$
$$= \left(\frac{wh}{2} \times h\right) \text{lb.}$$

If ' P ' represents this resultant thrust,

$$P = \tfrac{1}{2}wh^2 \text{ lb.}$$

Position of the resultant thrust :

The resultant liquid thrust will act horizontally through the centre of gravity of the pressure-variation diagram. It will therefore act at *one-third* the depth of the retained liquid from the bottom of the wall.

EXAMPLE.—*The depth of water behind a retaining wall with a vertical back is* 10 *ft. Calculate the magnitude of the resultant water thrust against the wall per foot run. Obtain the overturning moment this thrust produces about the base of the wall. Density of water =* 62·5 *lb. per cu. foot.*

$$P = \tfrac{1}{2}wh^2$$
$$= \tfrac{1}{2} \times 62{\cdot}5 \times 10^2 \text{ lb.}$$
$$= 3125 \text{ lb. per foot run of wall (see Fig. 371).}$$

The resultant water thrust ' P ' will act at $\frac{h}{3} = \frac{10 \text{ ft.}}{3} = 3\frac{1}{3}$ ft.

above the base of the wall. The overturning moment (sometimes termed '*bending moment*') about any point on the wall base

$= 3125$ lb. $\times 3\frac{1}{3}$ ft.
$= 10416\frac{2}{3}$ lb. ft. per foot run of wall.

Case (ii).—**Retaining Wall with Sloping Back.**

Let 'l' ft. = slope length of wetted portion of wall in Fig. 372.

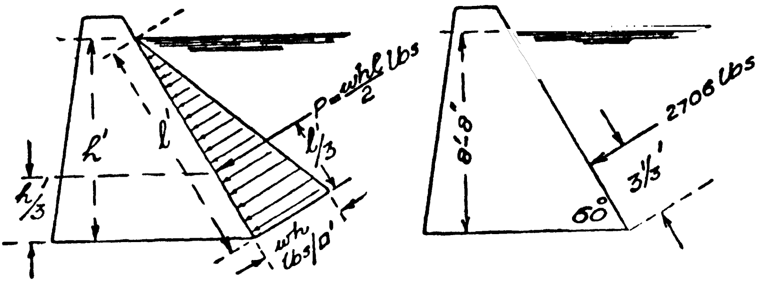


FIG. 372.—LIQUID THRUST ON INCLINED SURFACE.

Area of wetted surface per foot run of wall

$= (l \times 1)$ sq. ft. $= l$ sq. ft.

Intensity of pressure at bottom of wall, i.e. at a vertical depth of 'h' ft. $= wh$ lb. per sq. foot.

As before, the average pressure $= \frac{wh}{2}$ lb. per sq. ft.

$\therefore$ The resultant thrust $= \left(\frac{wh}{2} \times l\right)$ lb. per foot run of wall.

$$\therefore P = \frac{whl}{2} \text{ lb.}$$

Position of resultant thrust.—The resultant thrust will act, at right angles to the wall slope, at a point one-third of the length 'l' measured from the wall bottom up the slope.

EXAMPLE.—*The back of a retaining wall is inclined at* 60° *to the horizontal. The depth of water retained is* 8′ 8″. *Find the resultant thrust on the wall per foot run.*

The slope length of the wetted surface may be obtained by measurement of a scale diagram (see Fig. 372) or by calculation.

By calculation $l = \frac{8{\cdot}66'}{\sin 60°} = \frac{8{\cdot}66'}{{\cdot}866} = 10$ ft.

$$\text{Total resultant thrust} = \frac{whl}{2} \text{ lb.}$$

$$= \left(\frac{62{\cdot}5 \times 8{\cdot}66 \times 10}{2}\right) \text{lb.}$$

$$= 2706 \text{ lb. per foot run of wall.}$$

The thrust will act at right angles to the wall slope at $\frac{10}{3}$ ft. = $3\frac{1}{3}$ ft. from the bottom of wall, measured up the wall slope.

Earth Pressure

The particles of a granular material, like earth or sand, do not move freely amongst themselves when a force, such as the force of gravity, tends to produce relative movement. The surface of a tipped load of earth may be inclined to the horizontal at a considerable angle. This is possible because of the '*friction*' which exists between the particles. In liquids, the particles of which move over one another quite freely without frictional resistance, the free surface is always a horizontal plane.

The existence of friction in granular materials causes differences in properties as between liquid and earth pressures. To appreciate these differences it will be necessary to study some of the '*laws of friction.*'

Friction.—When one body tends to slide over another, the resistance to motion which is experienced at the surface of contact is termed '*frictional resistance.*'

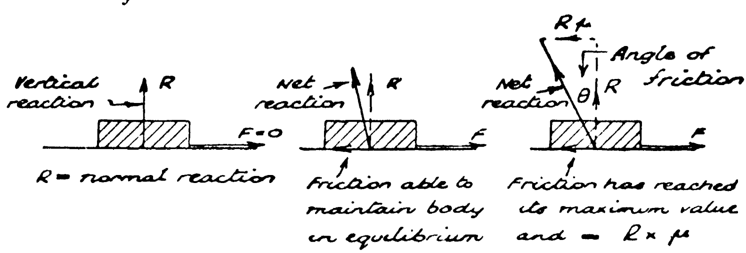


FIG. 373.—FORCE OF FRICTION.

As the force 'F' (Fig. 373) gradually increases in value from zero, the frictional resistance to motion increases just sufficiently to prevent motion. When, however, 'F' has attained a certain '*limiting value,*' the frictional resistance will be unable to prevent the body moving. The value of the frictional resistance when motion is about to take place is termed '*limiting friction.*'

It is found experimentally that two things govern the value of 'limiting friction': (i) the normal reaction between the two surfaces in contact, (ii) the nature of the two surfaces.

Law of limiting friction.—The value of the frictional resistance bears a constant ratio to the normal reaction between the two surfaces. This ratio is a physical constant for any given pair of surfaces in contact.

Let 'F' = the value of the maximum frictional resistance
'R' = the normal reaction between the surfaces
μ = the physical constant referred to above.

The law states that $\frac{F}{R} = \mu$, i.e. $F = R\mu$.

'μ' is termed the '*coefficient of friction*' for the two surfaces involved. Thus for 'wood on wood' an average value of the coefficient of friction would be ·35. 'μ' is a Greek letter, and is pronounced 'mu.'

It will be noted that the area of the surfaces in contact does not affect the frictional-resistance value.

EXAMPLE.—*A masonry block weighing* 100 *lb. rests on a wooden floor. Assuming the coefficient of friction for masonry on timber to be* ·4, *calculate the least horizontal force which will move the block.*

$$\begin{aligned} \text{The normal reaction} &= 100 \text{ lb.} \\ F &= R\mu \\ &= (100 \times \cdot 4) = 40 \text{ ,,} \\ \text{Necessary horizontal force} &= 40 \text{ ,,} \end{aligned}$$

Angle of Friction

When there is no force urging the block forward (Fig. 373), i.e. when $F = 0$, the total reaction between the two surfaces in contact is normal to the surfaces. But when 'F' has a small value, friction is introduced and the total reaction between the surfaces makes a small angle with the normal. When 'F' reaches '*limiting friction*' value, the maximum frictional resistance is brought into play and the total reaction makes its maximum angle with the normal. This maximum angle 'θ' is termed the '*angle of friction*' for the particular surfaces in contact.

It will be noted that $\tan\theta = \frac{R\mu}{R} = \mu$, i.e. the coefficient of friction is equal to the *tangent* of the 'angle of friction.'

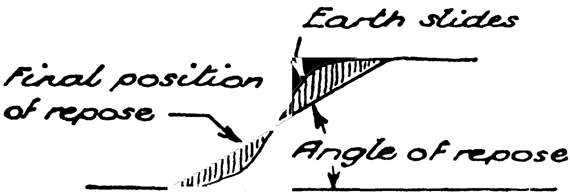


FIG. 374.—ANGLE OF REPOSE.

Angle of Repose.—Imagine a big open rectangular box full of granular material to be turned upside down suddenly and then the box to be removed vertically. The sides of the granular mass would begin to crumble down but a state of equilibrium would finally be established (Fig. 374). The angle which the surface slope makes with the horizontal in this condition of repose is termed the '*angle of repose*' for the given granular material. This 'angle of repose' may be considered to be the '*angle of friction*' for one portion of the material tending to slide over another (see Fig. 377). In the case of water, in which no friction exists, the angle of repose is zero (see Fig. 375).

CHARACTERISTICS OF BACKING MATERIALS.

Material.	Angle of Repose.	Weight in Lb. per Cu. Ft.
Sand (dry)	30°	90 to 100
(moist)	35°	100 to 110
(wet)	25°	110 to 125
Vegetable earth (dry)	30°	90 to 100
(moist)	45°	100 to 110
(wet)	15°	110 to 120
Gravel	40°	90
Rubble stone	45°	100 to 110
Gravel and sand	25° to 30°	100 to 110
Clay (dry)	30°	120 to 140
(moist)	45°	120 to 160
(wet)	15°	
Mud	0°	105 to 120
Ashes	40°	40

FIG. 375.

Earth-pressure Theories

There are various theories of earth pressure resulting in a number of different earth-pressure formulæ and graphical constructions. Space permits of reference to two theories only.

Coulomb's Wedge Theory

In this theory a wedge of earth behind the retaining wall (Fig. 376) is assumed to be responsible for the thrust against the wall.

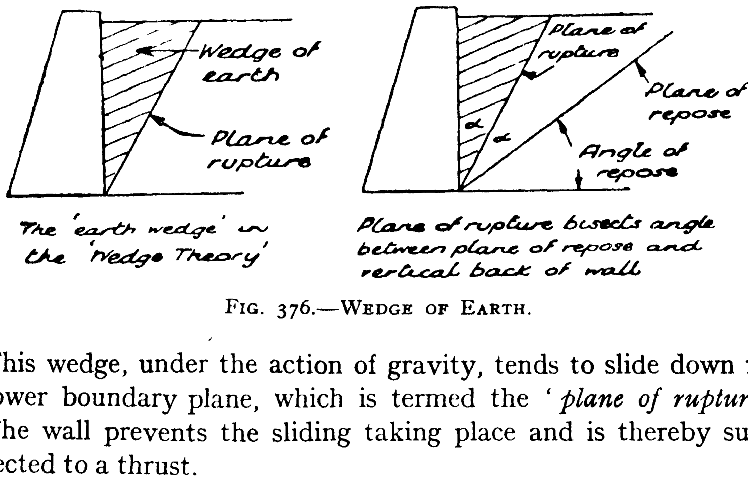


FIG. 376.—WEDGE OF EARTH.

This wedge, under the action of gravity, tends to slide down its lower boundary plane, which is termed the '*plane of rupture.*' The wall prevents the sliding taking place and is thereby subjected to a thrust.

In the following development of the theory we will assume the back of the wall to be vertical, the surface of the earth to be horizontal and no friction to exist between the earth and the back of the wall. Under these conditions it can be shown that the wall will be subjected to a maximum thrust if the plane of rupture bisect the angle between the back of the wall and the plane of repose.

The wedge of earth (Fig. 377) is in equilibrium under the action of three forces :

(i) Its own weight acting vertically downwards through the C.G. of the wedge.

(ii) The reaction of the wall, which will be horizontal on the no-friction assumption previously laid down.

(iii) The reaction of the earth beneath the plane of rupture. Assuming the angle of friction for earth on earth to be equal to

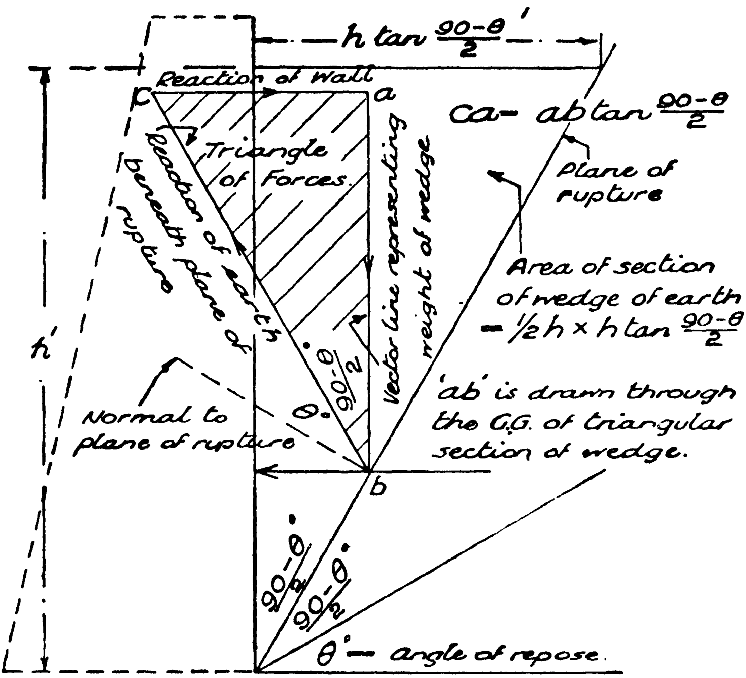


FIG. 377.—THE WEDGE THEORY.

' θ° ' (the ' *angle of repose* '), this reaction will make an angle θ° with the normal to the plane of rupture—over which sliding is assumed to be just taking place.

We can therefore draw a *triangle of forces* for these three forces and graphically find force (ii) above. This force, reversed in direction, will be the required earth thrust.

Just as in the case of water, earth pressure increases uniformly with the depth, giving a straight-line pressure-variation diagram. The resultant earth thrust will act at one-third the height of earth retained, from the bottom of the wall.

EXAMPLE.—*Using the graphical method for the wedge theory, find the resultant earth thrust against a retaining wall per foot of length, under the following circumstances:*

Height of wall (vertical back) = 13 ft.
Horizontal earth, 1 ft. from top of wall.

Density of earth = 90 *lb. per cu. ft.*
Angle of repose of the earth = 30°.

The graphical solution is given in Fig. 378.

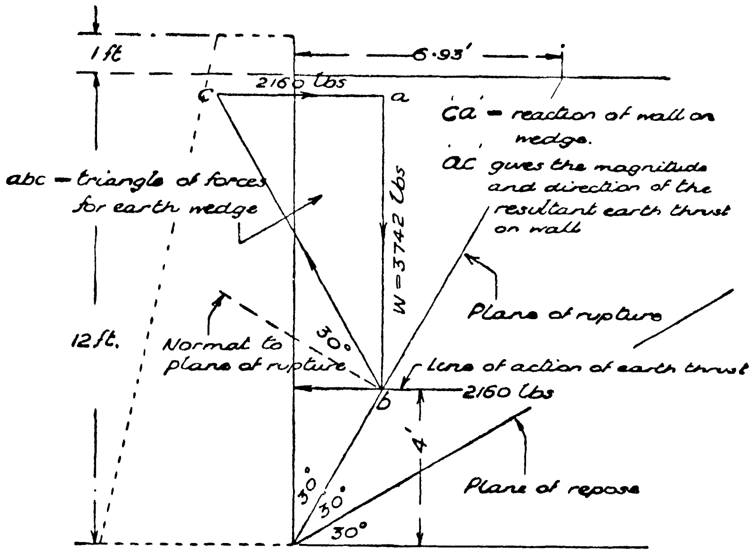


FIG. 378.—EARTH THRUST BY WEDGE THEORY.

The plane of rupture makes 30° $\left(=\frac{60^\circ}{2}\right)$ with the back of the wall. The horizontal dimension of the triangular section of the wedge may be obtained by measurement or calculation. It is (12 tan 30°) ft. = (12 × ·5774) = 6·93 ft.

∴ Area of section of wedge = (½ × 12 × 6·93) sq. ft. = 41·58 sq. ft.

Volume of wedge per foot run = (41·58 × 1) = 41·58 cu. ft.

Weight of wedge per foot run = (41·58 × 90) lb. = 3742 lb.

Vector line '*ab*' is drawn to represent 3742 lb. It is drawn through the C.G. of the wedge section.

Vector line '*bc*' is drawn at '30°' to the normal to the plane of rupture (30° = angle of friction), and '*ac*' is drawn horizontally.

The triangle '*abc*' is the triangle of forces for the wedge of earth. Vector line '*ca*' scales 2160 lb.

The resultant earth thrust *per foot run of wall* is 2160 lb., acting horizontally at 4 ft. above the bottom of the wall.

Formula for Resultant Thrust

Using the symbols given in Fig. 377, the volume of earth in the wedge $= \frac{1}{2}\left(h \times h \tan \frac{90 - \theta}{2}\right) = \frac{1}{2}h^2 \tan \frac{90 - \theta}{2}$.

$\therefore$ As 'w' = density of the earth, the weight of the wedge per foot run $= \frac{wh^2}{2} \tan \frac{90 - \theta}{2} = W$.

From the triangle of forces for the wedge we find P = earth thrust $= W \tan \frac{90 - \theta}{2} = \frac{wh^2}{2}\left(\tan \frac{90 - \theta}{2}\right)^2$.

It can be shown by trigonometrical deduction that

$$\left(\tan \frac{90 - \theta}{2}\right)^2 = \frac{1 - \sin \theta}{1 + \sin \theta}$$

$$\therefore P = \tfrac{1}{2}wh^2 \frac{1 - \sin \theta}{1 + \sin \theta}.$$

In this form the formula for the earth thrust is usually known as '*Rankine's formula.*' With the particular assumptions made in the present investigation the '*wedge theory*' and '*Rankine's theory*' yield the same result. Applying Rankine's formula to the previous numerical example :

$$P = \tfrac{1}{2}wh^2 \frac{1 - \sin \theta}{1 + \sin \theta} = \tfrac{1}{2}wh^2 \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}$$

$$= \tfrac{1}{2} \times 90 \times 12 \times 12 \times \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} \text{ lb.} = (\tfrac{1}{2} \times 90 \times 12 \times 12 \times \tfrac{1}{3}) \text{ lb.}$$

$$= 2160 \text{ lb. (see Fig. 380).}$$

This value agrees with the result obtained by the graphical construction (Fig. 378).

Rankine's Theory

Imagine a very small cube of earth, with horizontal and vertical faces, to be subjected to a normal pressure 'p' on the two horizontal faces. If the cube were composed of liquid instead of earth we would require to exert horizontal pressure $= p$ on the vertical faces to maintain equilibrium (Fig. 379).

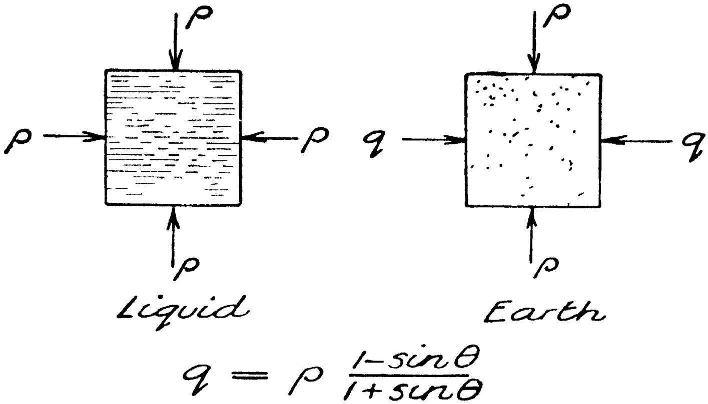


FIG 379 —LATERAL PRESSURES

Rankine showed that, in the case of an earth cube, a pressure less than 'p' on the vertical faces would be sufficient to prevent collapse of the cube. In the case of a granular material, owing to the frictional forces which the grains can exert on one another, disruption of the cube by slipping along internal planes is considerably hampered.

If 'q' be the necessary pressure applied horizontally to the vertical faces of the cube to prevent 'spreading' when a pressure of 'p' is applied vertically to the top and bottom faces, Rankine proved that 'p' and 'q' are connected by the following relationship: $q = p\,\dfrac{1 - \sin\theta}{1 + \sin\theta}$, where θ = angle of repose for the earth.

[If $\theta = 0$, as in liquids, $q = p$.]

In the practical application of the theory, 'q' lb. per sq. foot is taken to be the pressure exerted horizontally by earth which would exert a pressure 'p' lb. per sq. foot vertically. The value of 'p' is found as in liquid-pressure calculations, i.e. by multiplying the earth density by the depth of the point considered.

$$\therefore q = wh\,\frac{1 - \sin\theta}{1 + \sin\theta}.$$

The pressure-variation diagram will be as indicated in Fig. 380.

Taking 1-ft. length of wall, the total earth thrust will be $\frac{1}{2}wh\,\dfrac{1 - \sin\theta}{1 + \sin\theta} \times (h \times 1)$ lb.

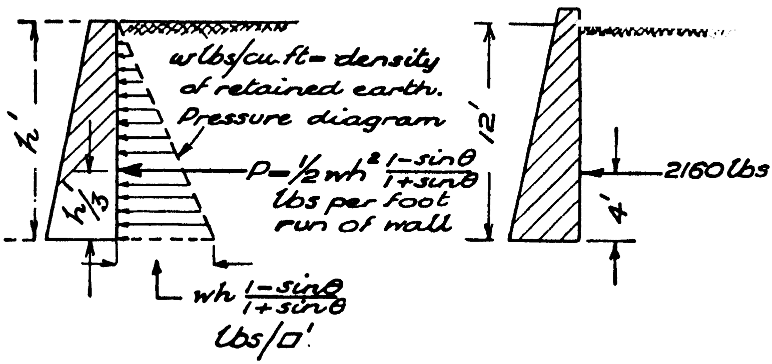


FIG. 380.—RANKINE'S FORMULA.

$$\therefore \text{P} = \tfrac{1}{2}wh^2 \frac{1 - \sin\theta}{1 + \sin\theta} \text{ lb. per foot run of wall.}$$

The resultant thrust will act through the C.G. of the triangular pressure diagram, i.e. '$\frac{1}{3}h$' from the bottom of wall.

The overturning moment of the resultant thrust about the base of wall $= \frac{1}{2}wh^2 \frac{1 - \sin\theta}{1 + \sin\theta} \times \frac{h}{3} = \frac{wh^3}{6} \frac{1 - \sin\theta}{1 + \sin\theta}$.

Stability of Gravity Retaining Walls

(i) Graphical Method

The resultant thrust which acts across the base of a retaining wall (and which has to be balanced by the force exerted by the foundation) is compounded of two forces :

(i) The thrust of the retained material behind the wall, and (ii) the self-weight of the wall. The resultant of forces (i) and (ii) is found by the usual vector-diagram method (Fig. 381).

It is upon the character of this resultant thrust—its *position, magnitude,* and *direction*—that the suitability of the wall design depends.

Position.—Assuming the wall to merely rest on its base, the resultant must obviously cut the base inside the toe of the wall if overturning is not to take place. If the resultant falls outside the base altogether, equilibrium is not possible. But it is necessary that the resultant thrust shall fall well inside the toe if all tendency to tilt is to be eliminated. It is usually assumed that the line of

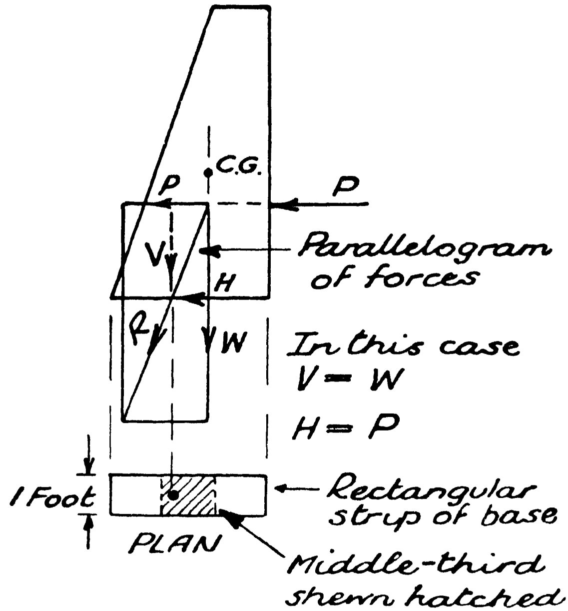


FIG 381 —RESULTANT THRUST.

action of the resultant must cut the base inside the '*middle-third*' portion of its width if all risk of uplift is to be prevented. The law—known as the *middle-third law*—states that when two rectangular surfaces are in contact the resultant thrust must not act outside the middle-third of the width of contact of the surfaces if tension is to be avoided. The base (or any horizontal section) of a typical retaining wall, taking 1-ft. run as hitherto, is a rectangular area, and hence the middle-third law is applicable. As a tendency for tension in masonry, or at mortar joints, is undesirable, the law is a test of satisfactory design. The law is considered mathematically a little later.

Magnitude.—The resultant is resolved into its vertical and horizontal components (Fig. 381). The problem of the vertical component usually is that of an eccentric load acting on a rectangular section. The stresses produced—sometimes referred to as '*normal stresses*'—are therefore computed by the method given on page 301. A more detailed consideration of these stresses, with the derivation of suitable formulæ for their calculation, is given in a subsequent paragraph.

The horizontal component of the resultant tends to cause the wall to slide over its base. Unless other forces are brought into play to assist in the stability from the sliding point of view, the frictional resistance at the base must be able to balance the horizontal component thrust.

The earth in front of the base of the wall assists in lateral stability and the base may be tilted upwards towards the toe to reduce the possibility of horizontal movement.

Direction.—The direction of the resultant thrust provides an alternative method of checking sliding tendency. If ' R ' makes a bigger angle with the normal to the base than the appropriate *angle of friction*, the wall will slide, assuming friction to be the only resisting force.

Foundation Pressures

The theory given overleaf is applicable to any horizontal section of the wall. We may have to investigate the stresses at the junction of a wall with its base. Similarly, stress computations may have to be effected for a section, say, at mid-depth of the wall to test the suitability of the width provided there. The formulæ

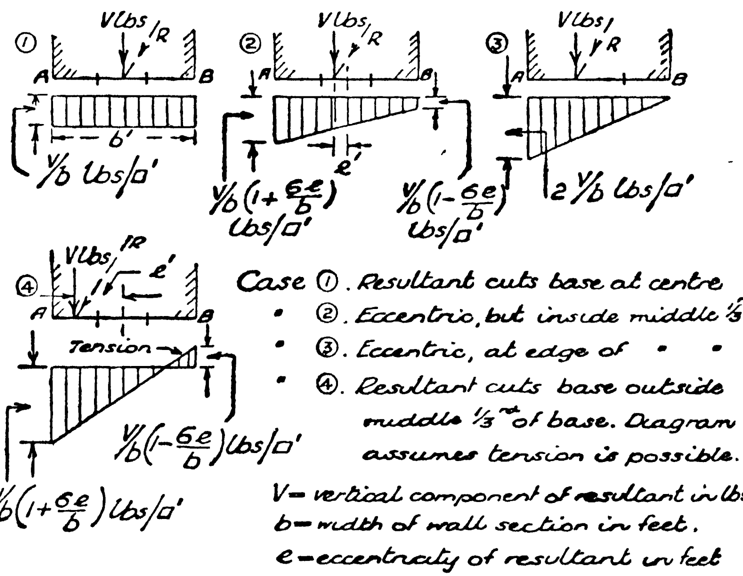


FIG. 382.—NORMAL STRESS DISTRIBUTION.

developed below will be just as applicable to the calculation of such stresses as they will be to the determination of the pressures produced at the wall base.

Let 'V' lb. = vertical component of resultant thrust
'b' ft. = width of base (or section considered)
'e' ft. = eccentricity of resultant, i.e. the distance, from the centre of base, at which 'V' acts (Fig. 382).

Taking 1-ft. length of wall:

$$\text{Area} = (b \times 1) = b \text{ sq. ft.}$$

$$\text{Section modulus (Z)} = \frac{1 \times b^2}{6}, \text{ i.e. } \left(\frac{breadth \times depth^2}{6}\right) = \frac{b^2}{6}.$$

$$\text{Direct stress} = \frac{\text{Load}}{\text{Area}} = \frac{V}{b} \text{ lb. per sq. foot}$$

$$\text{Bending stress} = \frac{M}{Z} = \frac{V \times e}{b^2/6} = \frac{6Ve}{b^2} \text{ lb. per sq. foot.}$$

$$\text{Total compressive stress at A} = \frac{V}{b} + \frac{6Ve}{b^2} = \frac{V}{b}\left(1 + \frac{6e}{b}\right)$$

$$\text{,, \quad ,, \quad ,, \quad ,, B} = \frac{V}{b} - \frac{6Ve}{b^2} = \frac{V}{b}\left(1 - \frac{6e}{b}\right).$$

The stress units are 'lb. per sq. foot' in this case.

Middle-third Law.—Considering the stress at 'B,' i.e. at the edge of section remote from the eccentric load, if there is to be no tension $\frac{V}{b}\left(1 - \frac{6e}{b}\right)$ must not yield a negative value,

$$\therefore \frac{6e}{b} \text{ must not exceed } 1$$

$$\text{i.e. } e \text{ ,, \quad ,, \quad ,, } \frac{b}{6}$$

i.e., 'V' must not deviate to either side of the centre of the section more than $\frac{'b'}{6}$ which confines its position to the '*middle-third*' portion of the width of the section.

The usually accepted forms of stress or pressure-variation diagrams are given in Fig. 382. These follow directly from the application of the formula: $\frac{V}{b}\left(1 \pm \frac{6e}{b}\right)$.

Special case.—At the base of a wall resting freely on its foundation no possibility of tension exists. As previously stated, tension anywhere in a masonry structure is undesirable. If unavoidable it should be limited to about 60 lb. per sq. inch. Where no tension is possible we have to amend the previous theory in order that the laws of equilibrium shall be satisfied.

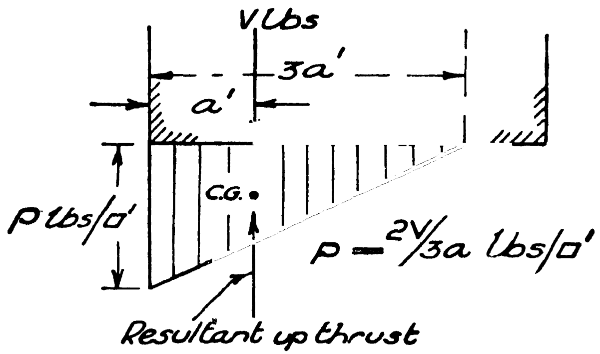


FIG. 383.—NO TENSION POSSIBLE.

The resultant up-thrust of the foundation must balance 'V' (see Fig. 383) and therefore must act in the same line. If 'V' act at distance 'a' from the toe of the wall, the form of the pressure triangle must be as indicated in order that its C.G. (through which the up-thrust acts) shall be positioned vertically below the point of application of 'V.'

If 'p' lb. per sq. foot be maximum pressure, the average pressure will be '$p/2$' lb. per sq. foot.

Area under pressure per foot run of wall

$$= (3a \times 1) = 3a \text{ sq. ft.}$$

$$\therefore \frac{p}{2} \times 3a = V$$

$$\therefore p = \frac{2V}{3a} \text{ lb. per sq. foot.}$$

Safe foundation pressures :

Typical values of safe foundation pressures are given in Fig. 384. These values must be taken only as a general guide to safe bearing capacities.

	Load on ground tons per square foot.
Alluvial soil, made ground, very wet sand	$\frac{1}{2}$
Soft clay, wet or loose sand	1
Ordinary fairly dry clay, fairly dry fine sand, sandy clay	2
Firm dry clay	3
Compact sand or gravel, London blue or similar hard compact clay	4

FIG. 384.—SAFE BEARING PRESSURES ON SUBSOILS.

Example of Simple Trapezoidal Wall

A retaining wall, 13 ft. high, has a vertical back. It is 2 ft. wide at the top and 8 ft. wide at the bottom. It retains water which is at a level of 1 ft. from the top of the wall. The average density of the wall material is 109 lb. per cu. foot. Find whether there will be any tendency for up-lift to be developed at the base. Calculate the pressures on the foundation subsoil at the toe and heel of the wall respectively.

Water thrust per foot length of wall

$= \frac{1}{2}wh^2 = \frac{1}{2} \times 62{\cdot}5 \times 12^2$ lb. $= 4500$ lb.

Weight of wall per foot length

$= \text{Volume} \times \text{Density} = \dfrac{2 + 8}{2} \times 13 \times 109$ lb. $= 7085$ lb.

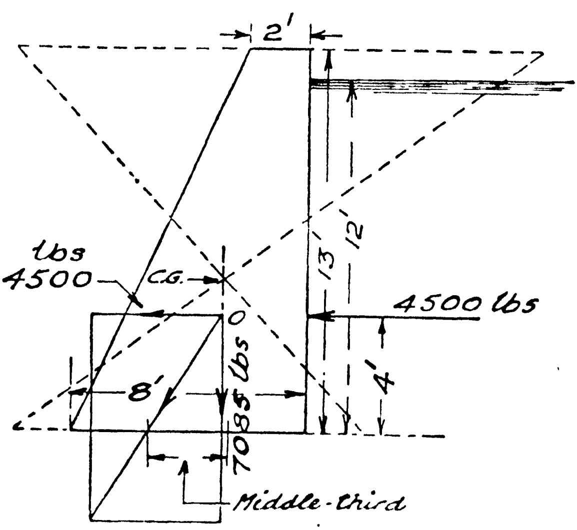


FIG. 385.—STABILITY OF RETAINING WALL.

The centre of gravity of the wall section is conveniently found by the graphical method given on page 107.

The line of action of the resultant water thrust cuts the vertical line, drawn through the C.G. of the wall section, in the point 'O' (see Fig. 385). From 'O' a vector line is drawn horizontally to the left to represent 4500 lb. From 'O' vertically downwards is drawn a vector line to represent 7085 lb. The parallelogram of forces is completed and the resultant of the two forces obtained. The resultant will be found to be a force of 8390 lb. Its line of action cuts the base at the outer middle-third point. As the resultant does not come *outside* the '*middle-third*' there will be no tendency for tension or up-lift at the base.

Foundation pressures.—The vertical component of the resultant is 7085 lb., i.e. the weight of the wall.

Eccentricity of loading (e) = $1\frac{1}{3}$ ft.

Pressure at toe of wall $= \frac{V}{b}\left(1 + \frac{6e}{b}\right) = \frac{7085}{8}\left(1 + \frac{6 \times 1\frac{1}{3}}{8}\right)$

lb. per sq. foot $= \frac{7085}{8} \times 2 = 1771 \cdot 25$ lb. per sq. foot.

Pressure at heel of wall $= \frac{V}{b}\left(1 - \frac{6e}{b}\right)$

$$= \frac{7085}{8}\left(1 - \frac{6 \times 1\frac{1}{3}}{8}\right) = \frac{7085}{8} \times 0 = 0, \text{ as we would expect.}$$

Note.—When the resultant thrust cuts the base at the outer middle-third point, the toe pressure is twice the average pressure, i.e. $2 \times \frac{V}{b}$, and the heel pressure is zero.

Example of Wall with Base

Show that the given retaining wall (*Fig.* 386) *is stable from the point of view of* (i) *tension at the base,* (ii) *sliding on its foundation. Assume the following data:*

Density of earth = 90 *lb. per cu. foot*
Angle of repose = 30°
Density of wall = 130 *lb. per cu. foot*
Coefficient of friction = ·4 (*between base and foundation*).

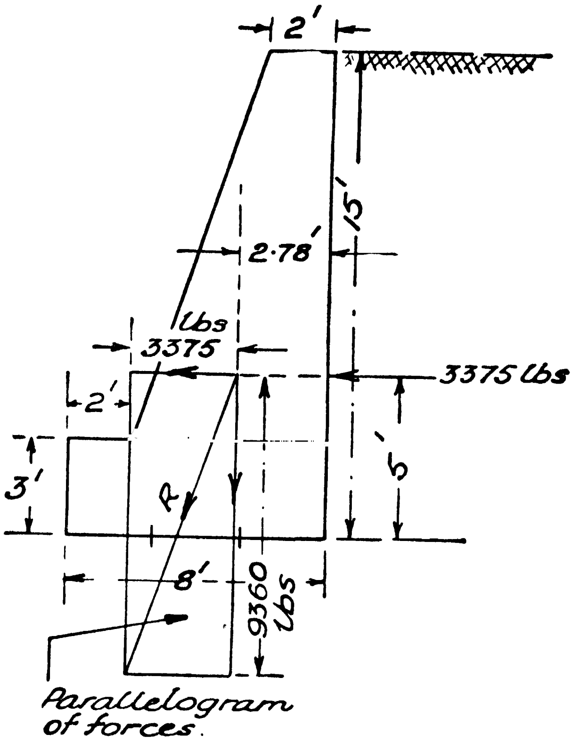


FIG. 386.—RETAINING WALL WITH BASE.

The first step in this example is to determine the centre of gravity of the wall section.

Let $\bar{x}$ = distance of C.G. from back of wall. Treating the upper wall as a rectangle plus a right-angled triangle

$$(24 + 24 + 24)\bar{x} = (24 \times 1) + (24 \times 3\tfrac{1}{3}) + (24 \times 4)$$
$$72\bar{x} = 200$$
$$\bar{x} = 2\tfrac{7}{9} \text{ ft.}$$

The line of action of 'W' will therefore act at $2\frac{7}{9}$ ft. from back of wall.

Earth thrust.—The earth thrust per foot run of wall

$$= \tfrac{1}{2}wh^2 \frac{1 - \sin\theta}{1 + \sin\theta}$$

$$\therefore P = \tfrac{1}{2} \times 90 \times 15^2 \times \frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}$$
$$= (\tfrac{1}{2} \times 90 \times 225 \times \tfrac{1}{3}) \text{ lb.} = 3375 \text{ lb.}$$

Weight of wall.—The weight of wall per foot run (W)
$$= 72 \text{ cu. ft.} \times 130 \text{ lb. per cu. foot} = 9360 \text{ lb.}$$

The resultant of ' P ' and ' W ' cuts the base inside the middle-third of the width, therefore there is no tendency for tension. The wall will exert a positive pressure all along the base.

Test for sliding.—In all cases of walls subjected to horizontal thrusts from retained materials, the *vertical* component of the resultant thrust equals the *weight* of the wall.

$\therefore$ V = 9360 lb., i.e. normal reaction = 9360 lb.

Frictional resistance to sliding = 9360 lb. $\times$ ·4 = 3744 lb.

Horizontal component of resultant = 3375 lb. As the frictional resistance exceeds the horizontal component, the wall is theoretically stable from the point of view of sliding.

The resistance to horizontal movement which is provided by the earth in front of the toe of the base (assuming the wall base to be sunken in the ground), and the increased normal pressure which the earth resting on the base projection would provide, are neglected in this problem. When all the resistances to sliding are added together the total resistance should not be much less than twice the horizontal component of the resultant thrust. For a full discussion of all the factors involved in the stability of retaining walls, the reader is referred to a publication issued by the Institution of Structural Engineers (see page 361).

(ii) Calculation Method

Consider the wall in Fig. 387, which is subjected to a thrust '**P**.' If ' P ' were not acting, the line of action of ' R ' would pass through the point ' A.' The effect of ' P,' therefore, is to displace or '*shift*' the resultant a distance '*s*,' i.e. from point ' A ' to point ' B.'

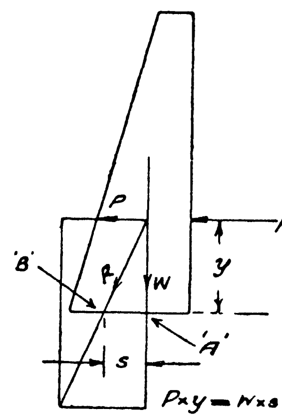


FIG. 387.—CALCULATION METHOD.

'W' and 'P' are the components of the resultant 'R.' The moment of force 'R' about 'B' is zero, therefore the moment of 'P' about point 'B' must be equal and opposite to that of force 'W.'

$$\therefore \mathrm{P} \times y = \mathrm{W} \times s$$

$$\therefore s = \frac{\mathrm{P} \times y}{\mathrm{W}}$$

'*s*' is sometimes referred to as the '*shift*' of the resultant force on the wall, due to the force 'P.'

EXAMPLE (i).—*Find the maximum height to which a* $13\frac{1}{2}''$ *brick wall may be built if it is subjected to a wind pressure of* 15 *lb. per sq. foot over its whole height. No tension is permissible in the wall. The density of the brickwork is* 125 *lb. per cu. foot.*

Let 'h' ft. = maximum height of wall.

Consider 1-ft. length of wall.

$$\text{Weight of wall} = \left(h \times 1 \times \frac{13\frac{1}{2}}{12}\right) \text{cu. ft.} \times 125 \text{ lb./cu. ft.}$$

$$= 141h \text{ lb.}$$

Total wind pressure on wall $= (15 \times h \times 1)$ lb. $= 15h$ lb.

The resultant wind force acts at $\frac{h}{2}$ ft. from the bottom.

$$\therefore \text{Overturning moment} = \left(15h \times \frac{h}{2}\right) = \frac{15}{2}h^2 = 7{\cdot}5h^2 \text{ lb. ft.}$$

In order that there shall be no tension, the eccentricity must not exceed $\frac{b}{6} = \frac{13{\cdot}5''}{6} = 2{\cdot}25$ ins.

$$\text{Shift of 'W'} = \frac{\mathrm{P} \times y}{\mathrm{W}} = \frac{\text{Overturning moment}}{\text{Weight of wall}} = \frac{7{\cdot}5h^2}{141h}$$

$$\frac{2{\cdot}25}{12} \text{ ft.} = \text{maximum permissible shift}$$

$$\therefore \frac{2{\cdot}25}{12} = \frac{7{\cdot}5h^2}{141h}$$

$$h = \frac{141 \times 2{\cdot}25}{12 \times 7{\cdot}5} = 3{\cdot}52 \text{ ft.}$$

EXAMPLE (ii).—*A retaining wall is to be built with a vertical back* 20 *ft. high. The widths at the top and bottom are to be* 3 *ft. and* 6 *ft. respectively. The earth to be retained weighs* 100 *lb. per cu. foot*

and has an angle of repose of 30°. *The earth is to be level with the top of the wall. Assuming the wall material to weigh* 120 *lb. per cu. foot, show that the proposed wall is liable to overturn.*

The following method may be found convenient for determining centre of gravity positions.

Let $\bar{x}$ = Distance of centre of gravity from back of wall.

Area		*Lever arm*		*Moment*
$20 \times 3 = 60$	$\times$	$1{\cdot}5$	=	90
$20 \times \frac{3}{2} = 30$	$\times$	$(3 + \frac{3}{3})$	=	120
90	$\times$	$\bar{x}$	=	210
		$\bar{x}$	=	$2{\cdot}33$ ft.

$$\text{Total earth thrust (P)} = \tfrac{1}{2}wh^2 \frac{1 - \sin\theta}{1 + \sin\theta}$$

$= \frac{1}{2} \times 100 \times 20^2 \times \frac{1}{3} = 6667$ lb. per foot of wall.

$$\text{Overturning moment} = P \times \frac{h}{3} = 6667 \times \frac{20}{3} \text{ lb. ft.}$$

$= 44446$ lb. ft.

Area of wall section per foot = 90 sq. ft.

∴ Weight of wall per foot run = (90 × 120) lb. = 10800 lb.

$$\text{`Shift' of `W'} = \frac{\text{Overturning moment}}{\text{Weight of wall}} = \frac{44446}{10800} \text{ ft.} = 4{\cdot}11 \text{ ft.}$$

∴ The point at which the resultant cuts the base = $(2{\cdot}33 + 4{\cdot}11)$ ft. = $6{\cdot}44$ ft. from back of wall.

This is outside the base, therefore the proposed wall is not stable against overturning.

EXAMPLE (iii).—*A retaining wall* 15 *ft. high,* 4 *ft. wide at the top, and* 9 *ft. wide at the base, retains water which is level with the top of the wall. If the wall material weighs* 150 *lb. per cu. foot, investigate the stability of the wall. Draw a diagram of soil pressure distribution at the base of the wall.*

Position of C.G. of wall ($\bar{x}$ = distance from back of wall) :

Area		*Lever arm*		*Moment*
$15 \times 4 = 60$	$\times$	2	=	120
$15 \times \frac{5}{2} = 37{\cdot}5$	$\times$	$(4 + \frac{5}{3})$	=	$212{\cdot}5$
$97{\cdot}5$	$\times$	$\bar{x}$	=	$332{\cdot}5$
		$\bar{x}$	=	$3{\cdot}41$ ft.

$$\text{Resultant water thrust} = \tfrac{1}{2}wh^2 = \tfrac{1}{2} \times 62{\cdot}5 \times 15^2 \text{ lb.}$$
$$= 7032 \text{ lb.}$$

$$\text{Overturning moment} = 7032 \times \frac{15}{3} = 35160 \text{ lb. ft.}$$

$$\text{Area of wall section} = 97{\cdot}5 \text{ sq. ft.}$$

$$\therefore \text{Weight of wall per foot run} = (97{\cdot}5 \times 150) \text{ lb.} = 14625 \text{ lb.}$$

$$\text{Shift of 'W'} = \frac{35160}{14625} \text{ ft.} = 2{\cdot}41 \text{ ft.}$$

$\therefore$ Point at which the resultant cuts the base $= (3{\cdot}41 + 2{\cdot}41)$ ft. $= 5{\cdot}82$ ft. from the back of wall, i.e. the resultant lies within the middle-third of the base width. No tendency for tension will occur at the wall base.

$$\text{Maximum soil pressure (at toe of wall)} = \frac{V}{b}\left(1 + \frac{6e}{b}\right).$$

'V' = 'W,' the weight of the wall = 14625 lb.

'b' = breadth of base = 9 ft.

'e' = eccentricity of resultant = $(5{\cdot}82 - 4{\cdot}5)$ ft. $= 1{\cdot}32$ ft.

$$\therefore \text{Maximum soil pressure} = \frac{14625}{9}\left(1 + \frac{6 \times 1{\cdot}32}{9}\right) \text{ lb. per sq. ft.}$$

$$= \frac{14625}{9}\left(1 + \frac{7{\cdot}92}{9}\right) = 3055 \text{ lb. per sq. ft.}$$

$$\text{Minimum soil pressure (at heel)} = \frac{V}{b}\left(1 - \frac{6e}{b}\right)$$

$$= \frac{14625}{9}\left(1 - \frac{7{\cdot}92}{9}\right) \text{ lb. per sq. ft.} = 195 \text{ lb. per sq. ft.}$$

The pressure-variation diagram is shown in Fig. 388.

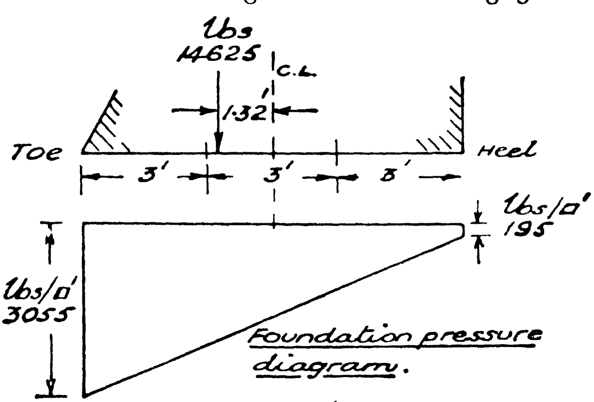


FIG. 388.

EXAMPLE (iv).—*Show by calculation method that the given retaining wall (Fig. 389) is just stable from the point of view of tension at the base. Earth density = 90 lb. per cu. foot. Angle of repose = 30°. Wall density = 130 lb. per cu. foot. Check the stability also by the graphical method. The foundation must not be subjected to a pressure exceeding 2 tons per sq. foot.*

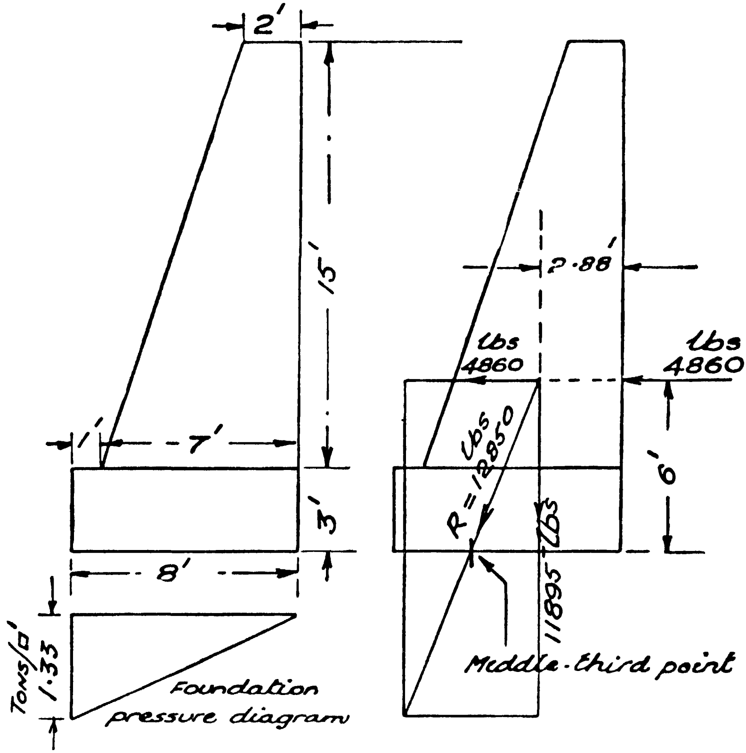


FIG. 389.—TEST OF WALL DESIGN.

The reader should verify that the following preliminary results are correct.

C.G. of wall is 2·88 ft. from the back.

P = 4860 lb. W = 11895 lb.

Taking these worked values :

$$\text{Shift of ` W '} = \frac{\text{Overturning moment}}{\text{Weight of wall}}$$

$$= \frac{4860 \times 6}{11895} \text{ ft.} = \frac{29160}{11895} \text{ ft.} = 2{\cdot}45 \text{ ft.}$$

$\therefore$ Resultant thrust cuts base at $(2{\cdot}45 + 2{\cdot}88)$ ft. $= 5{\cdot}33$ ft. from heel of wall.

But outer middle-third point is $\frac{2}{3} \times 8$ ft. $= 5{\cdot}33$ ft. from the heel.

$\therefore$ Resultant cuts base at outer middle-third point, i.e. the wall is just stable from the point of view of tension at base.

The maximum foundation pressure under the given conditions

$$= \frac{2V}{b} \text{ lb./sq. ft.} = \frac{2 \times 11895}{8 \times 2240} \text{ tons sq./ft.} = 1{\cdot}33 \text{ tons/sq. ft.}$$

This is less than the maximum permissible ground pressure.

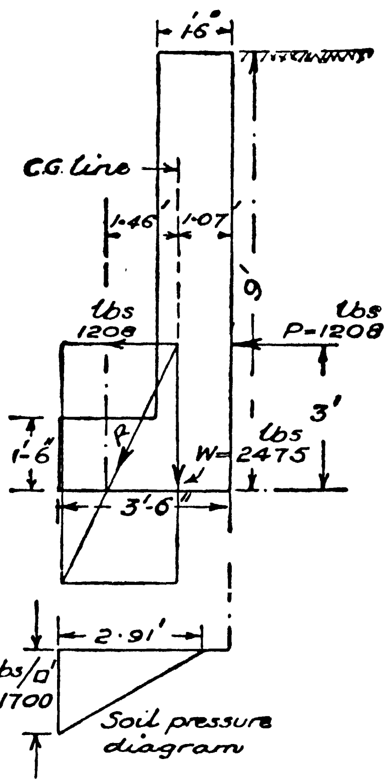


FIG. 390.—L-SHAPED RETAINING WALL.

EXAMPLE (v).—*An 'L-shaped' retaining wall is* 9 *ft. high* (*Fig.* 390). *The base is* 3′ 6″ *wide, and the wall is* 1 *ft.* 6 *ins. thick in each leg. The earth being retained weighs* 110 *lb. per cu. foot,*

and has an angle of repose of 35°. The earth is level with the top of the wall. The wall weighs 150 lb. per cu. foot. Test the wall for up-lift at the base. Draw the diagram of soil pressure distribution for the base of the wall. Construct the diagram of earth pressure variation for the back of the wall.

C.G. of wall section—from vertical back :

Area			*Lever arm*		*Moment*
$9 \times 1{\cdot}5 =$	$13{\cdot}5$	$\times$	$\cdot 75$	$=$	$10{\cdot}1$
$2 \times 1{\cdot}5 =$	$3{\cdot}0$	$\times$	$2{\cdot}5$	$=$	$7{\cdot}5$
	$16{\cdot}5$	$\times$	$\bar{x}$	$=$	$17{\cdot}6$

$$\bar{x} = \frac{17{\cdot}6}{16{\cdot}5} = 1{\cdot}07 \text{ ft.}$$

$$\text{Resultant earth thrust} = \tfrac{1}{2}wh^2 \frac{1 - \sin\theta}{1 + \sin\theta}$$

$$= \tfrac{1}{2} \times 110 \times 9^2 \times \frac{1 - \sin 35^\circ}{1 + \sin 35^\circ}$$

$$= \tfrac{1}{2} \times 110 \times 81 \times \frac{1 - \cdot 5736}{1 + \cdot 5736} = 1208 \text{ lb.}$$

Overturning moment = (1208 × 3) lb. ft. = 3624 lb. ft.
Area of wall section = 16·5 sq. ft.
∴ Weight of wall per ft. run = 16·5 × 150 = 2475 lb.

$$\text{Shift of 'W'} = \frac{3624}{2475} \text{ ft.} = 1{\cdot}46 \text{ ft.}$$

∴ Point at which the resultant cuts base is (1·07 + 1·46) ft. = 2·53 ft from back of wall, i.e. outside the middle-third.

The appropriate formula (proved on page 347) will be used for the soil pressure.

Distance from point at which the resultant cuts the base to toe of wall = a = (3·5 − 2·53) ft. = ·97 ft.

∴ 3 × a = 3 × ·97 = 2·91 ft.

$$\text{Maximum soil pressure} = \frac{2V}{3a} = \frac{2 \times 2475}{2{\cdot}91}$$

= 1700 lb. per sq. ft. (see Fig. 390).

Maximum earth pressure behind the wall

$$= wh\frac{1 - \sin\theta}{1 + \sin\theta} = \left(110 \times 9 \times \frac{1 - \cdot 5736}{1 + \cdot 5736}\right) \text{lb. per sq. ft.}$$

= 269 lb. per sq. ft.

The corresponding variation of pressure diagram is linear, of the type illustrated in Fig. 380. It is not shown in Fig. 390.

Retaining Walls Subjected to Inclined Thrusts

In Fig. 391 the thrust behind the wall is not horizontal. The line of action of the thrust is produced to cut the line of action of the weight and the parallelogram of forces is completed in the usual manner. A point to note in this case is that the value of 'V,' the vertical component of the resultant thrust, is not equal to 'W,' the weight of the wall. It is greater than 'W' and will be found by resolving 'R' vertically in the usual manner. When friction between the retained earth and the back of the wall is taken into account, we get the type of inclined thrust indicated in Fig. 391.

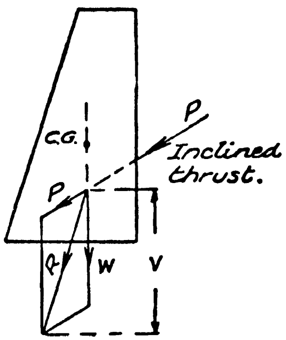


FIG. 391.—WALL WITH INCLINED THRUST.

Buttresses or Piers

The principles underlying the calculations in retaining wall problems may be applied to buttresses with horizontal or inclined thrusts. In such cases 1-ft. length of wall may not be representative, and it may be necessary to consider a portion of wall plus the buttress as a whole for calculation of weight and applied thrust.

Retaining Wall with Stepped Back.—In the example given in Fig. 392, the earth resting on the steps at the back of the wall assists in the stability of the wall. We may assume AB to be the *virtual* back of the wall. The value of 'W' in such a case would be the combined weight of the wall material and the supported earth. The line of action of 'W' would pass through the centre of gravity of the combined wall and earth masses.

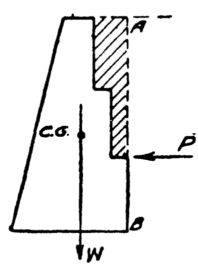


FIG. 392.—WALL WITH STEPPED BACK.

Retaining Wall with Superimposed Loading at the Back

In Fig. 393 the net-pressure diagram behind a wall in such a case is illustrated.

AB = the head of earth which would produce the given superimposed pressure of 'p_1' lb. per sq. foot.

If h_1 ft. = the head, $p_1 = wh_1$. $\therefore h_1 = p_1/w$ ft.

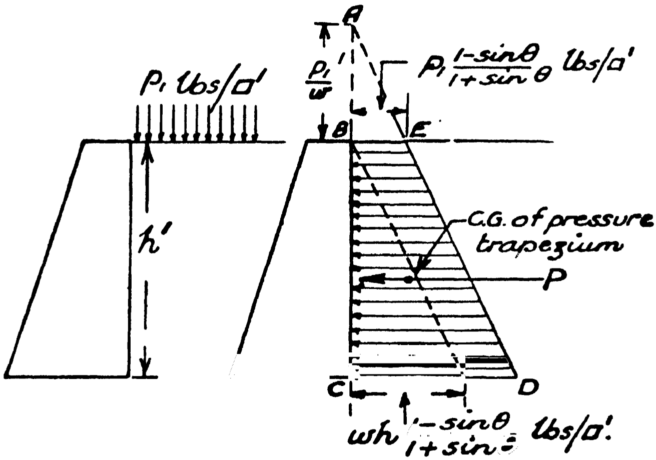


FIG. 393.—SUPERIMPOSED LOAD ON EARTH.

The pressure diagram ACD indicates that the pressure behind the wall is represented by the trapezoidal diagram BEDC. This may be directly constructed by drawing BE $= p_1 \dfrac{1 - \sin\theta}{1 + \sin\theta}$ and CD $= wh \dfrac{1 - \sin\theta}{1 + \sin\theta} + p_1 \dfrac{1 - \sin\theta}{1 + \sin\theta}$. The effect of the superimposed load is to increase the pressure everywhere on the back of the wall by an amount $p_1 \dfrac{1 - \sin\theta}{1 + \sin\theta}$ lb. per sq. foot.

EXAMPLE.—*A retaining wall is* 12 *ft. high* (*Fig.* 394). *The earth retained has a density of* 90 *lb. per cu. foot and angle of repose* 30°, *and is subjected to a superimposed load of* 150 *lb. per sq. foot. Calculate the overturning moment on the wall about the base per foot run of wall* (i) *by constructing the pressure diagram for the wall,* (ii) *by adding the component overturning moments due to retained earth and superimposed load respectively.*

Method (i) :

The net pressure-variation diagram is shown in Fig. 394. The horizontal pressure equivalent to the superimposed load of 150 lb. per sq. foot $= p_1 \times \frac{1 - \sin\theta}{1 + \sin\theta} = 150 \times \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}} = 150 \times \frac{1}{3}$ $= 50$ lb. per sq. foot.

$\therefore$ BE $= 50$ lb. per sq. foot.

The pressure at wall base against the wall, due to retained earth, $= wh\frac{1 - \sin\theta}{1 + \sin\theta} = 90 \times 12 \times \frac{1}{3} = 360$ lb. per sq. foot.

$\therefore$ CD $= (360 + 50) = 410$ lb. per sq. foot.

Employing the formula given on page 107 for the C.G. of a trapezium $\left(\bar{y} = \frac{a + 2b}{3(a + b)} \times 12 \text{ ft.}\right)$, the height of C.G. of trapezium above the base of wall

$$= \frac{410 + (2 \times 50)}{3(410 + 50)} \times 12 \text{ ft.} = \frac{510 \times 12}{3 \times 460} = \frac{102}{23} \text{ ft.}$$

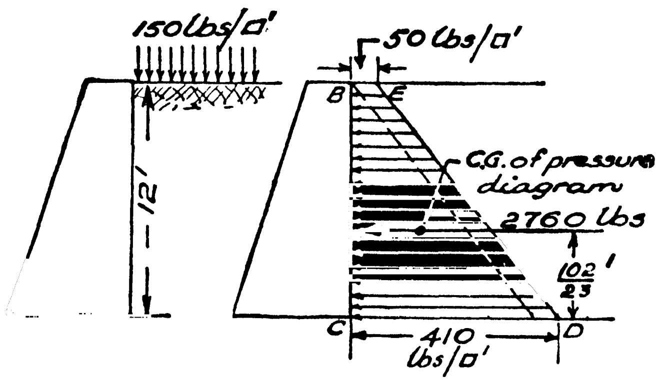


FIG. 394 —EXAMPLE INVOLVING SUPERIMPOSED LOAD.

Total thrust against wall = Average pressure $\times$ (12 sq. ft.)

$$= \tfrac{1}{2}(410 + 50) \times 12 \text{ lb.} = 2760 \text{ lb.}$$

Arm of overturning moment $= \frac{102}{23}$ ft.

$\therefore$ Overturning moment about base

$$= \left(2760 \times \frac{102}{23}\right) \text{lb. ft.} = 12240 \text{ lb. ft.}$$

Method (ii):

Overturning moment due to retained earth alone

$$= \tfrac{1}{2}wh^2 \frac{1 - \sin\theta}{1 + \sin\theta} \times \frac{h}{3} = (\tfrac{1}{2} \times 90 \times 144 \times \tfrac{1}{3} \times \tfrac{12}{3}) \text{ lb. ft.}$$

$= 8640$ lb. ft.

The superimposed load causes a uniform pressure of 50 lb. per sq. foot over the wall back.

$\therefore$ Total thrust per foot of wall $= (50 \times 12) = 600$ lb.

Arm of overturning moment $= \frac{12}{2} = 6$ ft.

$\therefore$ Overturning moment $= 600$ lb. $\times$ 6 ft. $= 3600$ lb. ft.

Total overturning moment $= (8640 + 3600) = 12240$ lb. ft. as before.

The second method is the quicker. If the position of the resultant thrust were required by this method, we would have to compound the two parallel thrusts due to retained earth and superimposed loads, acting respectively at $\frac{1}{3}$ and $\frac{1}{2}$ height of wall above the bottom.

Equivalent Liquid Pressure

Readers who take up the more advanced study of retaining walls will come across a method of treatment of earth pressures in which a liquid of calculated density is assumed to replace the given earth. By this means the horizontal pressure at any depth is obtained by the simple formula: pressure = density $\times$ depth. Taking the case of earth of density 100 lb. per cu. foot and angle of repose 35° we have:

$$p = wh \frac{1 - \sin\theta}{1 + \sin\theta} = w \frac{1 - \sin\theta}{1 + \sin\theta} \times h$$

$$= 100 \times \frac{1 - \sin 35^\circ}{1 + \sin 35^\circ} \times h = 100 \times \frac{1 - \cdot5736}{1 + \cdot5736} \times h$$

$\therefore p = 27{\cdot}1h$ lb. per sq. foot.

Horizontal pressures and thrusts may now be computed on the basis of a liquid of density 27·1 lb. per cu. foot.

Further treatment of retaining-wall problems is beyond the scope of this book. An excellent booklet entitled *Report on Retaining Walls* may be obtained, price 1s. 6*d*., from the

Institution of Structural Engineers, 11, Upper Belgrave Street, London, S.W.1.

The report deals with the question of friction between the retained material and the back of the wall. Walls are sometimes 'surcharged,' i.e. have to support material which is not level with the top of the wall. The surface inclination of the material to the horizontal may be any angle up to the 'angle of repose.' The practical consideration of this case and that of retaining walls with backs inclined to the vertical form part of the report. The characteristics of backing materials given in Fig. 375 are taken from the report by the kind permission of the Institution.

Exercises 15

(1) A vertical wall, 5 ft. high, is subjected to a uniform wind pressure of 20 lb. per sq. foot. Calculate the overturning moment about the wall base per foot run of wall.

(2) A retaining wall with a vertical back is 11 ft. high. It retains water the level of which is 2 ft. from the top of the wall. Taking the density of water to be 62·5 lb. per cu. foot, calculate the water thrust against the wall per foot run of wall.

(3) Earth, of density 90 lb. per cu. foot and angle of repose 30°, is retained by a wall with a vertical back. The wall is 12 ft. high and the earth is level with the top. Calculate, on the basis of 1-ft. length of wall, (i) the resultant earth thrust, (ii) the overturning moment about a point in the wall base.

Draw a diagram showing the variation of horizontal pressure on the back of the wall.

(4) A retaining wall has a section in the form of a right-angled triangle. The vertical back is 9 ft. high and the base is 'x' ft. The wall retains water which reaches to the top of the wall. Assuming the densities of the water and the wall to be 62·5 lb. per cu. foot and 130 lb. per cu. foot respectively, obtain the minimum permissible value of 'x' if there is to be no tendency for tension at the wall base.

(5) A retaining wall with a vertical back has a trapezoidal section. Taking the data given, obtain the resultant thrust at the wall base.

State whether or not the wall presses firmly on its foundation, from the toe to the heel of the base.

Width of wall at the top = 2 ft.

Width of wall at the bottom = 4 ft.

Height of wall = 9 ft.

Density of retained gravel, which is level with top of wall, = 90 lb./cu. ft.

Angle of repose = 40°.

Density of wall material = 140 lb./cu. ft.

Note.—Sin 40° = ·6428.

(6) (i) A wall of rectangular section, 1′ 6″ thick, is 5 ft. high. The density of the wall material is 100 lb. per cu. foot. Calculate the maximum permissible value of uniform horizontal wind pressure to which the wall may be subjected if no tension is to be developed at the wall base.

(ii) Calculate the maximum height to which a wall, of constant thickness 2 ft., may be built, without tension being developed, if the wind pressure be 20 lb. per sq. foot normal to the wall. The wall density is 120 lb. per cu. foot.

(7) Given that the subsoil under the retaining wall shown in Fig. 395 can safely take a pressure of $1\frac{1}{2}$ tons/ft.^{2} and that a co-

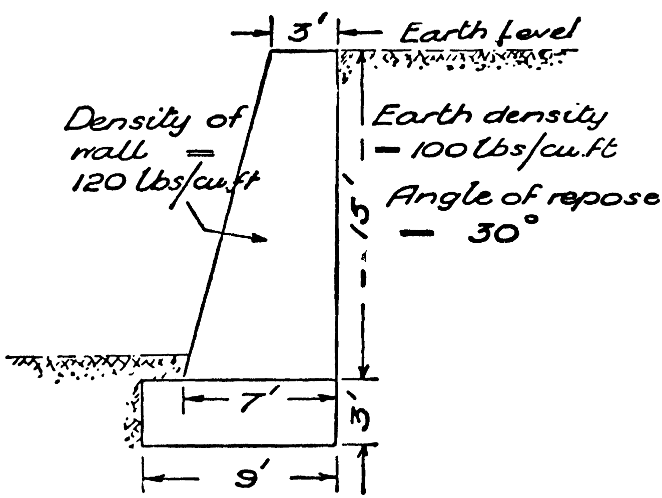


FIG. 395.

efficient of friction of ·5 may be assumed at the wall base, investigate the stability of the wall.

(8) Fig. 396 shows a rectangular base 'AB' subjected to a resultant thrust which has a vertical component of 9000 lb. per foot of wall. Obtain the foundation pressure at point 'A' of the base in each of the three cases given. No tension is possible at 'B.'

(9) Friction between the retained earth and the back of the wall is sometimes assumed. In this case the earth thrust is inclined to the horizontal.

Taking the example given in Fig. 397, obtain the maximum

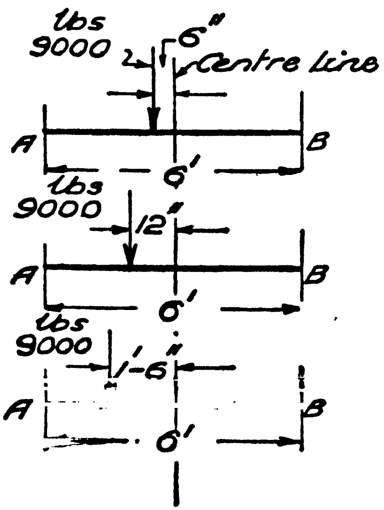


FIG. 396.

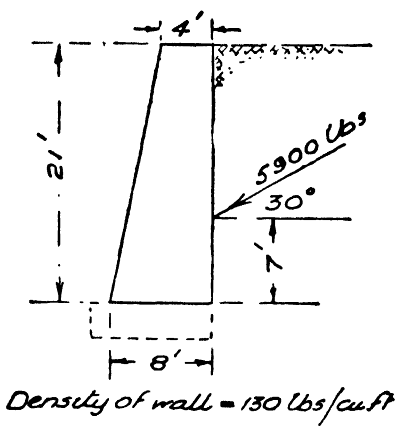


FIG. 397.

compressive stress at the junction of the upper wall and the base. Show that no tensile stress will be developed at this level in the wall.

[The earth thrust shown has been computed for a wall height of 21 ft., i.e. for the portion of wall *above the horizontal section being considered*. Find 'V' as in Fig. 391 and treat as a vertical force acting at the point in which the resultant 'R' cuts the junction of upper wall and base.]

CHAPTER XVI

REVISION EXAMPLES WITH ABRIDGED SOLUTIONS

In the examples given in this chapter, unless otherwise stated or suitable data be given, the self-weight of members is to be neglected. All beams are assumed to be simply supported.

The solutions are given in abridged form, with sufficient detail to enable the reader to check the intermediate stages in his calculations. Readers unacquainted with trigonometry should use graphical methods where the calculation method involves this branch of mathematics.

The page numbers given in the solutions will enable a reference to be made to examples involving similar principles, previously worked in greater detail in the text.

(1) A vertical brick wall, 5 ft. high, is 1′ 6″ thick. The wall is subjected to a horizontal wind pressure of 30 lb. per sq. foot of wall. Find the magnitude and direction of the resultant thrust at the wall base, taking the density of the brickwork to be 100 lb. per cu. foot. Calculations are to be based on 1-ft. length of wall.

(2) Verify that the withdrawal tendency on the bolt shown in Fig. 398 is wholly horizontal. Find the force of withdrawal.

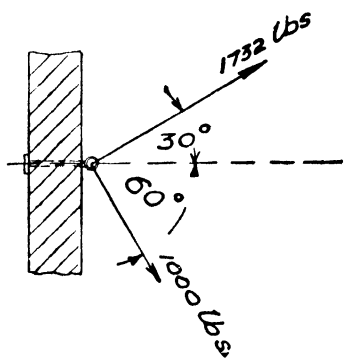


FIG. 398.

(3) A pier is subjected to a vertical thrust of 800 lb. It also supports a horizontal thrust of 'H' lb. Calculate 'H' if the resultant thrust on the pier is 1000 lb.

(4) Assuming a slate weighing $5\frac{1}{2}$ lb. to become loose on a roof of 30° pitch, obtain the gravitational force urging it down the roof slope.

(5) Fig. 399 shows a bracket connection to a steel column. The position of the bracket load is such as to cause rivet 'A' to

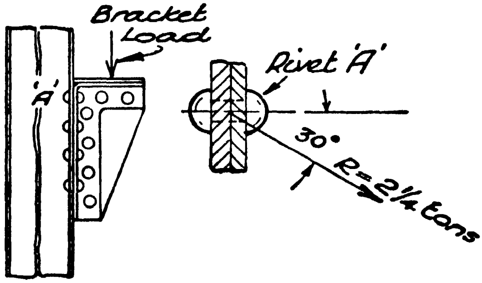


FIG. 399.

be subjected to a resultant load of $2\frac{1}{4}$ tons in the manner indicated. Find (i) the tensile load, (ii) the shear load, which the rivet has to carry.

(6) A simple triangular truss has the following dimensions: Bottom tie = 15 ft., left-hand rafter = 9 ft., right-hand rafter = 12 ft. It carries a vertical load of 'P' lb. at the apex. Assuming the left-hand rafter to be exerting a thrust of 1000 lb., obtain (i) the value of 'P', (ii) the thrust in the right-hand rafter.

(7) The vertical reaction at the left end of a truss is 2000 lb. The main rafter and bottom tie make angles of 30° and 10° respectively with the horizontal. Using Lami's theorem, calculate the given member forces. Check the results by constructing a triangle of forces.

(8) At a certain top boom joint in a braced girder the centre lines of four members meet. The two horizontal boom members exert thrusts, the left-hand member 12 tons, and the right-hand member 15 tons. Find the force in each of the other two members, one of which is vertical and the other inclined downwards towards the right, at 45° to the horizontal. There is no external load at the joint.

(9) A vertical wall, 40 ft. high, is subjected to horizontal wind pressure as follows: 15 lb. per sq. foot, up to a level 30 ft. above ground, and 20 lb. per sq. foot above this level. Calculate the overturning moment of this wind pressure about the base of the wall, per foot length of wall.

(10) A bolt, with its axis horizontal, is turned by a straight spanner. Assuming the maximum safe turning moment which may be applied to the bolt to be 2000 lb. ins., calculate the

greatest permissible distance, measured along the spanner from the centre of the bolt, at which a vertical load of 100 lb. may be applied, when the spanner makes an angle of 30° with the horizontal.

(11) In a testing machine a uniform straight bar with the fulcrum at the centre swings as a pendulum in a vertical plane. The bar carries two adjustable weights, 20 lb. near the bottom end and 10 lb. near the top. Calculate the moment of the couple which must be imposed on the pendulum to maintain it at an angle of 30° with the vertical, given that the bottom and top weights have their C.G.s respectively at 20 ins. and 10 ins. from the fulcrum.

(12) A beam, 15 ft. overall length, is simply supported at two points 'A' and 'B.' 'A' is 2 ft. from the left end and 'B' is 3 ft. from the right end of the beam. The beam carries a 9″ brick wall, of uniform height 4 ft., throughout its length. Taking the density of the brickwork to be 1 cwt. per cu. foot, calculate the reactions at 'A' and 'B' due to the weight of the wall.

(13) A beam (whose self-weight may be neglected) is simply supported at the left end and at a point 4 ft. from the right end. The overall length of the beam = 12 ft. The beam carries a point load of 2 tons at 2 ft. from the left end. What additional load per foot run can the beam support, if the left-end reaction is not to exceed 4·5 tons ?

(14) A simply supported beam 'AB,' of 20-ft. effective span, carries a point load of 8 tons at 4 ft. from 'A.' A uniform load of 1 ton per foot extends 12 ft. into the span from the end 'B.'

Determine the support reactions due to these loads :

(i) by calculation ;
(ii) by link polygon.

(15) The reactions for a beam, 15 ft. long, are at the left end and 5 ft. from the right end. The beam carries three loads : (i) 1 cwt. per foot over the whole beam length ; (ii) 2 cwts. per foot for a distance of 5 ft. from the left end ; and (iii) a point load of 30 cwts. at the extreme right end of the beam. Calculate the beam reactions.

(16) Fig. 400 shows a wall bracket. Calculate the magnitude

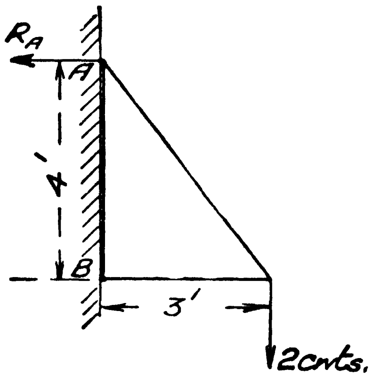


FIG. 400.

of the reaction at 'A' and the magnitude and direction of the reaction at 'B.'

Check the results by a graphical method.

(17) A king-post truss of 24-ft. span is subjected to a uniform positive wind pressure on the left slope of the truss. The length of the principal rafter of the truss is 14 ft. and the trusses are spaced at 8-ft. centres. The reaction at the right end of the truss is vertical. If this reaction have a value of 490 lb. due to the wind, calculate the intensity of the wind pressure normal to the roof slope.

(18) A trapezoidal retaining-wall section with a vertical back is 2 ft. wide at the top, 4 ft. wide at the bottom and 12 ft. high. Obtain the distance of the centre of gravity of the wall section from the back of the wall

(i) by calculation ;
(ii) by link polygon construction ;
(iii) by any other graphical construction.

(19) Find the centre of gravity of each of the structural sections given in Fig. 401.

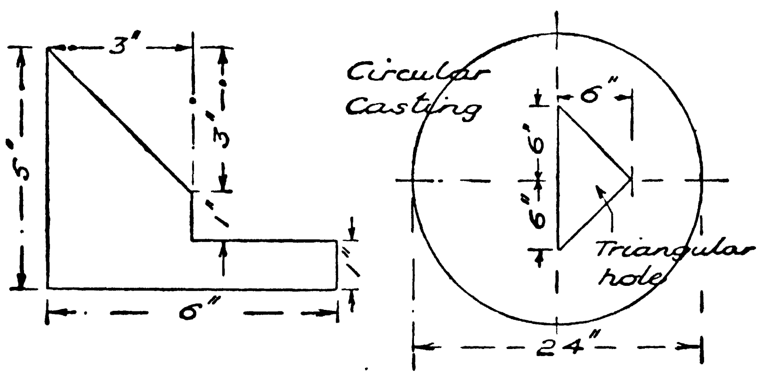


FIG. 401.

(20) A compound girder, 13″ overall depth, is composed of an I-section with a plate 8″ × ½″ on the top flange, and a plate 12″ × ½″ on the bottom flange. The dimensions of the I-section are:

Flange width	= 7″
„ thickness	= 1″
Depth	= 12″
Web thickness	= ½″.

Calculate the position of the neutral axis of the section, making no allowance for rivet holes.

(21) Draw the stress diagram for the frame given in Fig. 402. Draw up a table showing the force in each member of the frame, and the type of member.

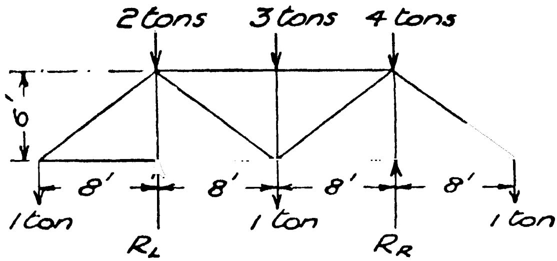


FIG. 402

(22) Forces of 1 cwt., 5 cwts., and 4 cwts. act vertically downwards at points 'A,' 'B,' and 'C' respectively on a horizontal beam. 'AB' = 4 ft. and 'BC' = 6 ft.

Find the magnitude, direction, and position (with respect to point 'A') of the resultant of the three forces:

(i) by calculation;
(ii) by means of a link polygon.

(23) In a travelling crane three wheel loads 'A,' 'B,' and 'C' traverse a girder. 'A' = 1 ton, 'B' = 3 tons, and 'C' = 2 tons. AB = 2 ft. and BC = 4 ft. Assuming the fact that the load system will produce its maximum bending moment when the '*3 tons*' load and the '*C.G. of the three loads*' lie equidistant on either side of the centre of the girder, find how far load 'A' is from the near end of the girder when the critical load setting

for maximum bending moment is attained. The girder has a span of 20 ft.

(24) A truncated right pyramid of square section is $4'' \times 4''$ at one end and $2'' \times 2''$ at the other and is used as a tie member. It is subjected to an axial pull of 360 lb. Calculate the stress at the mid-section of the pyramid and also determine the maximum stress produced.

(25) (i) How many $\frac{3}{4}''$-dia. rivets in single shear will be required to transmit a shear load of 53 tons? ($f_s = 6$ tons/in.^{2})

(ii) A tie-bar in a frame is connected to the boom angles by two gusset plates, one on either side of the tie-bar. Four $1''$-dia. turned and fitted bolts are used. Calculate the safe axial load for the tie-bar from the point of view of the shear in the bolts in its end connection ($f_s = 6$ tons/in.^{2}).

(26) A timber post, $3'' \times 3''$ in section and 4 ft. in length, is subjected to an axial thrust of 1800 lb. Taking Young's modulus for the timber to be 1,600,000 lb./in.^{2}, calculate the contraction in length of the post.

(27) An extensometer, employing an $8''$-gauge length, is attached to a mild steel tie-bar in a bridge truss. As a train passes over the bridge the extensometer registers a maximum extension of ·002 ins. Assuming the cross-sectional area of the tie-bar to be 6 sq. ins., estimate the live load created in it by the passing of the moving load. A suitable value for 'E' must be chosen.

(28) The concrete in a reinforced concrete circular water tank, on drying out after casting, has a shrinkage of ·01%, thus causing compressive stress in the steel reinforcement which is in the form of horizontal hoops. Calculate the value of the tensile stress which the filling-in of the water may tend to cause in the steel hoops without any residual tensile stress being created. E for steel = 30,000,000 lb./in.^{2}. It is assumed that there is no slip between the steel and surrounding concrete.

(29) A simply supported beam, of effective span = 'l,' carries a point load = 'W' at one-quarter of the span from one end. Prove that the maximum bending moment in the beam is given by the expression $\frac{3Wl}{16}$.

(30) A simply supported beam 'AB' has an effective span of 8 ft. At points 'C,' 'D,' and 'E' on the beam there are con-

centrated loads of 2 cwts., 4 cwts., and 1 cwt. respectively. AC = 1′, CD = 3′, DE = 2′.

Construct the bending moment and shear force diagrams for the beam.

(31) A simply supported beam of 10-ft. effective span carries a uniform load which develops a bending moment of 16 cwts. ft. at a section 2 ft. from the left end. Calculate the maximum bending moment and maximum shear force for the beam.

(32) A timber beam of 8-ft. effective span carries 10 sq. ft. of flooring. The inclusive floor load is 120 lb. per sq. foot. Design a suitable section for the beam, making the depth three times the breadth. The maximum fibre stress in the timber is not to exceed 1200 lb. per sq. inch.

(33) A timber beam of 10-ft. effective span has to carry a point load of 560 lb. at 4 ft. from the left end. Find a suitable depth for the beam assuming a breadth of 2″. The safe stress in bending for the timber is 1000 lb. per sq. inch.

(34) A timber beam has a breadth of 3″, a depth of 6″, and an effective span of 10 ft. In addition to a point load of 560 lb. at 4 ft. from the left end, it carries a uniform load of 300 lb. Calculate the maximum fibre stress in the timber.

(35) Calculate the maximum permissible span for a 3″ × 9″ timber beam which carries 100 lb. per foot run, given that the maximum stress must not exceed 1200 lb. per sq. inch and that the maximum deflection is not to be greater than 1/360th of the span.

'E' for the timber = 1,600,000 lb. per sq. inch.

(36) A balcony is supported outside a building by timber cantilevers. The width of the balcony is 4 ft. and the total inclusive load carried is 220 lb. per sq. foot of floor area. If the cantilevers be 3″ wide × 7″ deep, calculate their maximum permissible spacing, if the fibre stress in the timber be limited to 1000 lb. per sq. inch.

(37) A concrete footing to a retaining wall projects 18″ beyond the toe of the wall. The footing is 24″ deep. The net upward pressure at the edge of the footing is 2 tons per sq. foot and vertically beneath the toe of the upper wall the foundation pressure has fallen to 1·5 tons per sq. foot. Calculate the maximum tensile stress in the concrete.

(38) A B.S.B. has an effective span of 14 ft. It carries a

central point load of 3·5 tons and a uniform load of total value 7 tons. Taking 'f' = 10 tons per sq. inch, find the necessary section modulus for the beam.

(39) A 12″ × 6″ × 44 lb. B.S.B. having an effective span of 20 ft. carries point loads of 'W' tons at 5 ft. from each end, and, in addition, a uniform load of 8 tons. Calculate the value of 'W' if the maximum fibre stress in the steel = 10 tons per sq. inch.

Z_{XX} for the B.S.B. = 52·79 ins.^{3}.

(40) A steel beam carries three equal point loads of 6·4 tons each, at the quarter-points of an effective span of 16 ft. Assuming a working stress of 10 tons per sq. inch, select a suitable section from the following list:

12″ × 6″ × 44 lb. B.S.B. : Z_{XX} = 52·79 ins.^{3}
10″ × 8″ × 55 lb. B.S.B. : Z_{XX} = 57·74 ins.^{3}
12″ × 6″ × 54 lb. B.S.B. : Z_{XX} = 62·63 ins.^{3}

(41) Taking the case of a 9″ × 4″ × 21 lb. B.S.B., verify the rule that if a B.S.B. be fully loaded with uniform load so that f = 8 tons/in.^{2}, the effective span must not exceed 24 times the beam depth, if the maximum deflection is to be limited to $\frac{1}{325}$th part of the span. E = 13,000 tons/in.^{2}. I_{XX} for B.S.B. = 81·13 ins.^{4}.

(42) A compound girder, weighing 150 lb. per foot run, has an effective span of 20 ft. The girder carries a superimposed uniform load of total value 5960 lb. In addition the girder carries the reactions of secondary beams which are equivalent to two point loads of 23·75 tons each, respectively, at 6 ft. from the end supports. The girder is 17 ins. overall depth and I_{XX} = 1945 ins.^{4}. Calculate the maximum stress induced in the girder flanges.

(43) The steel corner post in a wire fence is of I-section. The flanges are $1\frac{1}{2}$″ wide and $\frac{1}{4}$″ thick. The web thickness is $\frac{1}{4}$″ and the overall depth is 2″. The fence wires contain an angle of 120° at the post and the 'YY' axis of the steel section bisects the angle between the wires. Calculate the maximum permissible tension in the wires, assuming there are three horizontal wire circuits at 1 ft., 2 ft., and 3 ft. respectively above the ground support. The post has no stays and the maximum stress must not exceed 8 tons/in.^{2}. The tension in each wire is the same.

(44) A beam, simply supported at the ends, carries a uniform load of 1 cwt. per foot run. The effective span = 10 ft.

Calculate the uniform load, in cwts. per foot run, which the beam can carry, in addition to the given load, assuming this new load to extend from the left support up to the centre of the span. The maximum bending moment in the beam is to be limited to 40 cwts. ft.

Draw the B.M. and S.F. diagrams for the beam when carrying thé calculated and given loads simultaneously.

(45) A beam of rectangular section, 2″ wide × 6″ deep, has an effective span of 8 ft. The load carried by the beam is uniformly distributed and of 'W' lb. total value. At 1 in. beneath the top of beam, at a section 2 ft. from the left end, the compressive stress is 600 lb. per sq. inch. Calculate the value of 'W.'

(46) An overhanging beam extends 2 ft. over the supports at both ends. It carries three equal concentrated loads, one at each end and one in the middle. Calculate the magnitude of the central span if the negative support moment is twice the maximum positive central-span moment.

(47) An overhanging beam of 15-ft. total length is supported at 3 ft. from each end. The beam carries a wall 9″ thick and of average density 100 lb. per cu. foot. The elevation of the wall is an equilateral triangle with 15-ft. base. Calculate (i) the support bending moment, (ii) the maximum central-span bending moment, due to the weight of the wall.

(48) What is the least radius of gyration of a solid circular column section $5\frac{1}{2}''$ diameter? Calculate the safe concentric load for a mild steel column of this section, assuming its effective length to be 13′ 9″, given the following 'F_a' values:

l/r	'F_a' (tons/in.^{2})
100	4·1
110	3·7
120	3·3

(49) A column has to support a concentric load of 49 tons. The effective length of the column is estimated to be 12 ft.

Select, from the sections given below, the most suitable joist column:

14″ × 6″ × 46 lb.	$I_{YY} = 21{\cdot}45$ ins.^{4}	Area = 13·59 sq. ins.
16″ × 6″ × 50 lb.	$I_{YY} = 22{\cdot}47$ ins.^{4}	Area = 14·71 sq. ins.

The following 'F_e' values are supplied:

l/r	F_e (*tons/in.*^{2})
110	3·7
120	3·3

(50) A short masonry pillar of square section, 9″ side, supports a vertical concentrated load of 'W' tons, having a single eccentricity of $1\frac{1}{2}''$ with respect to one of the principal axes of the section. Calculate the value of 'W' if the maximum compressive stress it induces in the masonry is 600 lb. per sq. inch.

(51) The resultant thrust at the base of a retaining wall is 8000 lb. per foot run of wall. It makes an angle of 30° with the vertical, and cuts the base at 3′ 6″ from the toe. The width at the base = 9′. Draw a diagram showing the distribution of normal pressure on the subsoil under the base.

(52) A retaining wall with a vertical back is 15 ft. high. Calculate the resultant thrust on the back of the wall, per foot run of wall, under each of the given circumstances:

(i) Uniform wind pressure of intensity 30 lb. per sq. foot.

(ii) Water, level with the top.

(iii) Earth, level with the top. The earth density is 90 lb. per cu. foot and angle of repose is 30°.

(53) The height of a retaining wall with a vertical back is 12 ft. It retains earth, level with the top of the wall. The earth density is 100 lb. per cu. foot and its angle of repose is 30°. There is a superimposed load of 210 lb. per sq. foot on the earth at the back of the wall. Calculate the resultant overturning moment about the base of the wall.

(54) A boundary wall of rectangular section, 6 ft. high, is subjected to a uniform horizontal wind pressure of 15 lb. per sq. foot. The density of the wall material is 120 lb. per cu. foot. Calculate the necessary thickness of the wall if no tension is to be developed at the base.

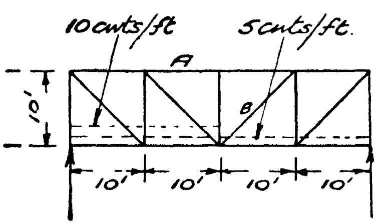


FIG. 403.

(55) Fig. 403 shows a braced girder carrying two uniform load systems, 5 cwts. per foot

for the length of the girder and 10 cwts. per foot extending over two bays. The loads are carried by a floor system which transmits the given loads to the panel points of the girder. Calculate the force in each of the members marked 'A' and 'B' respectively.

[*Note.*—The load at a given panel point will be the sum of the loads taken up by proceeding half-way to the adjacent panel points on either side.]

(56) A 12″ × 8″ × 65 lb. joist column section is used as a short column and is subjected to the following load system :

10 tons on the 'YY' axis, 8″ eccentric with respect to XX axis.
8 tons on the 'XX' axis, 3″ eccentric with respect to YY axis.
48 tons concentric load.

Given $Z_{XX} = 81{\cdot}3$ ins.^{3}, $Z_{YY} = 16{\cdot}3$ ins.^{3}, area of column section = 19·12 in.^{2}, calculate the maximum stress induced in the steel.

[For direct stress, add together the three loads and divide by area of section. For maximum bending stress, find the bending stress due to each eccentric load (with reference to the appropriate axis) and add the two stresses. Finally add the direct and total of bending stresses.]

(57) The column section given in Question 56 is used to carry a single load of 20 tons, which has an eccentricity of 6″ with respect to the 'XX' axis and 2″ with respect to the 'YY' axis. Calculate the maximum stress in the column.

[Add the direct stress to the combined bending stresses due to the two eccentricities.]

Solutions

[Page references are to examples involving similar principles.]

(1) Wind load = 150 lb. Weight of wall = 750 lb.

Resultant = $\sqrt{150^2 + 750^2} = 765$ lb. If 'θ' = inclination to vertical, $\tan\theta = 150/750 = \cdot 2$, $\theta = 11^\circ\ 18'$. [*Page* 14.]

(2) $1732 \times \cos 60^\circ = 1000 \cos 30^\circ$. $\therefore$ No vertical component.

Horizontal force on bolt = $1732 \cos 30^\circ + 1000 \cos 60^\circ =$ 2000 lb. [*Page* 25.]

(3) $H^2 + 800^2 = 1000^2$. $\therefore$ H = 600 lb. [*Page* 14.]

(4) $5\cdot5 \cos 60° = 5\cdot5 \times \frac{1}{2} = 2\cdot75$ lb. [*Page* 21.]

(5) (i) Tensile load $= 2\cdot25 \cos 30° = 1\cdot95$ tons.
(ii) Shear load $= 2\cdot25 \cos 60° = 1\cdot125$ tons. [*Page* 22.]

(6) Solution by 'triangle of forces' or by 'Lami's theorem.'
Lami's theorem reduces to: $\frac{P}{\sin 90°} = \frac{1000}{12/15}$. $\therefore$ P = 1250 lb.

Also $\frac{P}{\sin 90°} = \frac{Q}{9/15}$ which gives Q (force in right-hand rafter) = 750 lb. [*Pages* 35 *and* 325.]

(7) Reducing to three forces acting away from the point of concurrence, the contained angles are 120°, 80°, and 160°. $\frac{2000}{\sin 160°} = \frac{\text{Rafter force}}{\sin 80°} = \frac{\text{Tie force}}{\sin 120°}$. From this, we get **Rafter force = 5759 lb. Tie force = 5064 lb.** [*Page* 326.]

(8) Inclined member is a tie. T cos 45° = 3. T = **4·242 tons.**
Vertical member is a strut. S = T cos 45° = **3 tons.** [*Pages* 42 *and* 22.]

(9) Wind load on lower portion = 450 lb.
Wind load on upper portion = 200 lb.
Overturning moment = $(450 \times 15) + (200 \times 35) =$ **13750 lb. ft.** [*Page* 104.]

(10) $x'' =$ distance. $\therefore 100 \times x \times \cos 30° = 2000$.
$\therefore x =$ **23 ins.** [*Page* 48.]

(11) $(20 \times 20 \sin 30°) - (10 \times 10 \sin 30°) =$ **150 lb. ins.** [*Page* 49.]

(12) Total weight of brickwork = 45 cwts., acting at C.G., which is 5·5′ from 'A.'
$R_A \times 10 = 45 \times 4\cdot5$. $R_A =$ **20·25 cwts.**
$R_B \times 10 = 45 \times 5\cdot5$. $R_B =$ **24·75 cwts.** [*Page* 108.]

(13) If 'w' tons per foot be load
$4\cdot5 \times 8 = (2 \times 6) + (12w \times 2)$. $\therefore w =$ **1 ton per foot.** [*Page* 60.]

(14) $R_A \times 20 = (8 \times 16) + (12 \times 6)$. $\therefore R_A =$ **10 tons.**
$R_B \times 20 = (8 \times 4) + (12 \times 14)$. $\therefore R_B =$ **10 tons.**

[Treat uniform load as concentrated at the C.G. for link polygon construction.] [*Page* 125.]

(15) $(R_L \times 10) + (30 \times 5) = (10 \times 7{\cdot}5) + (15 \times 2{\cdot}5)$.

$R_L = -\ 3.75$ cwts., i.e. a downward force.

$(R_R \times 10) = (10 \times 2{\cdot}5) + (15 \times 7{\cdot}5) + (30 \times 15)$

$R_R = 58.75$ cwts. [*Page* 109.]

(16) $R_A \times 4 = 2 \times 3$. $\therefore R_A = 1{\cdot}5$ cwts.

$H_B = R_A = 1{\cdot}5$ cwts. $V_B = 2$ cwts.

$R_B = \sqrt{1{\cdot}5^2 + 2^2} = 2{\cdot}5$ cwts.

θ = inclination of 'R_B' to horizontal.

$\tan \theta = V_B/H_B = 2/1{\cdot}5 = 1{\cdot}33$

$\theta = 53°$ (nearly). [*Page* 84.]

(17) P = resultant wind load.

$P \times 7 = 490 \times 24$. $\therefore P = 1680$ lb.

Wind pressure $= \dfrac{P}{14 \times 8} = \dfrac{1680}{14 \times 8} = 15$ lb. per sq. foot. [*Page* 85.]

(18) $\bar{x}$ from back of wall.

$36\bar{x} = (24 \times 1) + (12 \times 2\frac{2}{3})$. $\therefore \bar{x} = 1\frac{5}{9}$ ft. [*Page* 101.]

(19) (i) Dividing into a triangle and two rectangles of respective areas 4·5, 6, and 3 sq. ins. and assuming C.G. at $\bar{x}$ from left edge and $\bar{y}$ from bottom:

$13{\cdot}5\bar{x} = (4{\cdot}5 \times 1) + (6 \times 1{\cdot}5) + (3 \times 4{\cdot}5)$. $\therefore \bar{x} = 2''$.

$13{\cdot}5\bar{y} = (4{\cdot}5 \times 3) + (6 \times 1) + (3 \times {\cdot}5)$. $\therefore \bar{y} = 1\frac{5}{9}''$. [*Page* 101.]

(ii) $\bar{x}$ = distance of C.G. from left edge:

$$416{\cdot}4\bar{x} = (452{\cdot}4 \times 12) - (36 \times 14)$$

$$\bar{x} = 11{\cdot}82''. \quad [\textit{Page } 102.]$$

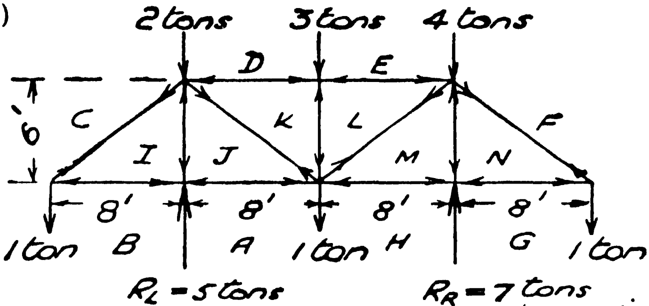
C.G. is 12″ from bottom of section.

(20) $\bar{y}$ = height of C.G. above bottom, i.e. height of NA. Area of I-section = 19 sq. ins. Total area = 29 sq. ins.

$29\bar{y} = (19 \times 6{\cdot}5) + (4 \times 12{\cdot}75) + (6 \times {\cdot}25)$

$\bar{y} = 6{\cdot}07$ ins. [*Page* 103.]

(21)



TONS

MEMBER	FORCE	TYPE
CI	1·66	TIE
DK	1·33	STRUT
EL	1·33	STRUT
FN	1·66	TIE
GN	1·33	STRUT
HM	1·33	STRUT
AJ	1·33	STRUT
BI	1·33	STRUT
IJ	5·00	STRUT
JK	3·33	TIE
KL	3·00	STRUT
LM	3·33	TIE
MN	7·00	STRUT

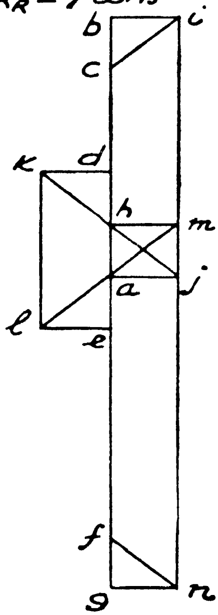


FIG 404.

(22) Magnitude $= (1 + 5 + 4) = 10$ cwts.
Direction $=$ vertically downwards.
Position $= x$ ft. from 'A'

$$10x = (5 \times 4) + (4 \times 10)$$
$$x = 6 \text{ ft.}$$ [*Pages* 72 *and* 119.]

(23) C.G. of load system is 3 ft. from 'A,' i.e. 1 ft. from 'B.' Therefore centre of span is 6" from 'B' and 6" from C.G. of system, measured in opposite directions. This setting will fix load 'A' as being 7·5 ft. from the end of the girder nearer to it. [*Page* 93.]

(24) Area at mid-section $= 9$ sq. ins.

Stress at mid-section $= \frac{360}{9} = 40$ lb. per sq. inch.

Smallest sectional area $= 4$ sq. ins.

$\therefore$ Maximum stress $= \frac{360}{4} = 90$ lb. per sq. inch. [*Page* 163.]

(25) (i) Value of one rivet $= 2{\cdot}65$ tons.

Number of rivets required $= \frac{53}{2{\cdot}65} = 20$.

(ii) Value of one bolt $= 9{\cdot}42$ tons.
$\therefore$ Safe load for 4 bolts $= 37{\cdot}68$ tons. [*Page* 186.]

(26) Stress $= \frac{1800}{9} = 200$ lb./in.^{2}.

Strain $= \frac{x''}{48''}$.

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{200}{x/48} = 1{,}600{,}000.$$

$x =$ contraction in length $= {\cdot}006$ ins. [*Page* 178.]

(27) Strain $= \frac{{\cdot}002''}{8''} = {\cdot}00025$.

Stress $=$ E $\times$ strain $= 13000 \times {\cdot}00025 = 3{\cdot}25$ tons/in.^{2}.
Load created $= (3{\cdot}25 \times 6) = 19{\cdot}5$ tons. [*Page* 178.]

(28) Strain $= {\cdot}01/100 = {\cdot}0001$.
Stress in steel $=$ E $\times$ strain $= (30{,}000{,}000 \times {\cdot}0001) = 3000$ lb./in.^{2}. This compressive stress must be neutralised before any tension is developed. It is assumed there is no relative slip between steel reinforcing bars and the concrete. [*Page* 178.]

(29) $R_L = \frac{W \times \frac{3}{4}l}{l} = \frac{3}{4}W$. B.M.$_{max.} = \frac{3}{4}W \times \frac{l}{4} = \frac{3Wl}{16}$.
[*Page* 209.]

(30) $R_A = 4$ cwts. $R_B = 3$ cwts.
B.M.$_C = (4 \times 1) = 4$ c.f.
B.M.$_D = (4 \times 4) - (2 \times 3) = 10$ c.f.
B.M.$_E = (3 \times 2) = 6$ c.f. [*Page* 211.]

(31) 'w' cwts. per foot $=$ uniform load.

$$R_A = \frac{10w}{2} = 5w \text{ cwts.}$$

$\therefore\ (5w \times 2) - (2w \times 2/2) = 16. \quad \therefore\ w = 2$ **cwts. per foot.**

$W = 2 \times 10 = 20$ cwts.

$$\text{B.M.}_{\text{max.}} = \frac{Wl}{8} = \frac{20 \times 10}{8} = 25 \text{ c.f.}$$

$$\text{S.F.}_{\text{max.}} = \pm\frac{W}{2} = \pm\ 10 \text{ cwts.}$$ [*Page* 214.]

(32) $$\text{B.M.}_{\text{max.}} = \frac{1200 \times 8 \times 12}{8} = 14400 \text{ lb. ins.}$$

$$1200 \times \frac{bd^2}{6} = 14400$$

$b = 2''$ and $d = 6''$. [*Page* 246.]

(33) $R_A = 336$ lb. $\text{B.M.}_{\text{max.}} = (336 \times 4) = 1344$ lb. ft.

$$1344 \times 12 = \frac{1000 \times 2 \times d^2}{6}.$$

$b = 2''$. $d = 7''$.

Suitable depth $= 7''$. [*Page* 246.]

(34) $R_A = 486$ lb.

$\text{B.M.}_{\text{max.}}$ occurs at the point-load position.

$\text{B.M.}_{\text{max.}} = (486 \times 4) - (120 \times 2) = 1704$ lb. ft.

$$1704 \times 12 = \frac{f \times 3 \times 6 \times 6}{6}. \quad \therefore f = 1136 \text{ lb./in.}^2.$$

[*Page* 266.]

(35) 'l' ft. = max. permissible span.

Strength: $$\frac{100l \times l \times 12}{8} = \frac{1200 \times 3 \times 81}{6}. \quad \therefore l = 18 \text{ ft.}$$

Deflection: $$y = \frac{5}{384}\frac{Wl^3}{EI}.$$

$$\frac{l \times 12}{360} = \frac{5}{384} \times \frac{100l \times l^3 \times 1728}{1{,}600{,}000 \times 182{\cdot}25}. \quad \therefore l = 16{\cdot}28 \text{ ft.}$$

$\therefore$ Max. span $= 16{\cdot}28$ ft. [*Page* 273.]

(36) Safe total load per cantilever $= W$ lb.

$$W \times 2 \times 12 = \frac{1000 \times 3 \times 7 \times 7}{6}. \quad \therefore W = 1020{\cdot}8 \text{ lb.}$$

'x' ft. = spacing (centres).

$$\therefore \frac{1020{\cdot}8}{4x} = 220. \quad x = 1{\cdot}16 \text{ ft.} = 13{\cdot}9 \text{ ins. (centres).}$$ [*Page* 267.]

(37) Distance of C.G. of pressure diagram from toe of upper wall, i.e. the arm of bending moment,

$$= \frac{a + 2b}{3(a + b)} \times l = \frac{1\cdot5 + 4}{3(3\cdot5)} \times 18 = 9\cdot43 \text{ ins.}$$

Upward thrust $= \left[\left(\frac{2 + 1\cdot5}{2}\right) \times (1\cdot5 \times 1)\right]$ tons $= 5880$ lb.

$\therefore 5880 \times 9\cdot43 = f \times \frac{12 \times 24 \times 24}{6}$.

$\therefore f = 48\cdot1$ lb./in.². [*Pages* 107 *and* 247.]

(38) Max. B.M. $= \left(\frac{3\cdot5 \times 14}{4}\right) + \left(\frac{7 \times 14}{8}\right) = 24\cdot5$ tons ft. $= 294$ tons ins.

$\therefore Z = \frac{294}{10} = 29\cdot4$ ins.³. [*Page* 257.]

(39) Available B.M. $= 10 \times 52\cdot79 = 527\cdot9$ tons ins.

$B.M._{max}$ due to uniform load $= \frac{8 \times 20 \times 12}{8} = 240$ tons ins.

$\therefore$ The two loads 'W' must create a B.M. $= (527\cdot9 - 240) = 287\cdot9$ tons ins.

$\therefore W \times 5 \times 12 = 287\cdot9$. $\therefore W = 4\cdot798$ tons. [*Page* 267.]

(40) $R_A = 9\cdot6$ tons.

$B.M._{max} = (9\cdot6 \times 8) - (6\cdot4 \times 4) = 51\cdot2$ tons ft.

$Z = \frac{51\cdot2 \times 12}{10} = 61\cdot44$ ins ³.

Use 12″ × 6″ × 54 lb. B.S.B. [*Page* 266.]

(41) $24 \times$ beam depth $= 24 \times 9'' = 216$ ins.

Safe load $=$ W tons. $M = f\frac{I}{y}$.

$\frac{W \times 216}{8} = 8 \times \frac{81\cdot13}{4\cdot5}$. $\therefore W = 5\cdot34$ tons. [*Page* 255.]

Max. deflection $= \frac{5}{384} \times \frac{5\cdot34 \times 216 \times 216 \times 216}{13000 \times 81\cdot13} = \cdot665$ ins.

Max. deflection permissible $= \frac{216}{325} = \cdot665$ ins. [*Page* 273.]

(42) Self-weight of girder $= 20 \times 150 = 3000$ lb.

Total U.D. load $= (3000 + 5960) = 8960$ lb. $= 4$ tons.

Total $B.M._{max.}$ $= \left(\frac{4 \times 20}{8}\right) + (23\cdot75 \times 6)$ tons ft. $= 152\cdot5$ tons ft.

$$Z = \frac{I}{y} = \frac{1945}{8\cdot5} = 228\cdot8 \text{ ins.}^3.$$

$$\therefore \; f = \frac{152\cdot5 \times 12}{228\cdot8} = 8 \text{ tons/in.}^2. \quad [\textit{Page } 269.]$$

(43)

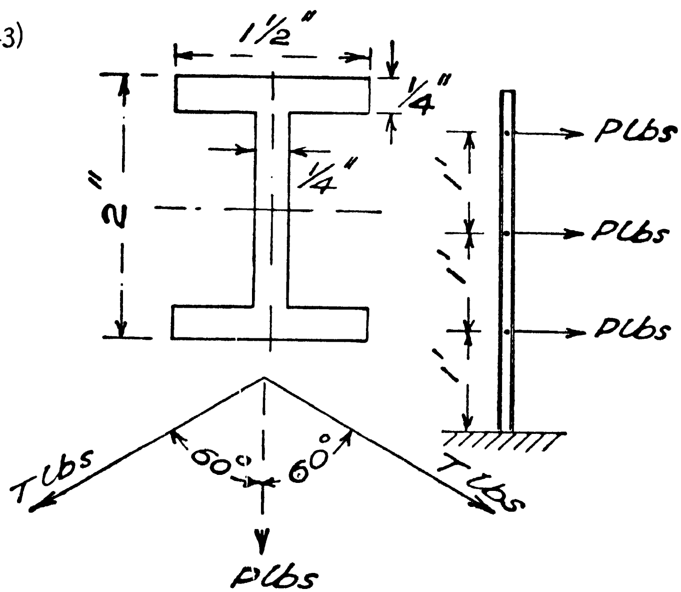


FIG. 405.

$$I_{XX} \text{ for section} = \frac{1\cdot5 \times 2^3}{12} - \frac{1\cdot25 \times 1\cdot5^3}{12} = \cdot648 \text{ ins.}^4.$$

$$Z_{XX} = \frac{I_{XX}}{y} = \frac{\cdot648}{1} = \cdot648 \text{ ins.}^3.$$

$(P \times 36) + (P \times 24) + (P \times 12) = \text{max. B.M.} = fZ = 8 \times \cdot648 \times 2240.$

$P = 161$ lb.

$2T \cos 60° = 161.$

$\therefore$ T = max. permissible tension = 161 lb. [*Page* 253.]

(44) $\text{B.M.}_{max.}$ will occur in portion of beam carrying the additional load.

Let 'w' cwts. per foot run = additional load.

$R_A \times 10 = (10 \times 5) + (5w \times 7\cdot5)$. $\quad \therefore \; R_A = (5 + 3\cdot75w)$ cwts.

Position of B.M.$_{max.}$ (from left end) $= \dfrac{5 + 3{\cdot}75w}{w + 1}$ ft.

$$\text{B.M.}_{max.} = \left[(5 + 3{\cdot}75w) \times \left(\frac{5 + 3{\cdot}75w}{w + 1}\right) - (5 + 3{\cdot}75w) \times \left(\frac{5 + 3{\cdot}75w}{2(w + 1)}\right)\right] = (5 + 3{\cdot}75w) \times \left(\frac{5 + 3{\cdot}75w}{2(w + 1)}\right) = 40.$$

$\therefore$ $w = 4$ **cwts. per foot.**

Max. B.M. due to 1 cwt./ft. load $= \dfrac{10 \times 10}{8} = 12{\cdot}5$ c.f.

Treating 4 cwts./ft. load as concentrated at centre (for construction purposes) B.M. at $2{\cdot}5'$ from left end $= \dfrac{20 \times 2{\cdot}5 \times 7{\cdot}5}{10}$ $= 37{\cdot}5$ c.f.

$R_L \times 10 = (20 \times 7{\cdot}5) + (10 \times 5)$. $\therefore R_L = 20$ cwts.

$R_R \times 10 = (20 \times 2{\cdot}5) + (10 \times 5)$. $\therefore R_R = 10$ cwts.

B.M. and S.F. diagrams are shown in Fig. 406. [*Page* 230.]

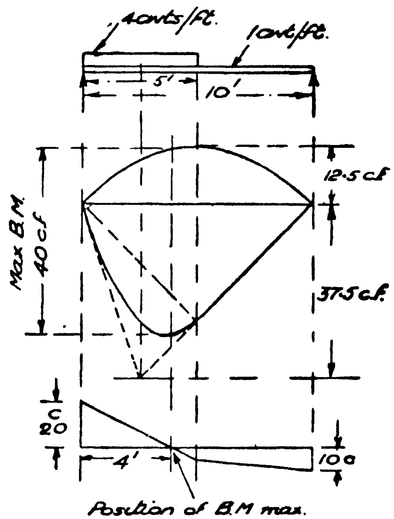


FIG. 406.

(45) If 'f' lb./in.^{2} be max. stress at given section,

$\dfrac{f}{3} = \dfrac{600}{2}$. $\therefore f = 900$ lb./in.^{2}.

$$\left(\frac{W}{2} \times 2 \times 12\right) - \left(\frac{W}{8} \times 2 \times 1 \times 12\right) = \frac{fbd^2}{6} = \frac{900 \times 2 \times 6 \times 6}{6}$$

W = 1200 lb. [*Page* 268.]

(46) W = each load. Central-span moment = $\frac{1}{2}$ the support moment.

If 'l' = central span,

$$\left[\left(\frac{3W}{2} \times \frac{l}{2}\right)\right] - \left[W \times \left(\frac{l}{2} + 2\right)\right] = \frac{W \times 2}{2}$$

$\therefore l =$ 12 ft. [*Page* 222.]

(47) Total weight of wall $= \frac{9}{12} \times \frac{15 \times \cdot 866 \times 15}{2} \times 100$
$=$ 7307 lb.

B.M. at mid-span $= (3653 \cdot 5 \times 4 \cdot 5) - \left(3653 \cdot 5 \times \frac{7 \cdot 5}{3}\right)$
$=$ 7307 lb. ft. Triangular load acts through its C.G.

B.M. at support. Height of wall at support $= 3 \tan 60° =$ 5·196 ft.

Weight of overhanging wall $= (\frac{1}{2} \times 3 \times 5 \cdot 196 \times \frac{9}{12} \times 100)$
$=$ 584·55 lb.

B.M. at support $= 584 \cdot 55 \times \frac{3}{3} = 584 \cdot 55$ lb. ft.

(48) Least radius $= \frac{d}{4} = \frac{5 \cdot 5''}{4} = 1 \cdot 375''$.

$\frac{l}{r} = \frac{13 \cdot 75 \times 12}{1 \cdot 375} = 120$. $F_a = 3 \cdot 3$ tons/in.^{2}.

Safe concentric load $= 3 \cdot 3 \times \frac{\pi \times 5 \cdot 5^2}{4} = 78 \cdot 3$ tons. [*Page* 296.]

(49) Try 14″ × 6″ × 46 lb. section.

$r_{YY} = 1 \cdot 26$ ins. $\frac{l}{r} = \frac{144}{1 \cdot 26} = 114 \cdot 28$.

F_a, by interpolation, $= 3 \cdot 53$ tons/in.^{2}.

Safe axial load $= 3 \cdot 53 \times 13 \cdot 59 = 48 \cdot 0$ tons.

$\therefore$ The section is not suitable.

Try 16″ × 6″ × 50 lb. section.

$r_{YY} = 1 \cdot 24$ ins.

$\frac{l}{r} = \frac{144}{1 \cdot 24} = 116 \cdot 1$.

F_a, by interpolation, $= 3 \cdot 456$ tons/in.^{2}.

Safe axial load $= 3 \cdot 456 \times 14 \cdot 71 = 50 \cdot 84$ tons.

Hence this section is suitable. [*Page* 297.]

(50) $\frac{W}{81} + \frac{W \times 1\frac{1}{2}}{Z} = 600$

$$Z = \frac{bd^2}{6} = \frac{9 \times 9 \times 9}{6} = 121 \cdot 5 \text{ ins.}^3$$

$\frac{W}{81} + \frac{W \times 1 \cdot 5}{121 \cdot 5} = 600. \quad \therefore$ **W = 24300 lb.** [*Page* 302.]

Alternatively, as the load is at the edge of the middle-third, the average stress $= \frac{600}{2} = 300$ lb./in.^{2}.

$\therefore W = 300 \times 81 =$ **24300 lb.** [*Page* 345.]

(51) Normal component of resultant thrust $= 8000 \cos 30°$ $= 6928$ lb.

Pressure at toe $= \frac{V}{b}\left(1 + \frac{6e}{b}\right) = \frac{6928}{9}\left(1 + \frac{6 \times 1}{9}\right)$

= **1283 lb. per sq. foot.**

Pressure at heel $= \frac{V}{b}\left(1 - \frac{6e}{b}\right) = \frac{6928}{9}\left(1 - \frac{6 \times 1}{9}\right)$

= **257 lb. per sq. foot.** [*Page* 354.]

(52) Wind thrust $= (30 \times 15 \times 1) =$ **450 lb.** acting at **7·5 ft.** from the bottom of the wall.

Water thrust $= \frac{1}{2}wh^2 = \frac{1}{2} \times 62 \cdot 5 \times 15^2 =$ **7031·25 lb.** acting at **5 ft.** from bottom of wall.

Earth thrust $= \frac{1}{2}wh^2 \frac{1 - \sin\theta}{1 + \sin\theta} = \frac{1}{2} \times 90 \times 15^2 \times \frac{1}{3} =$ **3375 lb.** acting at **5 ft.** from bottom of wall. [*Pages* 332, 333, 341.]

(53) Earth thrust = 2400 lb. (retained earth only).

Overturning moment = 2400 lb. × 4 ft. = 9600 lb. ft.

Thrust against wall due to superimposed load $= (210 \times \frac{1}{3} \times 12) = 840$ lb.

Overturning moment = 840 lb. × 6 ft. = 5040 lb ft.

Total overturning moment = **14640 lb. ft.** [*Page* 359.]

(54) x' = thickness of wall.

$$W = 6 \times x \times 1 \times 120 = 720x \text{ lb.}$$

$$P = 6 \times 1 \times 15 = 90 \text{ lb.}$$

$720x \times x/6 = 90 \times 3.$

x = 1·5 ft. = 18 ins. [*Page* 352.]

(55) $R_L \times 40 = (150 \times 30) + (100 \times 20) + (50 \times 10)$
$R_L = 175$ cwts. (net reaction).
$R_R \times 40 = (50 \times 30) + (100 \times 20) + (150 \times 10)$
$R_R = 125$ cwts. (net reaction).
$(F_A \times 10) + (150 \times 10) = (175 \times 20)$. [Moments about opposite joint.]
$\therefore F_A =$ **200 cwts.** Member 'A' is a strut.
$F_B = \text{S.F.} \times \sqrt{2} = (125 - 50)\sqrt{2} = 75\sqrt{2}$ cwts.
$=$ **106 cwts.** Member 'B' is a tie. [*Page* 314.]

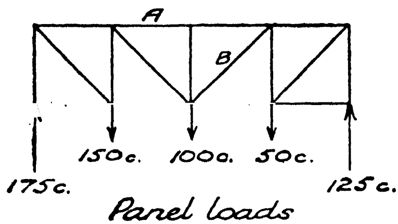


FIG. 407.

(56) Direct stress $= \dfrac{\text{Total load}}{\text{Area of section}} = \dfrac{66}{19{\cdot}12}$ tons/in.^{2} $= 3{\cdot}45$ tons/in.^{2}.

Bending stresses :

XX axis. $f = \dfrac{M}{Z} = \dfrac{10 \times 8}{81{\cdot}3}$ tons/in.^{2} $= {\cdot}984$ tons/in.^{2}.

YY axis. $f = \dfrac{M}{Z} = \dfrac{8 \times 3}{16{\cdot}3}$ tons/in.^{2} $= 1{\cdot}47$ tons/in.^{2}.

Max. compressive stress in steel
$= (3{\cdot}45 + {\cdot}984 + 1{\cdot}47)$ tons/in.^{2}
$=$ **5·91 tons/in.^{2}.** [*Page* 303.]

(57) Direct stress $= \dfrac{\text{Load}}{\text{Area}} = \dfrac{20}{19{\cdot}12}$ tons/in.^{2} $= 1{\cdot}05$ tons/in.^{2}.

Bending stresses :

XX axis. $f = \dfrac{M}{Z} = \dfrac{20 \times 6}{81{\cdot}3}$ tons/in.^{2} $= 1{\cdot}48$ tons/in.^{2}.

YY axis. $f = \dfrac{M}{Z} = \dfrac{20 \times 2}{16{\cdot}3}$ tons/in.^{2} $= 2{\cdot}45$ tons/in.^{2}.

Max. compressive stress $= (1{\cdot}05 + 1{\cdot}48 + 2{\cdot}45)$ tons/in.^{2}
$=$ **4 98 tons/in.^{2}.**

CHAPTER XVII

TEST PAPERS WITH WORKED SOLUTIONS

AT the end of the chapter the numerical answers to the questions set in the test papers are first given. The answers are then fully worked out but the calculations are given in concise form in order to keep the chapter within reasonable limits. When points of theory are raised in a question, page references are given in the solutions.

Test Paper No. 1

(1) Define the term 'centre of gravity.'

Calculate the position of the centre of gravity of the plated T-section given in Fig. 1. Check the calculated position by drawing a link polygon.

(2) A floor is to be composed of timber joists placed at 12" centres. The total inclusive floor load is 80 lb. per sq. foot of floor. The effective span of the joists is 10 ft. Assuming a working stress of 1000 lb. per sq. inch for the timber and a joist breadth of 2", calculate the necessary depth for the joists. Draw the bending moment and shear force diagrams for one of the joists.

(3) Briefly indicate the principal differences in properties between liquid pressure and earth pressure. Calculate the thickness of the wall (Fig. 2) so that it may be just stable from the point of view of tension at the base 'AB.' Verify the calculated thickness by a graphical method.

(4) Fig. 3 gives a sketch graph of the results of a tensile test on a mild steel specimen.

Calculate (i) Young's modulus, (ii) ultimate stress, (iii) the total extension expressed as a percentage of the gauge length.

Using a factor of safety of 4, obtain the safe axial load for a tie-bar of this quality steel, 4" wide × $\frac{3}{4}$" thick.

(5) Draw the stress diagram for the roof truss given in Fig. 4. Distinguish between 'struts' and 'ties' by writing, alongside the members, a plus sign for a strut and a minus sign for a tie.

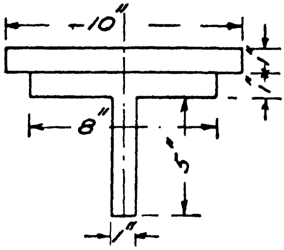
Write down the force in the member indicated by a cross.

(6) Determine the position and magnitude of the maximum bending moment in the beam shown in Fig. 5.

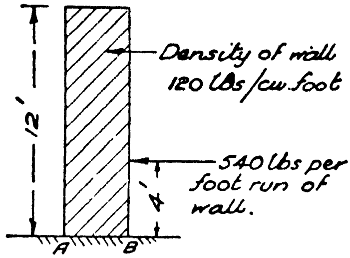
Draw the shear force diagram for the beam.

(7) Show, graphically and by calculation method, that the concurrent system of forces given in Fig. 6 has no resultant vertical effect. What force, introduced into the system, would reduce it to equilibrium?

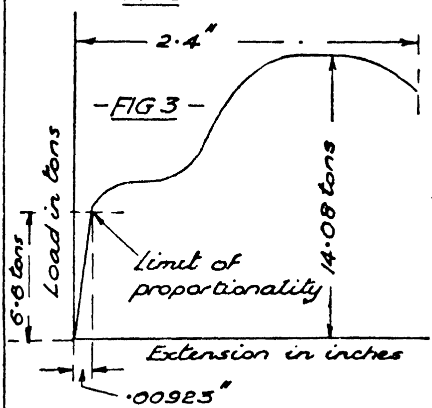
(8) Obtain, by means of a link polygon, the support reactions for the simply supported beam given in Fig. 7. Check the graphical results by calculation method.



- FIG I -

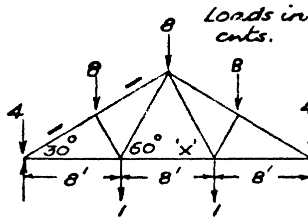


- FIG 2 -

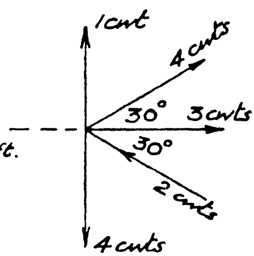


- FIG 3 -

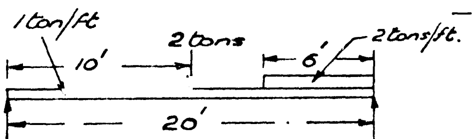
Sectional area of specimen = ·44 ins²
Gauge length for extensions = 8 ins



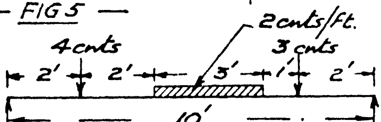
- FIG 4 -



- FIG 6 -



- FIG 5 -



- FIG 7 -

TEST PAPER NO. 1.

Test Paper No. 2

(1) Give the important assumptions upon which the theory of bending is based.

Draw the bending moment and shear force diagrams for the beam shown in Fig. 1.

Calculate the necessary section modulus for the beam. Maximum permissible stress = 10 cwts. per sq. inch.

(2) Draw the stress diagram for the cantilever truss given in Fig. 2.

Tabulate the forces in the respective members of the truss. Distinguish between the members which are in tension and those which are in compression.

(3) Explain briefly the meaning of the following: (i) compressive strain, (ii) shear stress, (iii) working stress, (iv) Young's modulus.

An extensometer, fitted for use with either 8″ or 10″ gauge length, was used to determine 'E' for a mild steel specimen. The following test results were recorded:

Sectional area of specimen	= ·66 sq. ins.
Axial load applied	= 2·145 tons
Corresponding extension	= ·002 ins.

Assuming 'E' to be 13000 tons/in.^{2}, find the gauge length actually used.

(4) Express, by calculation method, the conditions of equilibrium for a non-concurrent co-planar system of forces.

Calculate the force in each bar of the loaded frame given in Fig. 3.

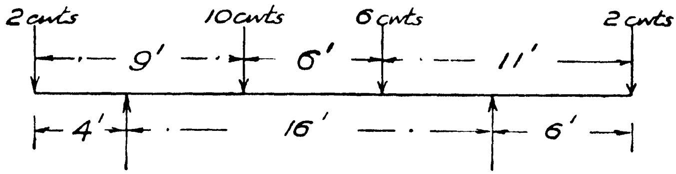
(5) The two steel beams given in Fig. 4 have the same sectional area.

Demonstrate the economy of the rolled steel joist type of section by comparing the total uniformly distributed loads the respective beams will carry for an effective span of 8 ft. Assume a working stress of 8 tons/in.^{2} in each case.

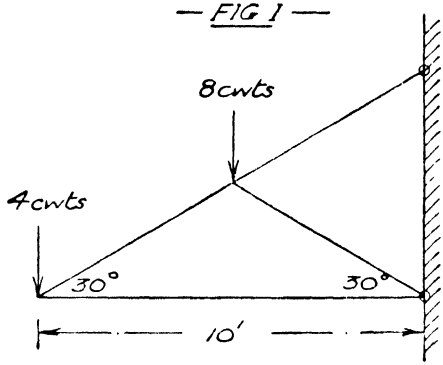
(6) Explain the meaning of the term 'bearing' as applied to rivets in a riveted joint.

Calculate the value of one rivet in the following circumstances: rivet dia. = $\frac{3}{4}$″, plate thickness = $\frac{5}{8}$″, rivet in double shear. $f_s = 6$ tons/in.^{2} and $f_b = 12$ tons/in.^{2}.

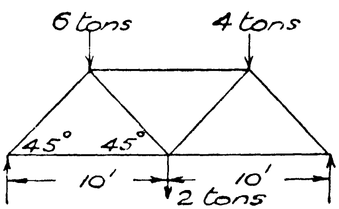
Obtain the maximum safe value for 'P' in the joint given in Fig. 5. Assume the rivet stresses given above and 8 tons/in.^{2} as the working stress in tension in the tie-bars.



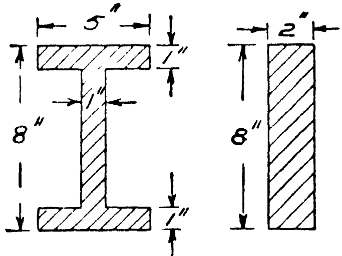
— FIG 1 —



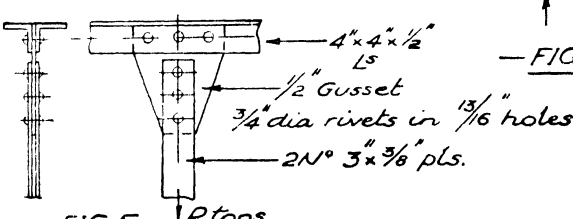
— FIG 2 —



— FIG 3 —



— FIG 4 —



— FIG 5 —

Test Paper No. 2.

Test Paper No. 3

(1) Fig. 1 shows a king-post roof truss carrying positive wind and dead loads. Draw the stress diagram for the truss.

Obtain the force in the member indicated by a cross. Is this member a strut or a tie ? Assuming the trusses to be spaced at 10-ft. centres, calculate the wind pressure (normal to roof slope) and the dead load respectively, in lb. per sq. foot of roof surface, corresponding to the joint loads given.

(2) A beam 'AB' (Fig. 2) rests on two supports. The left and right end supports are composed, respectively, of steel and concrete.

If 'A' and 'B' are still at the same (new) level when load 'W' is applied, calculate 'W' and 'x,' given that the load induces a stress of 100 lb. per sq. inch in the concrete.

$E_{steel} = 30{,}000{,}000$ lb./in.^{2}, $E_{concrete} = 2{,}000{,}000$ lb./in.^{2}.

(3) Find the value of 'w,' in the timber beam example given in Fig. 3, in order that the maximum fibre stress in the timber at a beam section, 2 ft. from the left end, shall be 8 cwts./in.^{2}. The beam is 3″ wide × 6″ deep.

Draw the bending moment and shear force diagrams for the beam, assuming the calculated load.

(4) Test the given retaining wall (Fig. 4) for tension at point 'B.' Obtain the compressive stress at 'A.'

Draw a diagram showing the variation of pressure on the subsoil under the wall.

(5) Fig. 5 shows a composite beam section.

Neglecting rivet holes, find the position of the centre of gravity of the section (i) by calculation, (ii) by any graphical method.

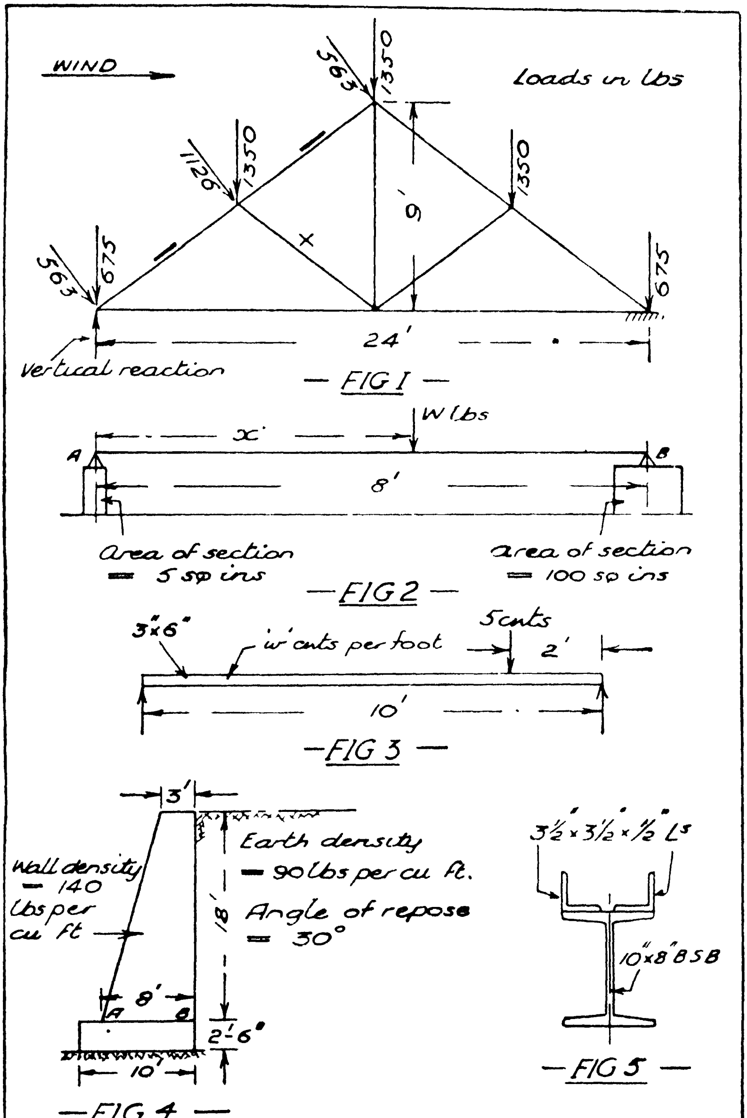
Properties of section :

B.S.B. Area = 16·177 sq. ins.
Angles. Area of one angle = 3·251 sq. ins.
C.G. from back of angle = 1·05 ins.

(6) Prove the formula for the maximum bending moment for a simply supported beam with uniformly distributed load.

A 9″ × 4″ × 21 lb. B.S.B. is used as a simply supported beam for an effective span of 8 ft. Inclusive of its own weight, it carries a uniform load of 6 tons. What additional point load could it carry if the load were placed (i) at the centre of span, (ii) at 2 ft. from the left end ? The maximum stress must not exceed 10 tons/in.^{2}.

Z_{xx} for the B.S.B. = 18·03 ins.^{3}.



TEST PAPER No 3.

Test Paper No. 4

(1) Design suitable timber beams for the floor shown in Fig. 1. Find the necessary section modulus for each of the steel beams 'AB' and 'CD' respectively. Select suitable B.S.B.s from the section tables given on pages 258–265.

(2) A stone wall and lintol (constructed in reinforced concrete) are of 1 ft. uniform thickness. Taking the particulars given in Fig. 2, construct the B.M. and S.F. diagrams for the lintol. The density of both wall and lintol may be assumed to be 140 lb. per cu. foot.

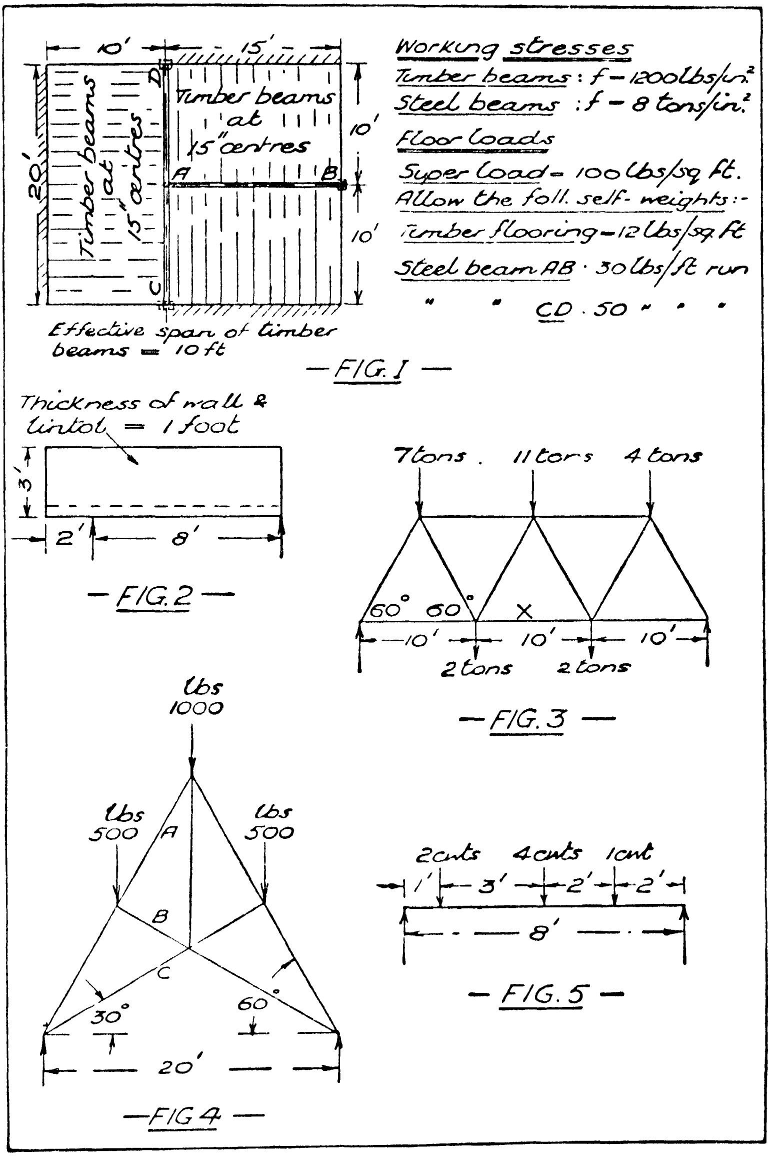
(3) Draw the stress diagram for the braced girder (Fig. 3). Obtain the force in the member indicated by a cross.

(4) Using the method of sections, find the force in each of the members 'A;' 'B,' and 'C' in the truss given in Fig. 4. Distinguish between struts and ties.

(5) Employing the method of the link polygon, construct the B.M. and S.F. diagrams for the simply supported beam (Fig. 5).

Write down the scales clearly, showing how the bending moment scale was derived.

(6) Calculate the necessary density for a vertical wall of rectangular section, 6 ft. high, if no tendency for tension is to be developed at the base joint, assuming it to be 1′ 6″ thick, and to be subjected to a horizontal wind pressure of 15 lb. per sq. foot.



Test Paper No. 4.

Test Paper No. 5

(1) For the given overhanging beam (Fig. 1) calculate (*a*) the support reactions, (*b*) the position and value of maximum bending moment.

Draw the shear force diagram for the beam.

(2) *Calculate* the support reactions for the truss shown in Fig. 2. The reaction at the left end is assumed to be vertical.

(3) A bolt, in double shear, is used with a plate thickness of $\frac{1}{2}''$. Assuming working stresses of 6 tons/in.^{2} and 12 tons/in.^{2} for shear and bearing respectively, calculate the bolt dia. in order that it shall be equally strong in ' bearing ' and ' shear.'

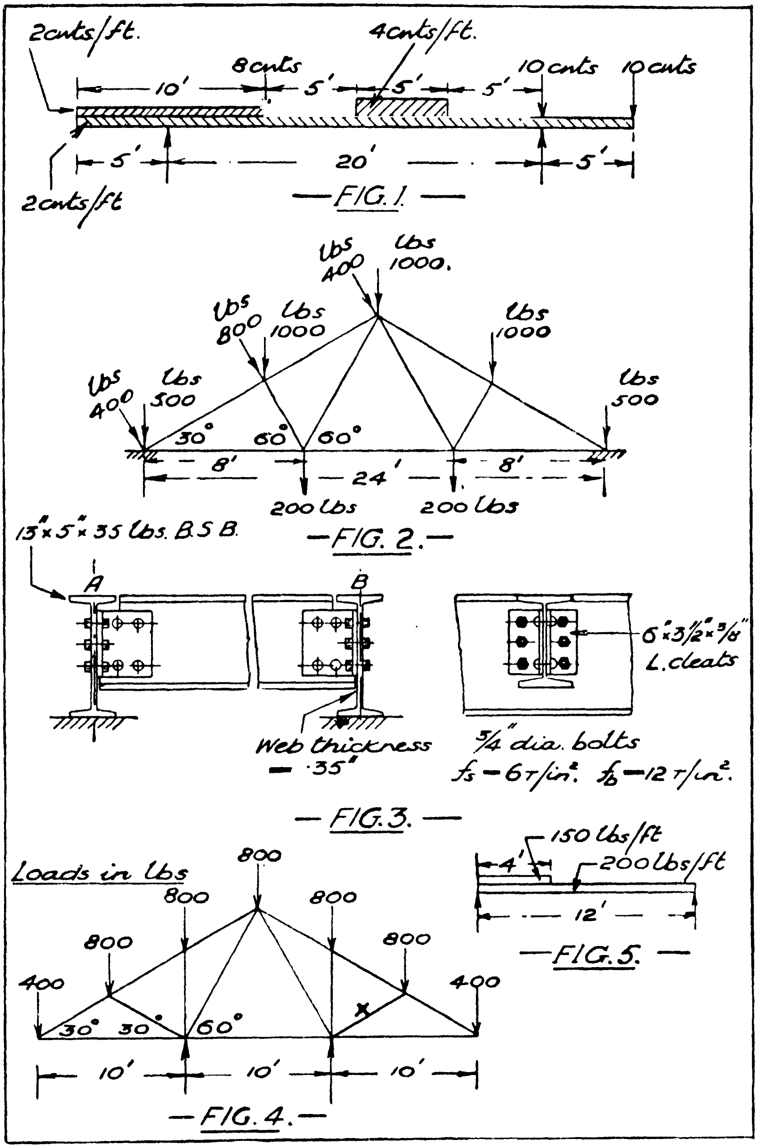
Calculate the safe uniform load for the beam ' AB ' (Fig. 3) from the point of view of the *bolts* in its end connections.

(4) Draw the stress diagram for the roof truss given in Fig. 4. Write, alongside the members, ' S ' for compression and ' T ' for tension. Find the force in the member indicated by a cross.

(5) A specimen of timber, $1'' \times 1''$ in section, was tested as a simple beam with central-point loading. The effective span was $24''$. Assuming the modulus of rupture of the timber to be 10000 lb./in.^{2}, calculate the load at fracture.

What factor of safety is represented by the beam shown in Fig. 5, if the same quality of timber is used ? The beam is $3''$ wide $\times$ $10'$ deep.

(6) Given the expression $\frac{5}{384}\,\frac{Wl^3}{EI}$, find the maximum deflection of a timber beam with simply supported ends, of width $= 2''$ and depth $= 6''$, and of 10-ft. effective span. The beam carries a uniform load of 960 lb. total value. ' E ' for the timber $= 1{,}600{,}000$ lb./in.^{2}.



Test Paper No. 5

Test Paper No. 6

(1) Calculate the maximum permissible depth of water behind the given retaining wall (Fig. 1) if there is to be no tension at the base 'AB.'

(2) Obtain the force in each of the members marked 'A,' 'B,' 'C,' and 'D' respectively in Fig. 2 (i) by calculation, (ii) by drawing a stress diagram.

(3) Calculate the support reactions and draw the stress diagram for the roof truss given in Fig. 3.

(4) Draw the bending moment and shear force diagrams for the cantilever shown in Fig. 4.

(5) Define the terms 'effective column length' and 'slenderness ratio.'

Calculate the safe concentric load for a mild steel solid circular column of 4″ dia. and 9-ft. effective height, given:

l/r	F_a (*tons/in.*^{2})
100	4·1
110	3·7

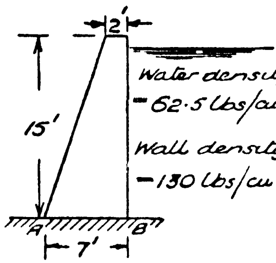
(6) Show that the floor, shown diagrammatically in Fig. 5, can safely support a total inclusive floor load of 160 lb. per sq. foot.

Working stresses: Timber = 1200 lb./in.^{2}
Steel = 10 tons/in.^{2}

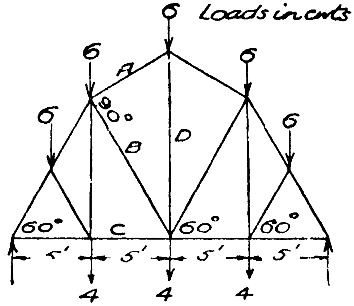
Z_{xx} for 10″ × 5″ × 30 lb. B.S.B. = 29·25 ins.^{3}.

(7) A short concrete pillar (Fig. 6), in addition to its own weight, carries the reactions of two steel beams. Draw the distribution of pressure diagram for the base 'ZZ.'

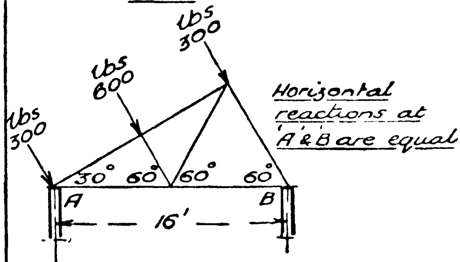
Density of concrete = 130 lb. per cu. foot.



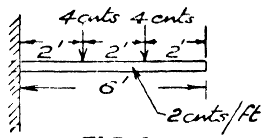
— FIG 1 —



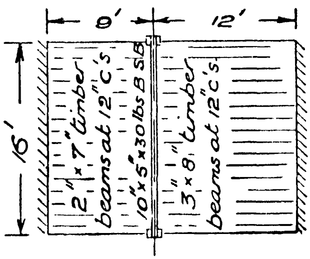
— FIG 2 —



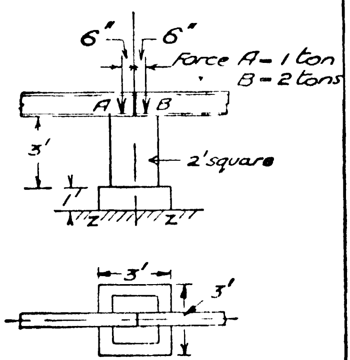
— FIG 3 —



— FIG 4 —



— FIG 5 —



— FIG 6 —

TEST PAPER No. 6.

Answers to Problems in Test Papers

Test Paper No. 1 (*page* 388)

Numerical Answers

(1) C.G. lies on the vertical axis of symmetry of the section, 5·28″ above the bottom.

(2) Necessary depth for joists = 6″.

B.M.$_{max.}$ = 12000 lb. ins.

S.F.$_{max.}$ = ± 400 lb.

(3) Width of wall = 3 ft.

(4) (i) E = 13000 tons/in.^{2}.

(ii) Ultimate stress = 32 tons/in.^{2}.

(iii) 30%.

Safe axial load = 24 tons.

(5) Force in given member = 15 cwts.

(6) B.M.$_{max.}$ occurs at 10·8 ft. from left end.

B.M.$_{max.}$ = 78·32 tons ft.

S.F. values : Left end = 12·8 tons.

Right ,, = 21·2 ,,

(7) Equilibrant = 4·732 cwts. acting horizontally towards the left.

(8) Left-end reaction = 6·5 cwts.

Right-end ,, = 6·5 ,,

Detailed Solutions

(1) Definition of C.G. (page 91).

Area of plate	= 10 ins.^{2}
,, flange	= 8 ,,
,, vertical leg	= 5 ,,
Total area	= 23 ins.^{2}.

Let $\bar{y}$ be height of C.G. above bottom of section.

$$23\bar{y} = (5 \times 2{\cdot}5) + (8 \times 5{\cdot}5) + (10 \times 6{\cdot}5)$$

$$\bar{y} = \frac{121{\cdot}5}{23} = 5{\cdot}28 \text{ ins.}$$

For link polygon construction, treat the areas as 'acting' towards the right (see page 121).

(2) Load per joist $= \left(10' \times \frac{12'}{12}\right) \times 80 \text{ lb./ft.}^2 = 800 \text{ lb.}$

$$\text{B.M.}_{max.} = \frac{Wl}{8} = \frac{800 \times 10 \times 12}{8} \text{ lb. ins.} = 12000 \text{ lb. ins.}$$

$$M = \frac{fbd^2}{6}. \quad \therefore 12000 = \frac{1000 \times 2 \times d^2}{6}.$$

$$d^2 = \frac{12000 \times 6}{1000 \times 2} = 36. \quad \therefore d = 6''.$$

(Forms of B.M. and S.F. diagrams as in Fig. 276.)

$$\text{B.M.}_{max.} = 12000 \text{ lb. ins.}$$

$$\text{S.F.}_{max.} = \pm \frac{W}{2} = \pm 400 \text{ lb.}$$

(3) Differences in liquid and earth pressures (Chapter XV).

Let 'b' ft = necessary thickness of wall.

Weight of wall per foot run = $(12 \times b \times 120) = 1440b$ lb.

Taking moments about outer middle-third point:

$$(540 \times 4) = 1440b \times b/6 = 240b^2.$$

$$\therefore b^2 = \frac{540 \times 4}{240} = 9. \quad \therefore b = 3 \text{ ft.}$$

Graphical method of solution (page 344).

(4) (i) Taking the stress and corresponding strain at the limit of proportionality:

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{6 \cdot 6}{\cdot 44} = 15 \text{ tons/in.}^2$$

$$\text{Strain} = \frac{\text{Extension}}{\text{Original length}} = \frac{\cdot 00923''}{8''} = \cdot 001154.$$

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{15}{\cdot 001154} \text{ tons/in.}^2 = 13000 \text{ tons/in.}^2$$

(ii) Ultimate stress $= \dfrac{\text{Max. load in test}}{\text{Original section area}} = \dfrac{14 \cdot 08}{\cdot 44} = 32$ tons/in.^{2}.

(iii) Percentage elongation $= \dfrac{2 \cdot 4''}{8''} \times 100 = 30.$

$$\text{Working stress} = \frac{\text{Ultimate stress}}{\text{Factor of safety}} = \frac{32 \text{ tons/in.}^2}{4} = 8 \text{ tons/in.}^2.$$

Sectional area of bar = 3 sq. ins.

$\therefore$ Safe axial load = (3×8) tons = 24 tons.

(5) (See Fig. 408.)

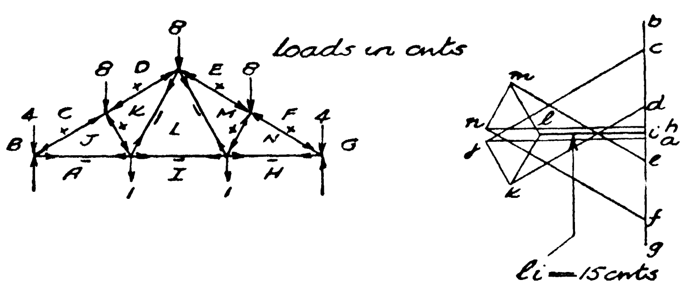


FIG. 408.

(6) $R_L \times 20 = (20 \times 10) + (2 \times 10) + (12 \times 3)$

$R_L = 12 \cdot 8$ tons.

$R_R \times 20 = (20 \times 10) + (2 \times 10) + (12 \times 17)$

$R_R = 21 \cdot 2$ tons.

Load on beam from left end up to a point 10·8 ft. from left end = (10·8 + 2) = 12·8 tons, hence B.M.$_{max.}$ occurs at 10·8 ft. from left end (see page 231).

B.M.$_{max.}$ = (12·8 × 10·8) − (10·8 × 5·4) − (2 × ·8)
= 78·32 tons ft.

Shear diagram.—S.F. at left end = 12·8 tons.
S.F. at right „ = 21·2 „

[Construction of S.F. diagrams, page 231.]

(7) Total up force = (1 + 4 cos 60° + 2 cos 60°) cwts. = 4 cwts.
Total down force = 4 cwts., hence there is vertical equilibrium.
Resultant force = (4 cos 30° + 3 − 2 cos 30°) cwts.
= (2 × ·866 + 3) = 4·732 cwts., acting towards the right, horizontally. Equilibrant of system is ' equal and opposite ' to the resultant.

(8)
$$R_L \times 10 = (4 \times 8) + (6 \times 4{\cdot}5) + (3 \times 2)$$
$$= 32 + 27 + 6 = 65$$
$$R_L = 6{\cdot}5 \text{ cwts.}$$
$$R_R \times 10 = (3 \times 8) + (6 \times 5{\cdot}5) + (4 \times 2)$$
$$= 24 + 33 + 8 = 65$$
$$R_R = 6{\cdot}5 \text{ cwts.}$$

Link-polygon construction (page 125).
Treat uniform load as ' 6 cwts.' concentrated at its centre of length for graphical solution.

Test Paper No. 2 (*page* 390)

Numerical Answers

(1) Maximum bending moment = 34·5 cwts. ft.
Max. positive shear force = 8·5 cwts.
„ negative „ „ = − 7·5 cwts.
Necessary section modulus = 41·4 ins.^{3}.

(2) (See Fig. 409.)

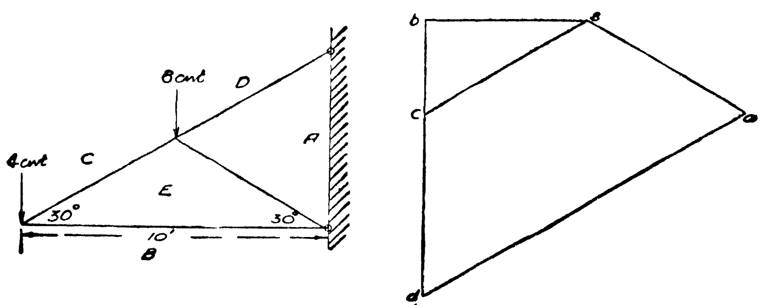


FIG. 409.

Member.	Force.	Type.
	cwts.	
CE	8·00	Tie
DA	16·00	Tie
AE	8·00	Strut
BE	6·93	Strut

(3) 8 ins.
(4) (See detailed solution below.)
(5) Safe uniform load for rectangular beam = 14·22 tons.
„ „ „ steel-beam type = 23·55 „
(6) Strength or value of one rivet = 5·3 tons.
Safe value of 'P' = 13·14 tons.

Detailed Solutions

(1) Assumptions in theory of bending (page 241).
Form of B.M. and S.F. diagrams as on page 222.
If 'A' and 'B' are reaction points and 'C,' 'D,' 'E,' and 'F' the load points in order from left:

$$(R_A \times 16) + (2 \times 6) = (6 \times 5) + (10 \times 11) + (2 \times 20)$$
$$16R_A + 12 = 30 + 110 + 40 = 180$$
$$16R_A = 180 - 12 = 168. \quad \therefore R_A = 10{\cdot}5 \text{ cwts.}$$
$$(R_B \times 16) + (2 \times 4) = (10 \times 5) + (6 \times 11) + (2 \times 22)$$
$$16R_B + 8 = 50 + 66 + 44 = 160$$
$$16R_B = 160 - 8 = 152. \quad \therefore R_B = 9{\cdot}5 \text{ cwts.}$$

$B.M._C = 0$. $B.M._A = (2 \times 4) = 8$ c.f. (negative).
$B.M._B = (2 \times 6) = 12$ c.f. (negative).
$B.M._D = (10{\cdot}5 \times 5) - (2 \times 9)$ c.f. $= (52{\cdot}5 - 18)$ c.f. $= 34{\cdot}5$ c.f.
$B.M._E = (9{\cdot}5 \times 5) - (2 \times 11)$ c.f. $= (47{\cdot}5 - 22)$ c.f. $= 25{\cdot}5$ c.f.
$B.M._F = 0$.

S.F. values: Between 'C' and 'A' = − 2·0 cwts.
„ 'A' „ 'D' = + 8·5 „
„ 'D' „ 'E' = − 1·5 „
„ 'E' „ 'B' = − 7·5 „
„ 'B' „ 'F' = + 2·0 „

Max. B.M. = 34·5 c.f. = 414 c. ins.

$$Z = \frac{M}{f} = \frac{414}{10} = 41{\cdot}4 \text{ ins.}^3$$

(2) (See diagram in numerical answers.)
(3) Definitions (Chapter IX).
Let x'' = gauge length.

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{2{\cdot}145}{{\cdot}66} \text{ tons/in.}^2 = 3{\cdot}25 \text{ tons/in.}^2$$

$\text{Strain} = \frac{\cdot 002''}{x''} = \frac{\cdot 002}{x}.$

$$\therefore 13000 = \frac{3\cdot 25}{\cdot 002/x}. \quad \therefore x = \frac{13000 \times \cdot 002}{3\cdot 25} \text{ ins.} = 8 \text{ ins.}$$

(4) Conditions of equilibrium (page 80).

Notation for braced girder: *Inclined members*, left to right, 1, 2, 3, and 4. *Flange members:* bottom left 5, bottom right 7, top flange member 6.

$R_L \times 20 = (6 \times 15) + (2 \times 10) + (4 \times 5) = 90 + 20 + 20 = 130.$
$\therefore R_L = 6\cdot 5$ tons.
$R_R \times 20 = (4 \times 15) + (2 \times 10) + (6 \times 5) = 60 + 20 + 30 = 110.$
$\therefore R_R = 5\cdot 5$ tons.

$\text{Reduction coefficient for inclined members} = \frac{\text{Length of diagonal}}{\text{Height of truss}}.$

$\text{Length of diagonal} = \sqrt{5^2 + 5^2} = \sqrt{50}.$

$\therefore \text{Reduction coeff.} = \frac{\sqrt{50}}{5} = \sqrt{2}.$

$F_1 = 6\cdot 5\sqrt{2} = 9\cdot 19$ tons (strut).
$F_2 = (6\cdot 5 - 6)\sqrt{2} = \cdot 5\sqrt{2}$ tons $= \cdot 707$ tons (tie).
$F_3 = (5\cdot 5 - 4)\sqrt{2} = 1\cdot 5\sqrt{2}$ tons $= 2\cdot 12$ tons (tie).
$F_4 = 5\cdot 5\sqrt{2} = 7\cdot 78$ tons (strut).
$F_5 \times 5 = 6\cdot 5 \times 5. \quad \therefore F_5 = 6\cdot 5$ tons (tie, moments about opposite joint).
$F_7 \times 5 = 5\cdot 5 \times 5. \quad \therefore F_7 = 5\cdot 5$,, (tie, ,, ,, ,, ,,).
$(F_6 \times 5) + (6 \times 5) = 6\cdot 5 \times 10. \quad \therefore F_6 \times 5 = 65 - 30 = 35.$
$\therefore F_6 = 7$ tons (strut).

(5) Rectangular beam: $Z = \frac{bd^2}{6} = \frac{2 \times 8^2}{6} = 21\cdot 33 \text{ ins.}^3.$

$$I_{XX} \text{ for R.S.J. beam type} = \frac{BD^3}{12} - \frac{bd^3}{12} = \left(\frac{5 \times 8^3}{12} - \frac{4 \times 6^3}{12}\right) \text{ ins.}^4$$
$$= 141\cdot 33 \text{ ins.}^4.$$

$Z_{XX} = \frac{141\cdot 33}{4} = 35\cdot 33 \text{ ins.}^3.$

For rect. beam: $\frac{Wl}{8} = fZ. \quad \therefore \frac{W \times 8 \times 12}{8} = 8 \times 21\cdot 33.$
$$\therefore W = 14\cdot 22 \text{ tons.}$$

For R.S.J. type: $\frac{Wl}{8} = fZ. \quad \therefore \frac{W \times 8 \times 12}{8} = 8 \times 35\cdot 33.$
$$\therefore W = 23\cdot 55 \text{ tons.}$$

The latter beam carries a much greater load for the same weight of steel.

(6) Bearing in rivets (page 183).

D.S. value of rivet $= 2 \times \frac{\pi d^2}{4} \times f_s$ tons $= \frac{2 \times \pi \times (\frac{3}{4})^2}{4} \times 6$ tons $= 5\cdot 3$ tons.

Bearing value $= d \times t \times f_b = \frac{3}{4} \times \frac{5}{8} \times 12$ tons $= 5{\cdot}625$ tons.

$\therefore$ Value of one rivet $= 5{\cdot}3$ tons.

Riveted joint :

(i) Connection between plates and gusset plate.

D.S. value of one rivet $= 5{\cdot}3$ tons.

Bearing value $= dtf_b = \frac{3}{4} \times \frac{1}{2} \times 12 = 4{\cdot}5$ tons.

$\therefore$ Actual value $= 4{\cdot}5$ tons.

$\therefore$ Rivet strength of connection $= 3 \times 4{\cdot}5 = 13{\cdot}5$ tons.

The weakest section for the plates is through the bottom rivet.

Tensile strength $= 2[3 - \frac{13}{16}] \times \frac{3}{8} \times 8$ tons

$= (2 \times 2{\cdot}19 \times 3)$ tons $= 13{\cdot}14$ tons.

(ii) *Connection between gusset plate and angles.*

Rivets in double shear and bearing in $\frac{1}{2}''$ plate.

Rivet strength $= 4{\cdot}5$ tons.

$\therefore$ Strength of connection $= 3 \times 4{\cdot}5 = 13{\cdot}5$ tons.

Safe value of ' P ' = smallest of calculated strengths $= 13{\cdot}14$ tons.

[It would be permissible in this example to use $\frac{13}{16}''$ as the diameter of the rivet in strength calculations.]

Test Paper No. 3 (*page* 392)

Numerical Answers

(1) (See Fig. 410.)

Member ' X ' is a strut. Force = 2300 lb.

Wind pressure = 15 lb. per sq. foot of roof surface.

Dead load = 18 " " " " "

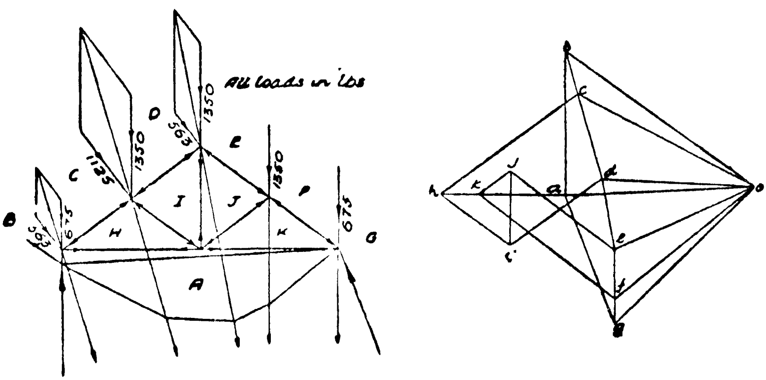


FIG. 410.

(2) W = 17500 lb.

x = 4·57 ft.

(3) w = $1\frac{1}{2}$ cwts. per foot.

Calculations for B.M. and S.F. diagrams given below.

S.M.—14

(4) No tension at point 'B.' (Eccentricity = 1 ft., nearly.)
Compressive stress at 'A' = 3032 lb. per sq. foot.

Foundation pressures :
At toe of wall = 2600 lb. per sq. foot.
,, heel ,, ,, = 871 ,, ,, ,,

(5) C.G. is 6·73 ins. above bottom of section.

(6) (i) 4·5 tons.
(ii) 7 ,,

Detailed Solutions

(1) *Wind pressure.*

Area of roof surface corresponding to a full joint $= \left(\frac{15'}{2} \times 10'\right) = 75$ sq. ft

$$\therefore \text{ Wind pressure} = \frac{1126 \text{ lb.}}{75 \text{ sq. ft.}} = 15 \text{ lb per sq. foot.}$$

Dead load :
Area of roof slope per joint = 75 sq. ft.

$$\therefore \text{ Dead load} = \frac{1350 \text{ lb.}}{75 \text{ sq. ft.}} = 18 \text{ lb. per sq. ft.}$$

(2) Load carried by concrete = stress × area = (100 × 100) lb = 10000 lb.

$$\text{Strain in concrete} = \frac{\text{Stress}}{\text{E}} = \frac{100}{2{,}000{,}000} = \frac{1}{20000}.$$

$$\therefore \text{ Strain in steel} = \frac{1}{20000}.$$

Stress in steel = E × strain $= \left(30{,}000{,}000 \times \frac{1}{20000}\right)$ lb./in.^{2} = 1500 lb./in.^{2}.

Load carried by steel = (1500 × 5) lb. = 7500 lb.
$W = R_A + R_B = (7500 + 10000)$ lb. = 17500 lb.
Moments about A :

$$R_B \times 8 = W \times x$$

$$\therefore x = \frac{10000 \times 8}{17500} \text{ ft.} = 4{\cdot}57 \text{ ft.}$$

(3) Left-end reaction $= \left(\frac{w \times 10}{2} + \frac{5 \times 2}{10}\right) = (5w + 1)$ cwts.

B.M. at 2 ft. from left end $= [(5w + 1) \times 2] - (2w \times 1)$
$= (10w + 2 - 2w)$ c.f. $= (8w + 2)$ c.f.

$$\therefore (8w + 2) \times 12 = \frac{fbd^2}{6} = \frac{8 \times 3 \times 6 \times 6}{6}$$

$$8w + 2 = 12$$

$$w = 1\tfrac{1}{4} \text{ cwts. per foot.}$$

Left-end reaction = 7·25 cwts.
Right-end reaction = 10·25 cwts.

B.M. diagram may be drawn thus: Construct a parabola of height 15·625 c.f. above base and a triangle of height (at load point) 8 c.f. below

base. The total depth of the diagram will represent bending moment values.

The S.F. diagram is directly constructed by usual rules (page 221).

(4) *Joint AB*—deal with portion of wall above AB only.

Earth thrust $= \frac{1}{2} \times 90 \times 18^2 \times \frac{1 - \cdot 5}{1 + \cdot 5} = 4860$ lb.

Weight of wall per foot $= \left(\frac{8+3}{2} \times 18 \times 140\right)$ lb. $= 13860$ lb.

The solution may be carried through graphically (as on page 344), or by calculation method.

Calculation method :

$$\bar{x} = \text{distance of C.G. from back of wall}$$
$$99\bar{x} = (54 \times 1\cdot 5) + (45 \times 4\tfrac{2}{3}) = 291$$
$$\bar{x} = 2\cdot 94 \text{ ft.}$$

'*Shift*' of resultant $= \frac{4860 \times 6}{13860}$ ft. $= 2\cdot 104$ ft.

Resultant cuts base at $(2\cdot 94 + 2\cdot 104)$ ft. $=$ (say) 5 ft. from back of wall.

Outer middle-third point $= (\frac{2}{3} \times 8)$ ft. $= 5\frac{1}{3}$ ft. from back of wall.

$\therefore$ Resultant cuts base just inside middle-third. $\therefore$ There will be no tension at 'B.'

Eccentricity $= (5 - 4) = 1$ ft.

Compressive stress at 'A' $= \frac{V}{b}\left(1 + \frac{6e}{b}\right) = \frac{13860}{8}\left(1 + \frac{6 \times 1}{8}\right) = 3032$ lb./ft.^{2}.

Total wall section :

Earth thrust $= \frac{1}{2} \times 90 \times 20\cdot 5^2 \times \frac{1}{3} = 6303\cdot 75$ lb.

Weight of wall $= (99 + 25) \times 140 = 17360$ lb.

$(54 + 45 + 25)\bar{x} = (54 \times 1\frac{1}{2}) + (45 \times 4\frac{2}{3}) + (25 \times 5)$

$$\bar{x} = 3\cdot 35 \text{ ft.}$$

Shift $= \left(6303\cdot 75 \times \frac{20\cdot 5}{3}\right) \div 17360 = 2\cdot 48$ ft.

Resultant cuts base at $(3\cdot 35 + 2\cdot 48)$ ft. $= 5\cdot 83$ ft. from back of wall.

$\therefore$ Eccentricity $= \cdot 83$ ft.

Foundation pressures :

At toe : $\frac{V}{b}\left(1 + \frac{6e}{b}\right) = \frac{17360}{10}\left(1 + \frac{6 \times \cdot 83}{10}\right) = 2600$ lb./ft.^{2}.

" heel : $\frac{V}{b}\left(1 - \frac{6e}{b}\right) = \frac{17360}{10}\left(1 - \frac{6 \times \cdot 83}{10}\right) = 871$ lb./ft.^{2}.

The pressure-variation diagram is linear, as in Fig. 388. There is no uplift at the base.

(5) The C.G. is on the vertical axis of symmetry.

Let $\bar{y}$ = height above base.

Total area of section $= (16\cdot 177 + 3\cdot 251 + 3\cdot 251)$ sq. ins.

$= 22\cdot 679$ sq. ins.

$$\therefore 22\cdot 679\bar{y} = (16\cdot 177 \times 5) + (6\cdot 502 \times 11\cdot 05) = 152\cdot 732$$
$$\therefore \bar{y} = 6\cdot 73 \text{ ins.}$$

Graphical solution may be effected by the method given on page 106, or by link-polygon construction.

(6) Formula for given beam example is proved on page 213.

(i) Moment of resistance of section $= fZ = (10 \times 18{\cdot}03)$ tons ins. $= 180{\cdot}3$ tons ins.

B.M. due to 6 tons load $= \dfrac{6 \times 8 \times 12}{8} = 72$ tons ins.

$\therefore$ Available B.M. for central load $= (180{\cdot}3 - 72)$ tons ins. $= 108{\cdot}3$ tons ins.

Let W tons = load

$$\frac{W \times 8 \times 12}{4} = 108{\cdot}3$$

$$W = \frac{108{\cdot}3}{24} = 4{\cdot}5 \text{ tons.}$$

(ii) Note that the B.M.$_{max}$ values due to the two component load systems must not be added in this case, as the max. values do not occur at the same beam section.

Let 'W' tons be the safe additional load. Assume, as a first trial, that the B.M.$_{max.}$ will occur at the concentrated-load point.

$$R_L \times 8 = (W \times 6) + (6 \times 4)$$
$$8R_L = 6W + 24$$
$$R_L = (\cdot 75W + 3) \text{ tons.}$$

B.M. at load point $= [(\cdot 75W + 3) \times 2 - (1{\cdot}5 \times 1)]$ tons ft.

$$\therefore (\cdot 75\,W + 3) \times 2 - 1{\cdot}5 = \frac{180{\cdot}3}{12}$$
$$1{\cdot}5W + 6 = 15 + 1{\cdot}5 = 16{\cdot}5$$
$$1{\cdot}5W = 10{\cdot}5$$
$$W = 7 \text{ tons.}$$

Using the zero shear rule for position of max. B.M., we find that our assumption for B.M $_{max.}$ position was correct.

For the B.M.$_{max.}$ to occur in the portion of span between the load point and right end, the right-end reaction must be less than (6 ft. $\times \cdot 75$ tons/ft.) $= 4{\cdot}5$ tons., i.e. $\left(\dfrac{W \times 2}{8} + 3\right)$ less than $4{\cdot}5$.

$\therefore \cdot 25W$ less than $1{\cdot}5$

$\therefore$ 'W' less than 6 tons.

The beam can safely carry an additional 7 tons at 2 ft. from the left end.

Test Paper No. 4 (*page* 394)

Numerical Answers

(1) Suitable dimensions for timber beams, 3″ wide × 6″ deep.

Steel beam AB.—Necessary $Z = 21{\cdot}66$ ins.^{3}, say, a 10″ × 4½″ × 25 lb. B.S.B. ($Z = 24{\cdot}47$ ins.^{3}). An 8″ × 5″ × 28 lb. has a nearer section modulus but is a heavier section.

Steel beam C.D.—Necessary $Z = 49{\cdot}3$ ins.^{3}, say, a $12'' \times 6'' \times 44$ lb. ($Z = 52{\cdot}79$ ins.^{3}), or a $15'' \times 5'' \times 42$ lb. ($Z = 57{\cdot}1$ ins.^{3}).

(2) B.M. at left support = 840 lb. ft. (negative).

Max. B.M. (for right span) occurs at 3·75 ft. from the right-end reaction and = 2953 lb. ft.

S.F. at left support = − 840 lb. (to left) and 1785 lb. (to right).

S.F. „ right „ = − 1575 lb.

(3) Force in given bar = 15 tons (tension).

(See Fig. 411.)

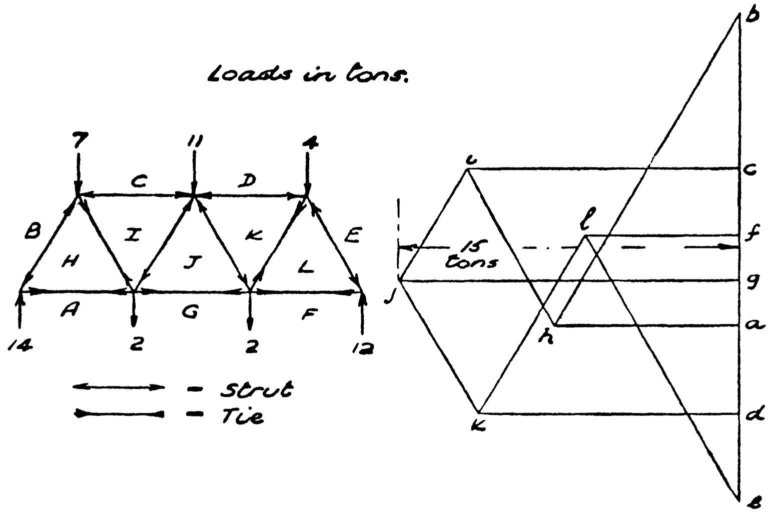


FIG 411.

(4) Member A. Strut. Force = 1299 lb.
„ B. „ „ = 250 „
„ C. Tie. „ = 1000 „

(5) B.M. at 2 cwts. load = 4 cwts. ft.
„ „ 4 „ „ = 10 „ „
„ „ 1 „ „ = 6 „ „

[Method of construction given on page 234.]

(6) Necessary density of wall = 120 lb. per cu. foot.

Detailed Solutions

(1) *Timber beams :*

Area supported by one beam $= 10' \times \frac{15'}{12} = 12{\cdot}5$ sq. ft.

U.D. load carried $= (12{\cdot}5 \times 112)$ lb. $= 1400$ lb.

Max. B.M. $= \frac{Wl}{8} = \frac{1400 \times 10 \times 12}{8}$ lb. ins. $= 21000$ lb. ins.

$M = \frac{fbd^2}{6}$. $\therefore 21000 = 1200 \times \frac{bd^2}{6}$. $\therefore bd^2 = 105$.

If 'b' $= 3''$, $d^2 = 35$. $\therefore d =$ say $6''$.

Steel beam 'AB':

Loads carried: U.D. load + self-weight
$= (150 \text{ sq. ft.} \times 112 \text{ lb./sq. ft.}) + (15 \times 30)$
$= (16800 + 450) \text{ lb.} = 17250 \text{ lb.} = 7{\cdot}7$ tons.

Max. B.M. $= \frac{Wl}{8} = \frac{7{\cdot}7 \times 15 \times 12}{8}$ tons ins. $= 173{\cdot}25$ tons ins.

$M = fZ$. $\therefore 173{\cdot}25 = 8 \times Z$. $\therefore Z = 21{\cdot}66 \text{ ins.}^3$.

Steel beam CD:

Loads carried: U.D. load from left floor bay + self-weight + central-point load ($= R_A$).

Load from left bay $= (5 \times 20 \times 112)$ lb. $= 11200$ lb.

Self-weight $= 50 \times 20 = 1000$ lb.

$\therefore$ Total U.D. load $= 12200$ lb. $= 5{\cdot}45$ tons.

Max. B.M. due to U.D. load $= \frac{Wl}{8} = \left(\frac{5{\cdot}45 \times 20 \times 12}{8}\right)$ tons ins.
$= 163{\cdot}5$ tons ins.

B.M.$_{max.}$ due to central load $= \left(\frac{3{\cdot}85 \times 20 \times 12}{4}\right)$ tons ins. $= 231{\cdot}0$ tons ins. $\left(R_A = \frac{W}{2} = \frac{7{\cdot}7 \text{ tons}}{2} \doteq 3{\cdot}85 \text{ tons.}\right)$

$\therefore$ Total max. B.M. $= (163{\cdot}5 + 231{\cdot}0)$ tons ins. $= 394{\cdot}5$ tons ins.

$Z = \frac{M}{f} = \frac{394{\cdot}5}{8} = 49{\cdot}3 \text{ ins.}^3$.

(2) Total weight of wall plus lintol $= (10 \times 3 \times 1 \times 140)$ lb. $= 4200$ lb.
Weight per foot run $= 420$ lb.
$R_L \times 8 = 4200 \times 5$. $\therefore R_L = 2625$ lb.
$R_R \times 8 = 4200 \times 3$. $\therefore R_R = 1575$ lb.
B.M. at left support $= [(420 \times 2) \times \frac{2}{2}] = 840$ lb. ft.

B.M.$_{max.}$ in right span occurs at x' from right support, where $420 \times x = R_R = 1575$.

$\therefore x = \frac{1575}{420}$ ft. $= 3{\cdot}75$ ft.

B.M.$_{max.}$ $= \left[(1575 \times 3{\cdot}75) - \left(1575 \times \frac{3{\cdot}75}{2}\right)\right]$ lb. ft. $= 2953{\cdot}125$ lb. ft.

'$\frac{Wl}{8}$' in right span for construction purposes $= \frac{(420 \times 8) \times 8}{8}$ lb. ft. $= 3360$ lb. ft.

(3) *By calculation* (moments about opposite joint)
$R_L \times 30 = (7 \times 25) + (2 \times 20) + (11 \times 15) + (2 \times 10) + (4 \times 5)$
$R_L = 14$ tons.
$(F_X \times 8{\cdot}66) + (2 \times 5) + (7 \times 10) = (14 \times 15)$. $F_X = 15$ tons.

(4) *Member 'A.'*—Take moments about the intersection point of members 'B' and 'C.'

$$(F_A \times \text{arm}) + (500 \times 5) = R_L \times 10$$
$$(F_A \times 10 \tan 30^\circ) + 2500 = 1000 \times 10 = 10000.$$
$$\therefore F_A = \frac{7500}{5{\cdot}774} \text{ lb.} = 1299 \text{ lb. (strut).}$$

Member 'B.'—Take moments about the intersection point of members 'A' and 'C.'

$$(F_B \times 10) = 500 \times 5$$
$$\therefore F_B = 250 \text{ lb. (strut).}$$

Member 'C.'—Take moments about the intersection point of members 'A' and 'B.'

$$(F_C \times 10 \sin 30^\circ) = R_L \times 5$$
$$\therefore F_C \times 5 = 1000 \times 5$$
$$\therefore F_C = 1000 \text{ lb. (tie).}$$

Struts and ties are determined as explained on page 309.

(5) See page 234 for method by link polygon.

$R_L \times 8 = (2 \times 7) + (4 \times 4) + (1 \times 2) = 14 + 16 + 2 = 32$

$R_L = 4$ cwts.

$R_R \times 8 = (1 \times 6) + (4 \times 4) + (2 \times 1) = 6 + 16 + 2 = 24$

$R_R = 3$ cwts.

B.M. at 2 cwts. load $= 4 \times 1 \quad = 4$ c.f

" " 4 " " $= (4 \times 4) - (2 \times 3) = 16 - 6 = 10$ c.f.

" " 1 " " $= (3 \times 2) = 6$ c.f.

(6) Let 'x' lb. per cu. foot = density.

Weight of wall per foot run $= (6 \times 1\frac{1}{2} \times x) = 9x$ lb.

Total wind thrust per foot run $= (6 \times 1 \times 15) = 90$ lb.

Taking moments about outer middle-third point:

$$9x \times \frac{1{\cdot}5}{6} = 90 \times 3$$
$$2{\cdot}25x = 270$$
$$x = 120.$$

Density = 120 lb. per cu. foot.

Test Paper No. 5 (*page* 396)

Numerical Answers

(1) (See Fig. 412.)

Left-end reaction = 61 cwts.

Right-end " = 67 "

Max. B.M. occurs at 10·5 ft. from left support.

$B.M._{max} = 145{\cdot}75$ cwts. ft.

(2) $R_L = 3123{\cdot}7$ lb.

$R_R = 2779$ lb. making an angle of $73\frac{1}{2}^\circ$ (nearly) with the horizontal.

(3) Bolt diameter = ·636″, say $\frac{5}{8}$″.

Safe load for beam = 31·8 tons.

(4) Force in member = 800 lb. Member is a strut.

(See diagram in detailed solutions later.)

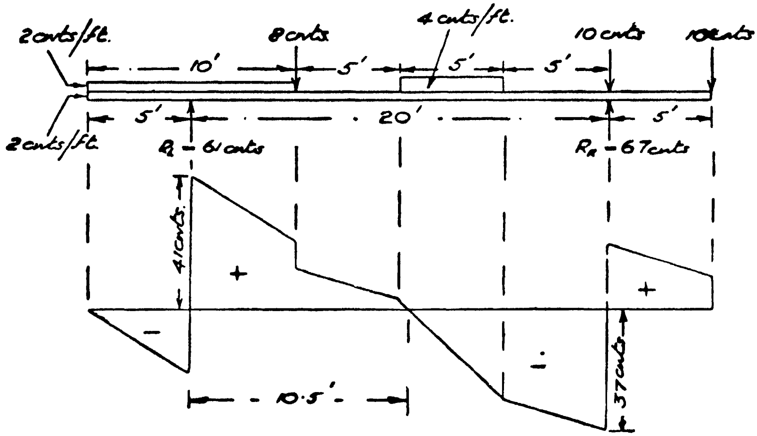


FIG. 412.

(5) Central load at fracture = 278 lb.
Factor of safety = 10 (nearly).
(6) Max. deflection = ·375 ins.

Detailed Solutions

(1) $(R_L \times 20) + [(2 \times 5) \times 2{\cdot}5] + (10 \times 5) = [(2 \times 25) \times 12{\cdot}5] + [(2 \times 10) \times 20] + (8 \times 15) + [(4 \times 5) \times 7{\cdot}5]$

$20R_L + 25 + 50 = 625 + 400 + 120 + 150 = 1295$

$20R_L = 1220. \quad \therefore R_L = 61$ cwts.

$(R_R \times 20) + [(2 \times 5) \times 2{\cdot}5] = [(2 \times 25) \times 12{\cdot}5] + (10 \times 25) + (10 \times 20) + [(4 \times 5) \times 12{\cdot}5] + (8 \times 5)$

$20R_R = 1340. \quad \therefore R_R = 67$ cwts.

$R_L + R_R = 128$ cwts., which is correct.

B.M. at left support = $(4 \times 5 \times 2{\cdot}5) = 50$ c.f. (−).

" " *right* " = $(10 \times 5) + (10 \times 2{\cdot}5) = 75$ c.f. (−).

B.M.$_{max.}$ *in central span* (using zero shear rule).

Up to beginning of 4 cwts./ft. load, load on span

$= [(15 \times 2) + (10 \times 2) + 8] = 58$ cwts.

$(61 - 58) = 3$ cwts. more required.

At 6 cwts./ft. run, this means ·5 ft.

∴ B.M.$_{max.}$ occurs at 10·5 ft. from the left-end support.

$$\text{B.M.}_{max.} = (61 \times 10{\cdot}5) - \left(2 \times 15{\cdot}5 \times \frac{15{\cdot}5}{2}\right) - (20 \times 10{\cdot}5) - (8 \times 5{\cdot}5) - ({\cdot}5 \times 4 \times {\cdot}5/2)$$

$$= 640{\cdot}5 - 494{\cdot}75 = 145{\cdot}75 \text{ cwts. ft.}$$

Hence absolute maximum bending moment = 145·75 cwts. ft.
Construct S.F. diagram by rules given on page 221.

(2) *Reactions due to vertical loads :*

Total load on truss = 4400 lb

$\therefore$ $R_L = R_R = 2200$ lb. (acting vertically).

Reactions due to inclined loads :

The resultant inclined load = 1600 lb. We may assume this resultant force to act at any point in its line of action. A convenient point is where it cuts the lower tie, i.e. at the point where the left '200 lb.' acts.

Resolving vertically, V = 1600 cos 30° = 1385·6 lb.

$$V_L = \frac{1385 \cdot 6 \times 16}{24} = 923 \cdot 7 \text{ lb}$$

$$V_R = \frac{1385 \cdot 6 \times 8}{24} = 461 \cdot 9 \text{ lb.}$$

$$H_L = 0. \quad H_R = 1600 \cos 60° = 800 \text{ lb.}$$

Total reactions :

$R_L = (2200 + 923 \cdot 7) = 3123 \cdot 7$ lb.

Total $V_R = (2200 + 461 \cdot 9) = 2661 \cdot 9$ lb.

„ $H_R = 800$ lb.

$\therefore R_R = \sqrt{2661 \cdot 9^2 + 800^2} = 2779$ lb.

If reaction make 'θ' with horizontal

$$\tan \theta = \frac{V}{H} = \frac{2661 \cdot 9}{800} = 3 \cdot 3274$$

$$\theta = 73\tfrac{1}{2}° \text{ nearly.}$$

(3) For equal strengths $\dfrac{2\pi d^2}{4} f_s = dtf_b$

$$\therefore \frac{2 \times \pi \times d^2 \times 6}{4} = d \times \tfrac{1}{2} \times 12.$$

$$\therefore d = \frac{4}{2\pi} = \cdot 636 \text{ ins., say } \tfrac{5}{8}''.$$

In the given connection the bolts are in single shear, or bearing in either $\frac{3}{8}''$ angle thickness or ·35″ web thickness, i.e. a lesser thickness of ·35″.

Single-shear value of one bolt $= \dfrac{\pi d^2}{4} f_s = \dfrac{\pi \times \cdot 75^2 \times 6}{4} = 2 \cdot 65$ tons.

Bearing value in ·35″ plate thickness $= dtf_b$

$= \cdot 75 \times \cdot 35 \times 12$ tons $= 3 \cdot 15$ tons.

$\therefore$ Value of one bolt = 2·65 tons.

$\therefore$ Strength of 6 bolts = 15·9 tons.

$\therefore$ Safe load for beam = 2 × 15·9 = 31·8 tons.

(4) (See Fig. 413.)

(5) Let 'W' lb. = central-point load at fracture.

$$\frac{Wl}{4} = \frac{fbd^2}{6}. \quad \therefore \frac{W \times 24}{4} = \frac{10000 \times 1 \times 1^2}{6}.$$

$$\therefore W = 278 \text{ lb.}$$

$R_L \times 12 = (600 \times 10) + (2400 \times 6) = (6000 + 14400) = 20400$

$R_L = 1700$ lb.

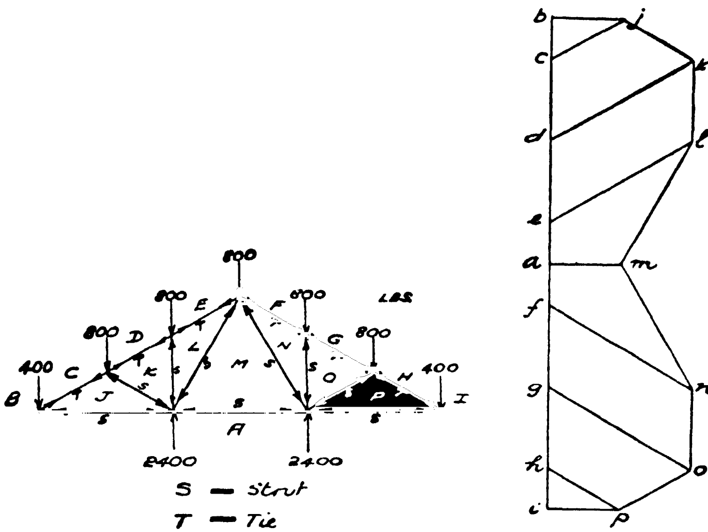


FIG. 413.

$$R_B \times 12 = (600 \times 2) + (2400 \times 6) = (1200 + 14400) = 15600$$
$$R_B = 1300 \text{ lb.}$$

B.M.$_{max}$ will occur at $\frac{1300}{200} = 6{\cdot}5$ ft. from right end.

$$\text{B.M.}_{max} = (1300 \times 6{\cdot}5) - \left(1300 \times \frac{6{\cdot}5}{2}\right) \text{ lb. ft.}$$
$$= (1300 \times 3{\cdot}25) \text{ lb. ft.} = 4225 \text{ lb. ft.}$$
$$4225 \times 12 = \frac{f \times 3 \times 10 \times 10}{6}$$
$$f = \frac{24 \times 4225}{100} \text{ lb./in.}^2$$
$$= 1014 \text{ lb./in.}^2.$$

$$\text{Factor of safety} = \frac{10000 \text{ lb./in.}^2}{1014 \text{ lb./in.}^2} = 10 \text{ (nearly).}$$

(6) Max. deflection $= \frac{5}{384} \frac{Wl^3}{EI}$.

$W = 960$ lb., $l = 120''$, $E = 1{,}600{,}000$ lb./in.^{2}.

$$I = \frac{bd^3}{12} = \frac{2 \times 6 \times 6 \times 6}{12} \text{ ins.}^4 = 36 \text{ ins.}^4.$$

$$\text{Max. deflection} = \frac{5}{384} \times \frac{960 \times 120 \times 120 \times 120}{1{,}600{,}000 \times 36} = \tfrac{3}{8}''.$$

Test Paper No. 6 (*page* 398)

Numerical Answers

(1) Depth of water permissible = 12·25 ft.

(2) 'A' = 11·5 cwts. Strut.
 'B' = ·87 ,, ,,
 'C' = 10·4 ,, Tie.
 'D' = 5·5 ,, ,, (See Fig. 414.)

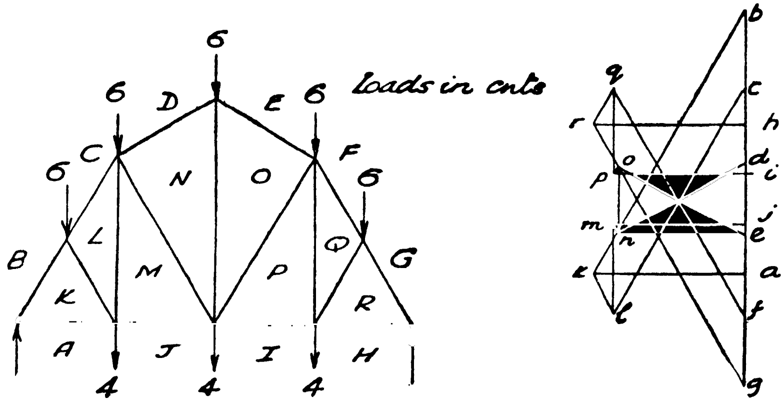


FIG. 414.

(3) $R_A = R_B = 600$ lb. (See Fig. 415.)

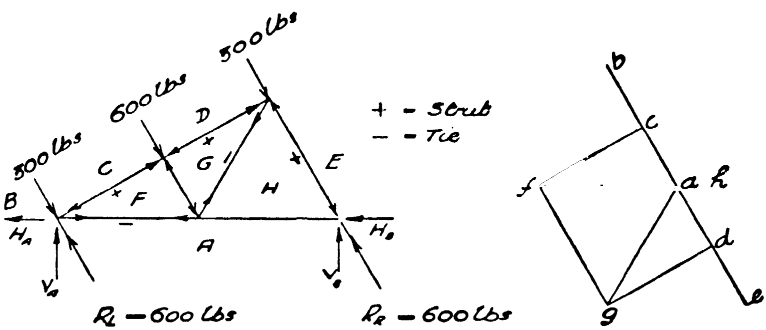


FIG. 415.

Each reaction makes 60° with horizontal.

(4) B.M. at support = 60 cwts. ft.
 S.F. ,, ,, = 20 cwts.

(See Fig. 416.)

Details of calculation given in solutions later.

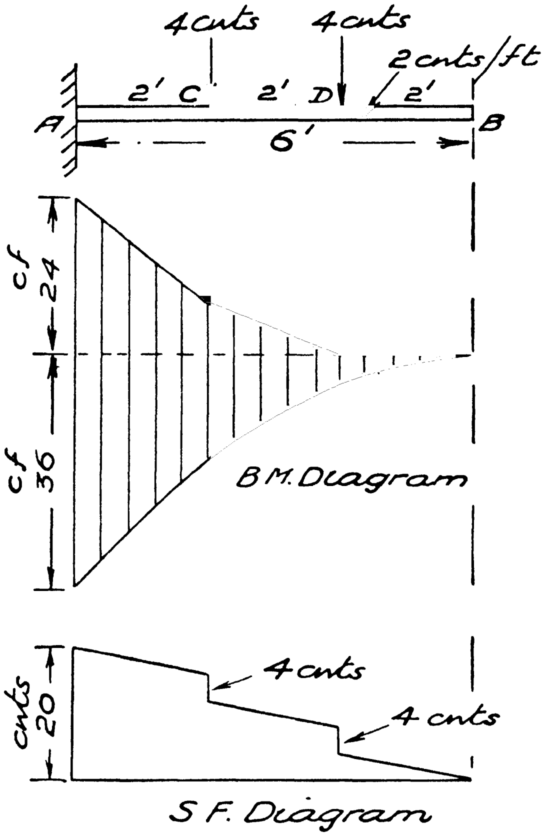


FIG 416

(5) 47·5 tons. Definitions (see pages 287 and 288)

(6) Max stress in 2″ × 7″ beams = 1190 lb /in ².

" " " 3″ × 8″ " = 1080 "

" " " steel beam = 9·85 tons/in ².

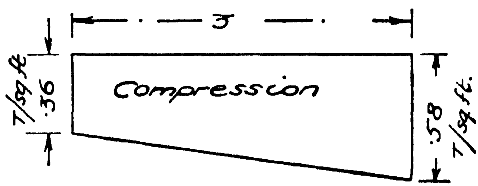


FIG. 417.

(7) Max. pressures. Left edge of base = ·36 tons per sq. foot.
Right " " " = ·58 " " "

(See Fig. 417.)

Detailed Solutions

(1) Weight of wall per foot $= \frac{1}{2}(7 + 2) \times 15 \times 130 = 8775$ lb.

If 'h' ft. = depth of water, water thrust per foot run of wall $= \frac{1}{2}wh^2$ $= \frac{1}{2} \times 62{\cdot}5 \times h^2$ lb.

Let $\bar{x}$ = distance of C.G. of wall from back

$$67{\cdot}5\bar{x} = (30 \times 1) + (37{\cdot}5 \times 3\tfrac{2}{3}) = 167{\cdot}5$$
$$\bar{x} = 2{\cdot}481 \text{ ft.}$$

Taking moments about outer middle-third point:

$$\tfrac{1}{2} \times 62{\cdot}5 \times h^2 \times h/3 = 8775 \times \text{'shift'}$$
$$= 8775 \times [(\tfrac{2}{3} \times 7) - 2{\cdot}481]$$
$$\therefore h^3 = \frac{2 \times 3 \times 8775 \times 2{\cdot}185}{62{\cdot}5} = 1840{\cdot}6$$
$$\therefore h = 12{\cdot}25 \text{ ft.}$$

(2) $R_L = R_R = \frac{42}{2} = 21$ cwts.

Imagine a section plane cutting members 'A,' 'B,' and 'C.'

Member A.—Moments about mid-point of bottom tie.

$(F_A \times 10) + (6 \times 5) + (4 \times 5) + (6 \times 7{\cdot}5) = 21 \times 10.$

$10F_A + 30 + 20 + 45 = 210.$

$10F_A = 210 - 95 = 115. \quad \therefore F_A = 11{\cdot}5$ cwts.

Member 'A' is a strut.

Member 'B.'—Moments about intersection point of 'A' and 'C,' i.e. a point level with the bottom tie and 10 ft. to the left of the left reaction point.

Arm of moment for $F_B = 20 \cos 30° = 17{\cdot}32$ ft.

Assuming member 'B' to be pulling at the section plane:

$(F_B \times 17{\cdot}32) + (6 \times 15) + (4 \times 15) + (6 \times 12{\cdot}5) = 21 \times 10.$

$17{\cdot}32F_B + 225 = 210. \quad \therefore 17{\cdot}32F_B = -15.$

$F_B = -\,{\cdot}87$ cwts.

The negative sign indicates that F_B is not pulling at the section plane, hence member 'B' is a strut.

Member C.—Moments about intersection point of members 'A' and 'B.'

Arm of moment $= 5 \tan 60° = 5 \times \sqrt{3} = 8{\cdot}66$ ft.

$$\therefore F_C \times 8{\cdot}66 + (6 \times 2{\cdot}5) = 21 \times 5$$
$$8{\cdot}66F_C = 105 - 15 = 90$$
$$F_C = 10{\cdot}4 \text{ cwts.}$$

Member 'C' is a tie.

Member D.—Consider vertical equilibrium at the top end of the member.

$$2 \times F_A \cos 60° = 6 + F_D$$
$$\therefore F_D = (2 \times 11{\cdot}5 \times \tfrac{1}{2}) - 6 = 5{\cdot}5 \text{ cwts.}$$

As member 'D' pulls downwards it is a tie.

(3) Resultant wind load = 1200 lb.

Resolving the resultant, assuming it to act at the middle point of bottom tie :

$$V = 1200 \cos 30° = 1200 \times \cdot 866 = 1039 \cdot 2 \text{ lb.}$$

$$\therefore V_A = V_B = 519 \cdot 6 \text{ lb.}$$

$$H_A = H_B = \frac{1200 \cos 60°}{2} = 300 \text{ lb.}$$

$$R_A = R_B = \sqrt{519 \cdot 6^2 + 300^2} = 600 \text{ lb.}$$

If 'θ' be inclination of each reaction to horizontal,

$$\tan \theta = \frac{V}{H} = \frac{519 \cdot 6}{300} = 1 \cdot 732.$$

$$\therefore \theta = 60°.$$

(4) B.M. at support due to U.D. load = 12 × 3 = 36 cwts. ft.

" " D " " point loads = 0

" " C " " " " = 8 cwts. ft.

" " A " " " " = (4 × 4) + (4 × 2) = 24 cwts. ft.

The B.M. diagrams for the load systems are shown in Fig. 416, one being drawn above the base line and the other below. The net B.M. diagram is therefore the addition of the two diagrams, i.e. B.M. values are obtained by scaling the total depth of the combined diagrams. If desired a new diagram may be obtained by plotting ordinates afresh from a horizontal base line, the ordinate at any given point being the sum of the corresponding ordinates in the component diagrams. The S.F. diagram is plotted directly.

$$\text{S.F.}_{max.} = [(2 \times 6) + 4 + 4] \text{ cwts.} = 20 \text{ cwts.}$$

(5) Least radius of gyration $= \frac{\text{Diameter}}{4} = \frac{4''}{4} = 1''.$

Slenderness ratio $= \frac{108''}{1''} = 108.$

l/r	F_a (*tons/in.*^{2})
100	4·1
110	3·7
Diff. = 10	Diff. = ·4
" = 8	" $= \frac{\cdot 4}{10} \times 8 = \cdot 32.$

$\therefore$ For $l/r = 108$, $F_a = (4 \cdot 1 - \cdot 32) = 3 \cdot 78$ tons/in.^{2}.

Sectional area of column $= \frac{\pi \times D^2}{4} = \frac{\pi \times 4^2}{4} = 12 \cdot 57$ in.^{2}.

$\therefore$ Safe axial load $= (12 \cdot 57 \times 3 \cdot 78) = 47 \cdot 5$ tons.

(6) *Timber beams. 9-ft. bay :*

Assuming 160 lb. per sq. foot total load :

Load carried by one beam = (160 × 9) = 1440 lb.

$$\therefore \frac{1440 \times 9 \times 12}{8} = \frac{f \times 2 \times 7 \times 7}{6}$$

$$f = \frac{1440 \times 9 \times 12 \times 6}{8 \times 2 \times 7 \times 7} \text{ lb./in.}^2 = 1190 \text{ lb./in.}^2.$$

Timber beams. 12-ft. bay :

Load carried by one beam $= (160 \times 12) = 1920$ lb.

$$\therefore \frac{1920 \times 12 \times 12}{8} = \frac{f \times 3 \times 8 \times 8}{6}$$

$$f = \frac{1920 \times 12 \times 12 \times 6}{8 \times 3 \times 8 \times 8} \text{ lb./in.}^2 = 1080 \text{ lb./in.}^2.$$

Steel beam :

$$\text{Load carried} = \left(\frac{21}{2} \times 16 \times 160\right) \text{ lb.} = 26880 \text{ lb.} = 12 \text{ tons.}$$

$$M = fZ. \quad \therefore f = \frac{M}{Z}.$$

$$M = \frac{12 \times 16 \times 12}{8} \text{ tons ins.} = 288 \text{ tons ins.}$$

$$\therefore f = \frac{288}{29{\cdot}25} = 9{\cdot}85 \text{ tons/in.}^2.$$

The given floor load is therefore safe.

(7) Volume of concrete in pier $= (3 \times 2 \times 2) + (3 \times 3 \times 1) = 21$ cu. ft.
Weight $= (21 \times 130)$ lb. $= 2730$ lb. $= 1{\cdot}22$ tons.

$$\text{Direct stress on base 'ZZ'} = \frac{\text{Total load}}{\text{Area}}$$

$$= \frac{1 + 2 + 1{\cdot}22}{9} \text{ tons/sq. ft.} = \frac{4{\cdot}22}{9} \text{ tons/sq. ft.}$$

$$= {\cdot}47 \text{ tons/sq. ft.}$$

$$\text{Bending stress} = \frac{M}{Z}.$$

B.M. $= [(2 \times 6) - (1 \times 6)]$ tons ins. $= 6$ tons ins. $= {\cdot}5$ tons ft.

$$Z = \frac{bd^2}{6} = \frac{3 \times 3^2}{6} \text{ ft.}^3 = 4{\cdot}5 \text{ ft.}^3.$$

$$\therefore \text{Bending stress} = \frac{{\cdot}5}{4{\cdot}5} \text{ tons/sq. ft.} = {\cdot}11 \text{ tons/sq. ft.}$$

Max. compressive stress $= ({\cdot}47 + {\cdot}11) = {\cdot}58$ tons/sq. ft.
Min. „ „ $= ({\cdot}47 - {\cdot}11) = {\cdot}36$ „ „

APPENDIX

BRITISH STANDARDS

THE following list of B.S. has been extracted, by kind permission of the British Standards Institution, from the sectional lists of British Standards dealing with 'Building,' etc. Copies of the B.S. may be obtained from the British Standards Institution, Sales Branch, 2, Park Street, London, W.1.

'*Add.*' signifies that an Amendment is issued with the particular standard.

B.S. Materials for Bridges and Buildings, etc.

§4: 1932 **Channels and Beams for Structural Purposes,** Dimensions and Properties of. *Add.* April, 1934. (Partly superseding No. 6: 1924.)

§4A: 1934 **Equal Angles, Unequal Angles and Tee Bars for Structural Purposes,** Dimensions and Properties of. (Partly superseding No. 6: 1924.)

6: 1924 (Extract from.) **Bulb Angles and Bulb Plates for Structural Purposes,** Dimensions and Properties of. (See Nos. 4 and 4A.)
(N.B.—Approximate Formulae for all Sections are included in the Extract from No. 6: 1924.)

§15: 1948 **Structural Steel.**

153: — **Girder Bridges.**
153-Parts 1 and 2—1933.
1—Materials. *Add.* March, 1941.
2—Workmanship.
153-Parts 3A—1954. Loads.
3B—1937. Stresses. *Add.* Oct., 1938, [Dec., 1954.
153-Parts 4 and 5—1937.
4—Details of Construction.
5—Erection.

§405: 1945 **Expanded Metal (Steel) for General Purposes.**

§449: 1948 The use of **Structural Steel in Building.**

§548: 1934 **High Tensile Structural Steel for Bridges,** etc., and General Building Construction. *Add.* May, 1936, Feb., 1938, and June, 1942.

§648: 1949 Schedule of Weights for Building Materials.

§785: 1938 **Rolled Steel Bars and Hard Drawn Steel Wire for Concrete Reinforcement.** *Add.* Jan., 1952.

B.S. Cement, Lime and Plasters

§12: 1947 Portland Cement (ordinary and rapid-hardening). *Add.* May, 1948, Jan., 1950, Aug., 1950, Jan., 1952, Feb., 1952, Aug., 1952.

§146: 1947 Portland-blastfurnace cement, details of. *Add.* May, 1948, Jan., 1950, Aug., 1950, May, 1952, Aug., 1952.

§890: 1940 Building Limes.

§915: 1947 High Alumina Cement. *Add.* Jan., 1950, Aug., 1950, Aug., 1952.

§1014: 1942 Pigments for Colouring Cement Magnesium Oxy-chloride and Concrete.

§1191: 1955 Gypsum building plasters.

1370: 1947 Low heat Portland cement. *Add.* Dec., 1948, Jan., 1950, Aug., 1950, Aug., 1952.

PD 572 Specification for a typical vibration machine for compacting mortar cubes for testing cement. Specification and four drawings.

1881: 1952 Methods of testing Concrete.

B.S. Aggregates

812: 1951 Sampling and testing of mineral aggregates, sands and fillers, methods for the. *Add.* May, 1952, Feb., 1953 and April, 1954.

§877: 1939 Foamed blastfurnace slag for concrete aggregate. *Add.* April, 1947.

§882, 1201: 1954 Concrete aggregates from natural sources. *Add.* Jan., 1956, July, 1957.

§1198, 1199, 1200 : 1955 Building sands from natural sources.

§1047 : 1952 Air-cooled blastfurnace slag coarse aggregates.

§1165 : 1957 Clinker aggregate for plain and precast concrete.

B.S. **Timber**

373: 1957 Testing small clear specimens of timber, methods of.

§565: 1949 Glossary of Terms Applicable to Timber, Plywood and joinery.

§881 & 589: 1955 Nomenclature of commercial timbers (including sources of supply).

§1186: Part 1: 1952 Quality of timber in joinery.

§1860: 1952 Structural softwood. Measurement of characteristics affecting strength. *Add.* July, 1952.

Codes of Practice

CP. 3: — Chapter V: 1952. Loading.

CP. 101. 1948 Foundations and substructures for houses, flats and schools of not more than two storeys.

CP. 111: 1948 Structural recommendations for loadbearing walls.

CP. 112: 1952 The structural use of timber in buildings.

CP. 113: 1948 The structural use of steel in buildings.

CP. 114: 1957 The structural use of normal reinforced concrete in buildings

§ A summary of this standard is included in Handbook No. 3.

APPENDIX II

FABRICATION OF STEELWORK

In various parts of the book we have referred to steel frames, built-up steel girders, etc. The reader may be interested to learn how such steelwork is fabricated.

Fig. 418 shows a flange plate being riveted on to a girder by a '*Hydraulic Riveter.*' In this riveter water pressure is utilised to force a ' die ' on to the soft rivet shank while the other end of the rivet is held firmly by the stationary die.

Site riveting is carried out by a pistol-shaped compressed-air machine known as the '*Pom-Pom*' or '*Pneumatic Hammer*' (Fig. 419). The head is formed in this case by a rapid succession of blows.

Figs. 420 and 421 illustrate the joining of units together by the metal-arc welding process. The metal required for deposition at the weld forms part of an electrical circuit. It is in the form of rods known as '*electrodes.*' In the photographs the welding operator is holding the electrode a small distance away from the joint being welded. An electric arc completes the circuit and the electrode metal is gradually being deposited to form the necessary connection.

FIG. 418.—HYDRAULIC RIVETING.
(Rivet head about to be formed.)

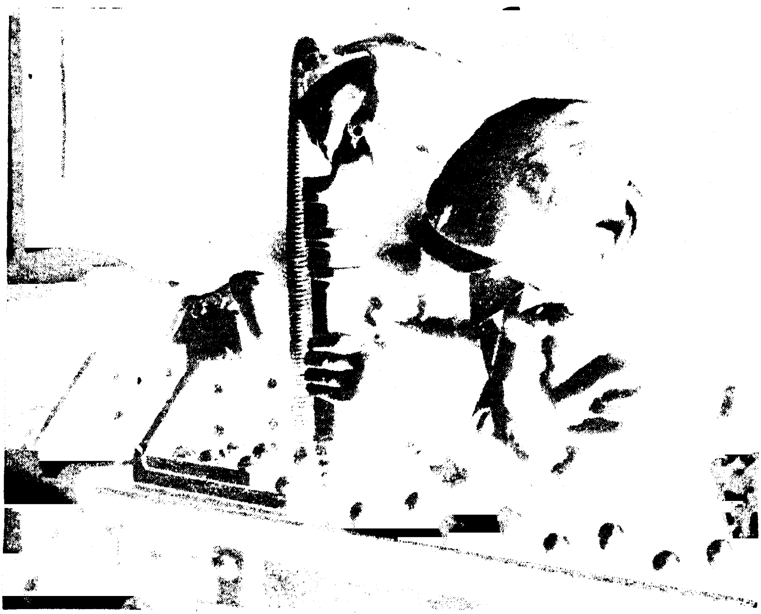


FIG. 419.—PNEUMATIC HAMMER RIVETING.

Figs. 418 and 419 reproduced by permission and courtesy of Messrs. Dawnays Ltd.

FIG 420.

FIG. 421.

Figs 420 and 421 reproduced by permission and courtesy of Messrs. The Quasi-Arc Company, Limited.

Answers to Chapter Exercises

Exercises (1) (*page* 14)

(1) 52 lb. 32° (nearly) to vertical.
(2) 100 lb. 53° (nearly).
(3) Vertical load = 500 lb.
(4) 7·39 cwts., bisecting angle between rope directions.
(5) (i) 20 cwts. 37° (nearly) to horizontal.
(ii) 174 lb. 13° ,, ,, ,,
(iii) 908 ,, $82\frac{1}{2}^\circ$,, ,, ,,
All the above resultants act downwards towards the right.
(6) 2126 lb. $13\frac{1}{2}^\circ$ (nearly) to vertical.
(8) $2\frac{1}{2}$ cwts. vertically downwards.
(9) 2000 lb. (nearly).
(10) 2·05 tons.
(11) 5910 lb. at 24° (nearly) to the vertical. Resultant cuts base at 3·61 ft. from back of wall.

Exercises (2) (*page* 28)

(1) (i) 448 lb. acting upwards towards the right.
366 ,, ,, downwards ,, ,, ,,
(ii) 8·66 cwts., vertically downwards.
5·00 ,, horizontally towards right.
(iii) 2 tons, upwards towards left.
2·83 ,, vertically downwards.
(2) Forward pull = 103·9 lb.
(3) 154·5 lb.
(4) 44·64 ,,
(5) (i) H = 4 cwts. (to right). V = 6·93 cwts. (upwards).
(ii) H = 500 lb. ,, ,, V = 866 lb. (downwards).
(iii) H = 1·532 tons (to left). V = 1·286 tons (downwards).
(6) H = 2·134 tons (to right).
V = 1·77 ,, (downwards).
(7) H = 646·4 lb. (to right).
V = 1519·6 lb. (downwards).
Resultant = 1651 lb. making an angle of 67° (nearly) with the horizontal.
(8) Horizontal sliding tendency = 3420 lb.
$\therefore$ frictional resistance should be 5130 lb.
(9) 60°.
(10) 3000 lb.

Exercises (3) (*page* 43)

(1) Left-hand rafter. 500 lb. Strut.
Right-hand ,, 866 ,, ,,
(2) 433 lb.

(3) Pull in tie = 4 cwts.
Thrust in jib = 7 cwts.
Jib is in compression.

(4) Pull in tie = 4 cwts.
Thrust in jib = 12 cwts.

(5) (i) $63\frac{1}{2}°$ with horizontal.
(ii) Reaction at upper hinge = 20 lb.
,, ,, lower ,, = 44·72 lb.

(6) Left reaction = 462 lb. acting downwards towards the left at 30° to the horizontal.
Right reaction = 462 lb. acting vertically downwards.

(7) Tension in rope = 60 lb.
Reaction at hinge = 60 lb. acting upwards towards the left at 30° to the horizontal.

(8) Force polygon closes. By calculation method $\Sigma H = 0$ and $\Sigma V = 0$.

(9) X = 256 lb. Y = 1079 lb.
Both members are struts.

(10) Member DE is a strut. Force = 9 tons.
,, EA ,, tie. ,, = 1·414 tons.

Exercises (4) (*page* 66)

(1) 200 lb. ft. A.C.W.
17·32 cwts. ft. C.W.
2·4 tons ft. A.C.W.

(2) (*a*) 14·14 lb.
(*b*) 10 lb.

(3) 40 lb. ft. C.W.

(4) 1″.

(5) 30 lb.

(6) Reaction = 1212·4 lb.

(7) (i) Left end = 8 tons. Right end = 10 tons.
(ii) ,, ,, = 2680 lb. ,, ,, = 2520 lb.
(iii) ,, ,, = $28\frac{1}{3}$ cwts. ,, ,, = $31\frac{2}{3}$ cwts.

(8) Left end = 68·25 cwts. Right end = 54·75 cwts.

(9) ,, ,, = 4050 lb. ,, ,, = 4250 lb.

(10) 3·75 ft.

(11) R_L = 12 cwts. R_R = 15 cwts.
Moment = 54 cwts. ft. Moments agree in magnitude, one is C.W. and the other A.C.W.

(12) The magnitude of the resultant = 2·88 cwts.

(13) A = $2\frac{3}{8}$ tons.
B = $1\frac{1}{4}$,,
C = $1\frac{1}{2}$,,
D = $\frac{7}{8}$,,
Total = 6 tons.

(14) $P_{max.}$ = 76·25 lb.

Exercises (5) (*page* 87)

(1) (i) 9 lb. vertically downwards at 12″ to right of 3-lb. force.
(ii) 17 tons „ „ „ $12\frac{4}{17}$ ft. to right of 2 ton-force.

(2) (i) 2 cwts. horizontally towards the right at 12 ft. vertically above the 2-cwt. force.
(ii) 90 lb. acting, parallel to system, towards the right in line with the 80-lb. force.

(3) Resultant = 12 tons in line with the 2-ton load.
Reactions: Left end = 7·2 tons. Right end = 4·8 tons.

(4) 2200 lb., acting parallel to system towards the left at 11 ft. from apex of truss.

(5) Resultant load acts at 6 ft. from 60 ton-load. Overall length = 16 ft.

(6) 10·2 ft. from right end.

(7) Force in couple = 12 lb.
Arm of „ = $9\frac{1}{3}$ ins.
Moment of „ = 112 lb. ins. C.W.

(8) C = T = 3 tons.

(10) Pull in string = 7·5 lb.
Hinge reaction: Vertical component = 3·25 lb.
Horizontal „ = 6·495 lb.
Magnitude = 7·26 lb. acting upwards towards the right at $26\frac{1}{2}$° (nearly) to the horizontal.

(11) W = 12 lb.
x = $4\frac{2}{3}$ ins.

(12) R_A = 2000 lb.
R_B = 10440 lb. acting at 73° 18′ to the horizontal, upwards towards the left.
[Vertical component = 10,000 lb.
Horizontal „ = 3,000 „]

Exercises (6) (*page* 112)

(1) 9″ from '4-lb.' mass.

(2) (*a*) 8·17″. (*b*) 2″.

(3) 1·183″ from back of each leg.

(4) (i) 1·685″ below top of section on vertical axis of symmetry.
(ii) ·827″ from left edge of angle section.
1·327″ „ bottom „ „ „
(iii) 1·018″ from left edge of section, on horizontal axis of symmetry.
(iv) 2·325″ from top of section, on vertical axis of symmetry.

(5) 5″ above bottom of section on vertical axis of symmetry.

(6) 3·25 ft. from back of wall.

(7) 1·34 ft. from left edge of section.
1·5 „ „ bottom „ „ „

(8) (i) R_L = 1440 lb.
R_R = 2880 „
(ii) R_L = 3520 „
R_R = 2240 „

(9) 2 ft. from line joining centres of the two left-hand columns.
6′ 8″ above horizontal line through the ' 10 tons ' column.

(10) 6·54″ above base, on vertical axis of symmetry.

(11) 12″ from left edge of section.
11·8″ from bottom edge of section.
The C.G. is at mid-thickness of the casting.

(12) 3·22 ft. from left edge of base.

Exercises (7) (*page* 128)

(1) Resultant = 185 lb. acting at 2·03 ft. from the top of mast at an angle of 22° (downwards towards the left) with the horizontal.

(2) 2·8 ft.

(3) 2·17″ from left edge of section and 1·17″ from bottom edge.

(5) 2·11 ft. from back of wall.

(7) (i) R_L = 6·5 tons. R_R = 7·5 tons.
(ii) R_L = 7 cwts. R_R = 6 cwts.

(8) R_L = 11·5 tons. R_R = 12·5 tons.

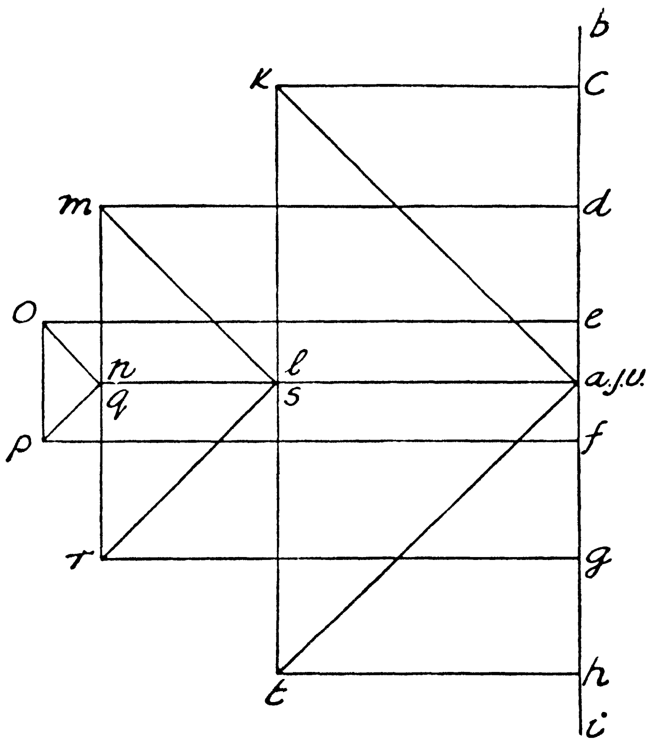


FIG. 422.

(9) Free-end reaction (vertical) = 1559 lb.
Fixed-end reaction = 2381 lb. acting at 41° to the horizontal.

(11) R_L = 5040 lb.
R_R = 7920 „

Exercises (8) (*page* 155)

(1) See Fig. 422.

Member.	Force.	Type.
JK & TU	28·28 tons	Tie
LM & RS	16·97 „	Tie
NO & QP	5·66 „	Tie
AJ & AU	0·00 „	—
CK & HT	20·00 „	Strut
AL & AS	20·00 „	Tie
DM & GR	32·00 „	Strut
AN & AQ	32·00 „	Tie
EO & FP	36·00 „	Strut
BJ & IU	24·00 „	Strut
KL & TS	20·00 „	Strut
MN & RQ	12·00 „	Strut
OP	8·00 „	Strut

(2) See Fig. 423.
6558 lb. (tie) in left member (AH).
8030 „ (strut) in right member (FK).

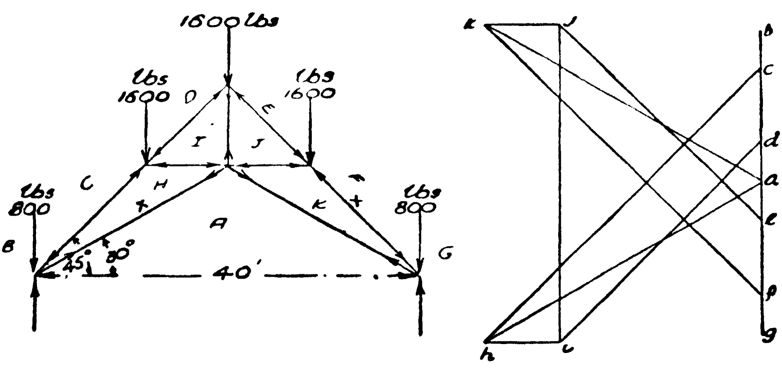


FIG. 423.

(3) See Fig. 424.
AF = 10·93 tons (tie).
IJ = 4·00 „ (strut).
JK = 2·83 „ (tie).

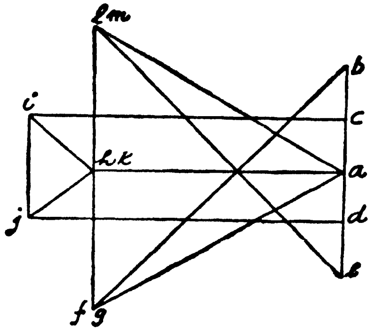
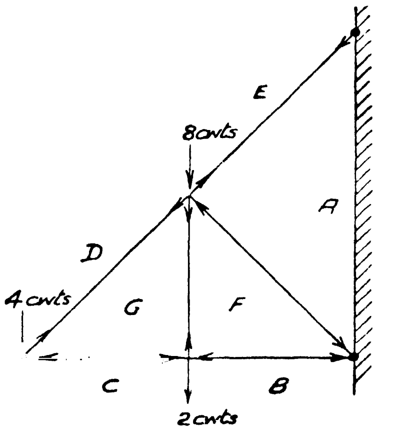
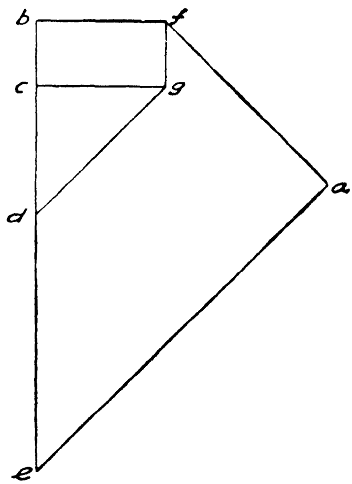


FIG. 424.

(4) See Figs. 425 and 426.





MEMBER	FORCE	TYPE
BF	4·00 cwts	STRUT
CG	4·00 "	STRUT
DG	5·66 "	TIE
EA	12·73 "	TIE
AF	7·07 "	STRUT
FG	2·00 "	TIE

FIG. 425.

(5) $R_L = 17\frac{1}{8}$ cwts.
$R_R = 20\frac{7}{8}$,,
Force in member = 1·9 cwts. (tie).
See Fig. 427.

(6) See Fig. 428.

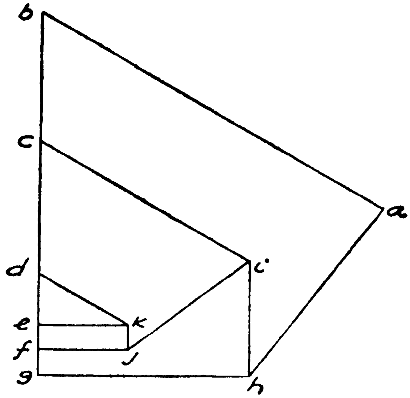


FIG 426

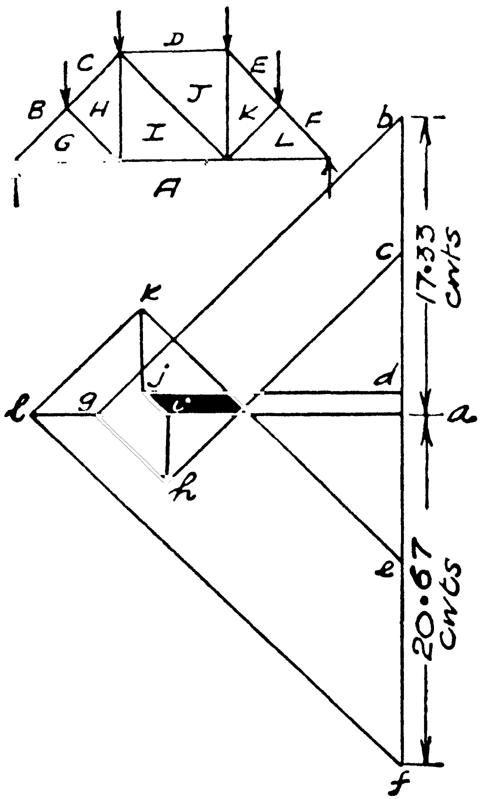


FIG 427.

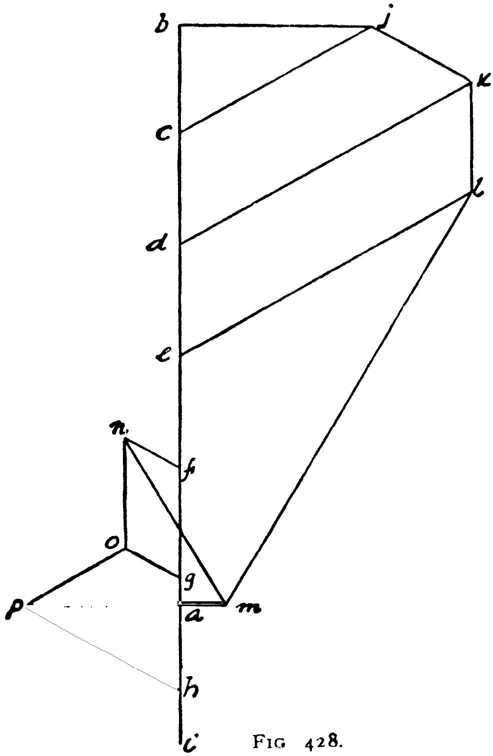


FIG 428.

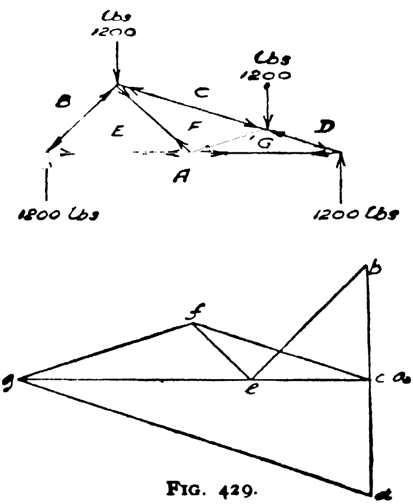


FIG. 429.

(7) Force in member = 1200 lb. (tie).
See Fig. 429.

(8) See Fig. 430.

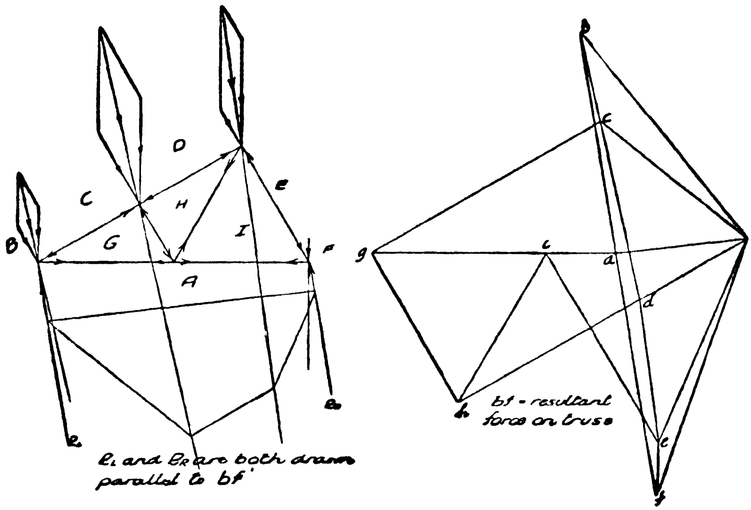


FIG 430.

(9) Dead loads: Full joint load = 960 lb ; ½ joint load (at eaves) = 480 lb. Wind loads at apex and eaves = 450 lb. Central joint = 900 lb.

(10) Forces in inclined members from left.
13·86T (strut), 5·775T (tie), 5 775T (strut), 6 93T (strut), 6·93T (tie), 11·55T (strut). See Fig 431.

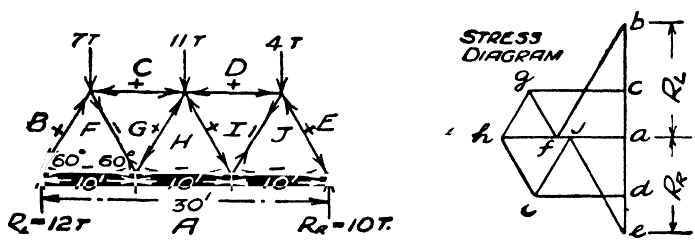


FIG 431.

Exercises (9) (*page* 191)

(1) 8 tons/in.².
1″ thickness.

(2) ⅞″.

(3) 7′ 6″ square.

(4) $13\frac{1}{2}''$ square.
(5) 10.
(6) 1,200,000 lb./in.^{2}.
(7) ·06 ins.
(8) (i) 9000 lb./in.^{2}.
(ii) 48000 lb.
(9) (i) 13,000 tons/in.^{2}.
(ii) 32 tons/in.^{2}.
(iii) $\frac{1}{8}''$.
(10) $\frac{5}{8}''$ dia.
(11) Concrete load = 1600 lb.
Timber ,, = 2160 ,,
(12) 11·82 tons.
(13) 17·72 ,,
[Rivet strength = 18·05 tons.]
(14) Value of one bolt = 3·61 tons.
Max. safe reaction load = 6 × 3·61 = 21·66 tons

Exercises (10) (*page* 217)

(1) $B.M._{max}$ = 6 cwts. ft. (—).
S.F. ,, = 2 cwts.
(2) (i) $B.M._{max.}$ = 25 c.f. (—).
(ii) S.F. ,, = 6 cwts.
(iii) 9 c.f. (—).
(iv) 6 cwts.
(3) 200 lb. per sq. foot.
Max. S.F. = 1800 lb.
(4) $B.M._{max.}$ = 400 lb. ft.
S.F. ,, = ± 100 lb.
(5) B.M.s at load points from left:
27·5 tons ft., 45 tons ft., 32·5 tons ft,
R_L = 5·5 tons. R_R = 6·5 tons.
(6) R_L = 7·4 tons. R_R = 7·6 tons.
B.M.s at load points from left:
29·6 tons ft., 51·2 tons ft., 56·8 tons ft., 30·4 tons ft.
B.M. at given section = 55·4 tons ft.
S.F. ,, ,, ,, = 1·4 tons.
(7) $B.M._{max.}$ = 36 c.f.
S.F. ,, = ± 12 cwts.
B.M. at given section = 32 c.f.
S.F. ,, ,, ,, = 4 cwts.
(8) Load = 2 cwts. per foot run.
$B.M._{max.}$ = 25 c.f.
(9) B.M. ,, = 800 lb. ft. in timber beam.
S.F. ,, = ± 400 lb. ,, ,, ,,
B.M. ,, = 25600 lb. ft. in steel beam.
S.F. ,, = ± 6400 lb. ,, ,, ,,

(10) B.M. „ = 4 tons ft.
(11) (i) 8 tons.
(ii) 4·5 tons.
(12) B.M. = 5400 lb. ft.
S.F. = 450 lb.
$B.M._{max.}$ = 5625 lb. ft.
S.F. „ = ± 2250 lb.
(13) B.M. (mid-height) = 24300 lb. ft.
S.F. „ „ = 1620 lb.
B.M. (base) = 97200 lb. ft.
S.F. „ = 3240 lb.
(14) 4 ft. and 6 ft.
(15) 4 tons.
(i) $5\frac{7}{9}$ tons.
(ii) 4 tons.

Exercises (11) (*page* 235)

(1) B.M. at 6 ft. = 11 c.f.
S.F. „ 6 „ = − ·5 cwts.
B.M.'s at load points from left:
0, − 10 c.f. (at reaction), 12 c.f., 10 c.f., − 8 c.f. (at reaction), 0.
Reactions: Left end = 7·5 cwts., right end = 6·5 cwts.
(2) Reactions: Left end = 5250 lb., right end = 8750 lb.
B.M.'s: Left support = − 5000 lb. ft. Right support = − 20000 lb. ft.
$B.M._{max.}$ occurs at 8·125 ft. from left support (for central span).
B.M. „ = 8203 lb. ft. Absolute $B.M._{max.}$ = 20000 lb. ft.
(3) Reactions: Left end = $13\frac{7}{8}$ tons, right end = $22\frac{1}{8}$ tons.
B.M.'s: Left support = − 18 tons ft., right support = − 36 tons ft.
B.M. at point load = 45 tons ft.
B.M. diagrams for overhangs are parabolæ. The diagram for central span consists of straight lines.
(4) Reactions: Left end = $5\frac{1}{3}$ cwts., right end = $2\frac{2}{3}$ cwts.
$B.M._{max.}$ occurs at $4\frac{2}{3}$ ft. from left end.
B.M. „ = $17\frac{7}{9}$ c.f.
(5) Reactions: Left end = 7 cwts., Right end = 5 cwts.
$B.M._{max.}$ = 12·25 c.f. at 3·5 ft. from left end.
(6) Reactions: Left end = 10 cwts., Right end = 8 cwts.
(6 + 8) cwts. exceeds left-end reaction value.
(7) Reactions: Left end = 21 tons, Right end = 15 tons.
$B.M._{max.}$ = 76·5 tons ft. at 9 ft. from left end.
(8) Reactions. Left end = 34·5 tons, Right end = 15·5 tons.
A $B.M._{max.}$ occurs at '5 tons' load and equals 32·5 tons ft.
Absolute $B.M._{max.}$ = 37·5 tons ft. at left support.
(9) Reactions: Left end = 17 cwts., Right end = 18 cwts.
$B.M._{max.}$ = 30·25 c.f. at 5·5 ft. from left support.

(10) Reactions : Left end = 27·87 cwts., Right end = 22·13 cwts.
B.M. at left support = − 50 c.f.
" " right " = − 32 "
" " 6 cwts. load = − 10·67 c.f.
" " 8 " " = − 1·33 "
[B.M. diagram is wholly beneath base line.]

(11) Reactions : Left end = 9 cwts., Right end = 6 cwts.
$B.M._{max.}$ occurs at 4′ from left end = 18 c.f.

(12) 375 c.f. = 18·75 tons ft. Each reaction = 150 cwts.
B.M. at each support = − 125 c.f.

(13) Total wind load = 120 lb.
B.M. at base = 480 lb. ft.
$S.F._{max.}$ = 120 lb.
[Treat as cantilever with partial uniform load.]

(14) $B.M._{max.}$ occurs at 6·93 ft. from left end.
B.M. " = 55·44 c.f.

(15) B.M. value at top of mast = 0.
" " " '300 lb.' = 400 lb. ft.
" " " '400 lb.' = 1400 "
" " " base = 800 "
Max. B.M. = 1400 lb. ft.
(See Fig. 432.)

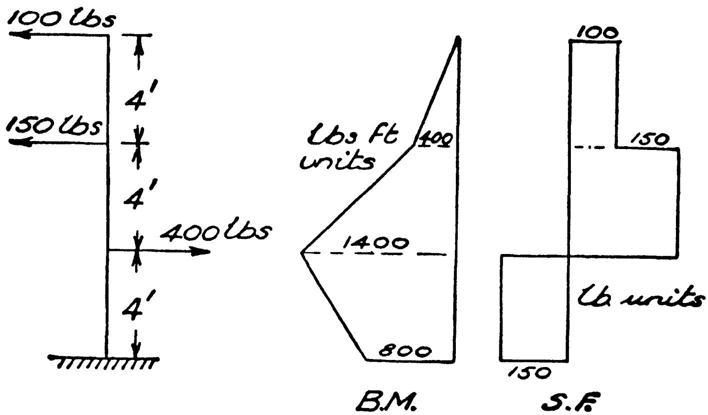


FIG. 432.

Exercises (12) (*page* 277)

(1) (i) Max. fibre stress = 10 cwts./in.^{2}.
(ii) 45 cwts.
(iii) 4 ins.

(2) 400 lb.

(3) 450 "

(4) 2″ × 7″.

(5) Reactions: Left end = 480 lb., Right end = 360 lb.
B.M.$_{max.}$ = 1040 lb. ft.
f = 1040 lb./in.^{2}.
(6) 229 lb. per sq. foot.
(7) B.M. available for end load = (M.R. − 52800 lb. ins.).
= 115200 − 52800 = 62400 lb. ins.
Load = 1300 lb.
(8) 15 tons.
7·5 tons central load.
(9) (i) Z_{XX} = 22·42 ins.^{3}.
(ii) I_{XX} = 705·6 ins.^{4}.
(iii) 16″.
(10) Reactions: Left end = 5·9 tons, Right end = 5·1 tons.
B.M.$_{max.}$ = 11·6 tons ft. at 3·93 ft. from left end.
Z = 17·4 ins.^{3}.
(11) Reactions: Left end = 5 tons, Right end = 5 tons.
B.M.$_{max.}$ occurs at '5 tons' load = 25·5 tons ft.
Z = 30·6 ins.^{3}.
(12) 7″ × 4″ × 16 lb. B.S.B. (Z required = 11·25 ins.^{3}).
(13) f = 5·97 tons/in.^{2}.
(14) B.M.$_{max.}$ is at '40 tons' load = 40 tons ft.
Necessary 'Z' for each beam = 13·3 ins.^{3}.
(15) B.M.$_{max.}$ = 1795·5 lb. ft.
b = 3″, d = 6″.
(16) For bay with 3″ × 7″ beams: 245 lb./sq. foot.
„ „ „ 3″ × 9″ „ 259 „ „
„ steel beam strength: 282 lb./sq. foot.
∴ safe floor load = 245 lb./sq. foot.
(17) Max. B.M. = 14040 lb. ins.
'Z' of section = 2·652 ins.^{3}.
f = 5295 lb./ins.^{2}.
(18) 'Z' of section = 1·43 ins.^{3}.
W = 572 lb.
(19) 'Z' of section = 44·93 ins.^{3}.
9·95 tons.
(20) 12 ins.

Exercises (13) (*page* 304)

(1) (i) 1″.
(ii) ·866″.
(iii) 1·84″.
(iv) 1·46″.
(2) (i) 72.
(ii) 60.
(3) F_a = 5·01 tons/in.^{2}.
Load = 110·5 tons.

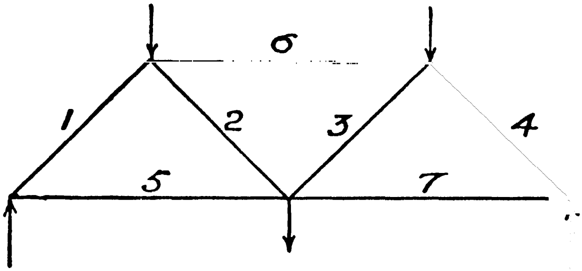
(4) 14 ft. (nearly).
(5) 12″ × 8″ × 65 lb. will carry 105 tons.
(6) Direct stress = 300 lb./in.^{2}.
Bending „ = 180 „ „
Max. comp. stress = 480 lb./in.^{2}.
Min. „ „ = 120 „ „
(7) Direct stress = 2 tons/in.^{2}.
Bending „ = 2 „ „
∴ total stress varies from 4 tons/in.^{2} to zero.
(8) Direct stress = 1·188 tons/sq. ft.
Bending „ = ·37 „ „
At 'A' total = 1·56 „ „
„ 'B' „ = ·818 „ „
(9) 3·57 ins.
(10) 10″ dia.
(i) ·0036 ins.
(ii) Z = 98·21 ins.^{3}.
W = 11·7 tons.
(11) Component at right angles to joint = 1879·4 lb.
Compressive stress at 'A' = 1645 lb./sq. ft.

Exercises (14) (*page* 328)

(1) A = 1600 lb. strut.
B = 800 „ „
C = 2078 „ tie.
(2) Reactions: Left end = 1450 lb., Right end = 1150 lb.
A = 1700 lb. strut.
B = 1270 „ tie.
C = 837 „ „
(3) See Fig. 433
(4) A = 1000 lb. strut.
(5) See Fig. 434.
(6) A = 2600 lb. tie.
B = 1300 „ „
C = 2800 „ strut.
(7) Left rafter = 200 lb.
Right „ = 346·4 lb.
(8) A = 3464 lb. strut.
B = 2000 „ tie.

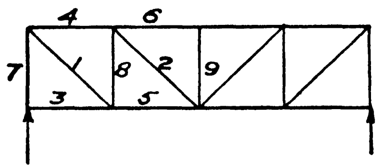
Exercises (15) (*page* 362)

(1) Wind load = 100 lb.
Overturning moment = 250 lb. ft.
(2) 2531·25 lb.



MEMBER	FORCE	TYPE
1	16·97 T	STRUT
2	2·83 T	TIE
3	5·66 T	TIE
4	14·14 T	STRUT
5	12·00 T	TIE
6	14·00 T	STRUT
7	10·00 T	TIE

FIG 433



MEMBER	FORCE	TYPE
1	10·61 T	TIE
2	3·54 T	TIE
3	0·00 T	—
4	7·50 T	STRUT
5	7·50 T	TIE
6	10·00 T	STRUT
7	9·50 T	STRUT
8	6·50 T	STRUT
9	4·00 T	STRUT

FIG. 434.

(3) (i) 2160 lb.
(ii) 8640 lb. ft.
Earth pressure at base = 360 lb per sq. foot.

(4) Water thrust = 2531·25 lb.
$x = 6{\cdot}24$ ft.

(5) Earth thrust = 792·6 lb.
Weight of wall = 3780 lb.
Resultant = 3860 lb. at 12° (nearly) to vertical.
Resultant cuts base at 2·185 ft. from back of wall.
Compression everywhere along base.

(6) (i) W = 750 lb.
Wind pressure = 15 lb. per sq. foot.
(ii) 8 ft.

(7) Earth thrust = 5400 lb.
Weight of wall = 12240 lb.
Resultant cuts base at 1·28 ft. from centre.
Max. ground pressure = 1·13 tons/sq. foot.
Resistance to sliding = 6120 lb.
Wall is stable, but resistance to sliding does not exceed the sliding tendency by a sufficient margin.

(8) (i) 2250 lb. per sq. foot.
(ii) 3000 " " " "
(iii) 4000 " " " "

(9) Weight of wall = 16380 lb.
C.G. from back of wall = 3·11 ft.
Eccentricity of resultant = ·49 ft.
V = 19330 lb.
Max. compressive stress = 3300 lb. per sq. foot.

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